New Advances in Quantum Information Science and Quantum Technology

Entangled States as Robust and Re-usable Carriers of Information

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Entangled states can be used as secure carriers of information

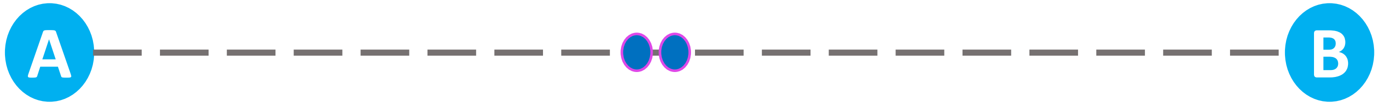
Conventionally in quantum information processes entanglement is used as a resource which is consumed at the end of a process and has to be renewed for a second round of use

- measurement-based quantum computation
- Teleportation
- quantum secret sharing
- Quantum Key Distribution
- Quantum State sharing

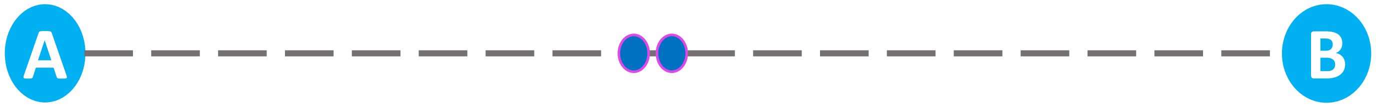
A

B

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



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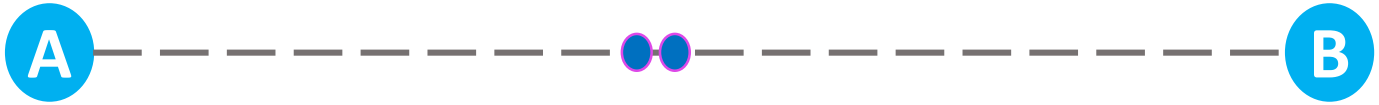

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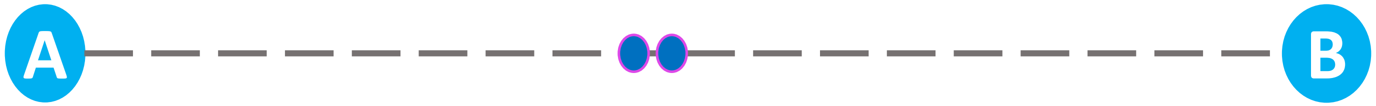


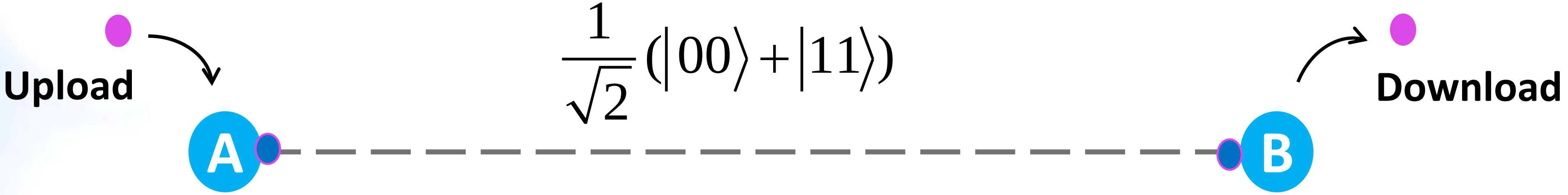
$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Using an entangled state between two remote points, as a reusable carrier of information

Y. S. Zhang, C. F. Li, and G. C. Guo; Phys. Rev. A, 64, 024302 (2001)

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



Alice goal is to send the quantum particle securely to Bob

Alice entangles (uploads) the state to the carrier and Bob disentangles (downloaded) the state from the carrier at the other end.

Using an entangled state between two remote points, as a reusable carrier of information

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

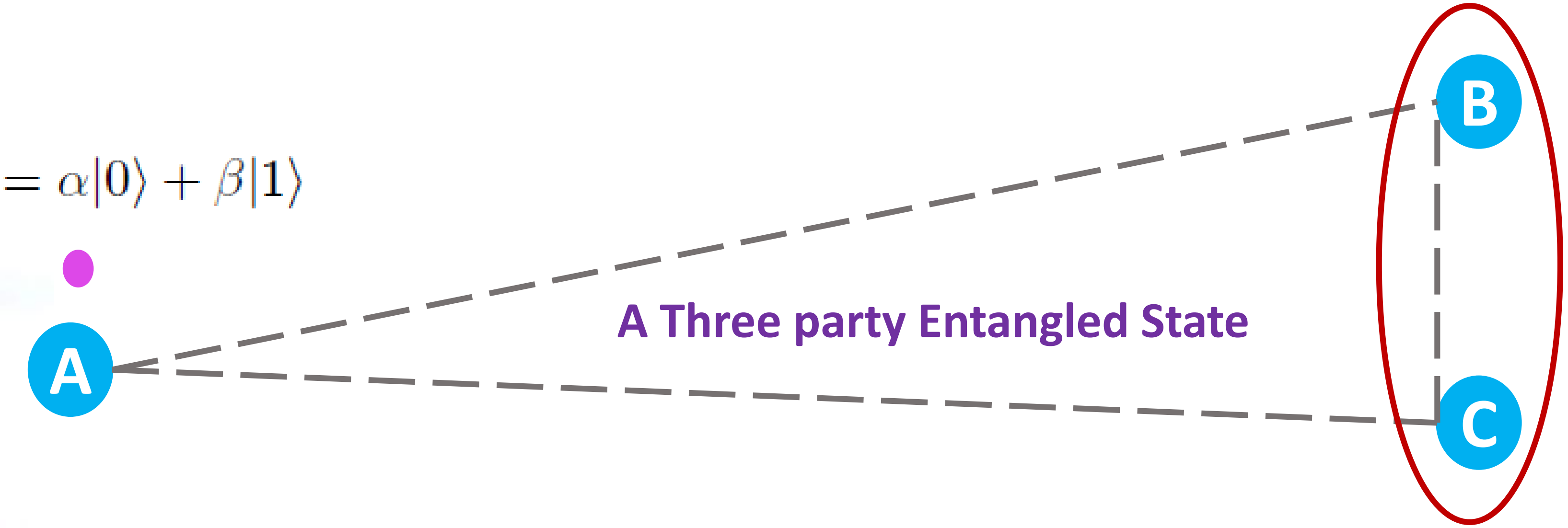
during transmission, the state is in a highly mixed state and is hidden from adversaries

At the end of each round, the carrier returns to its initial state and is ready for a second round of use.

Entangled States as Robust and Re-usable Carriers of Information

S. Bagherinezhad and V. Karimipour, Phys. Rev. A 67, 044302 (2003)

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$



The goal of Alice is to send a quantum state to Bob and Charlie so that they can retrieve it only if they collaborate with each other.

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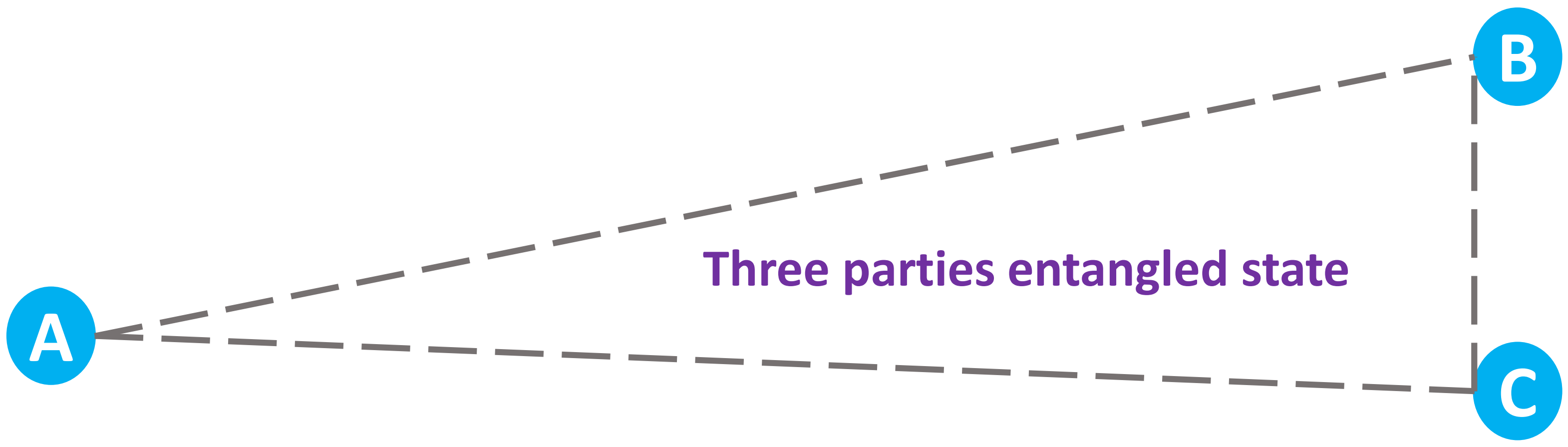

There are two types of carriers which alternate in even and odd rounds

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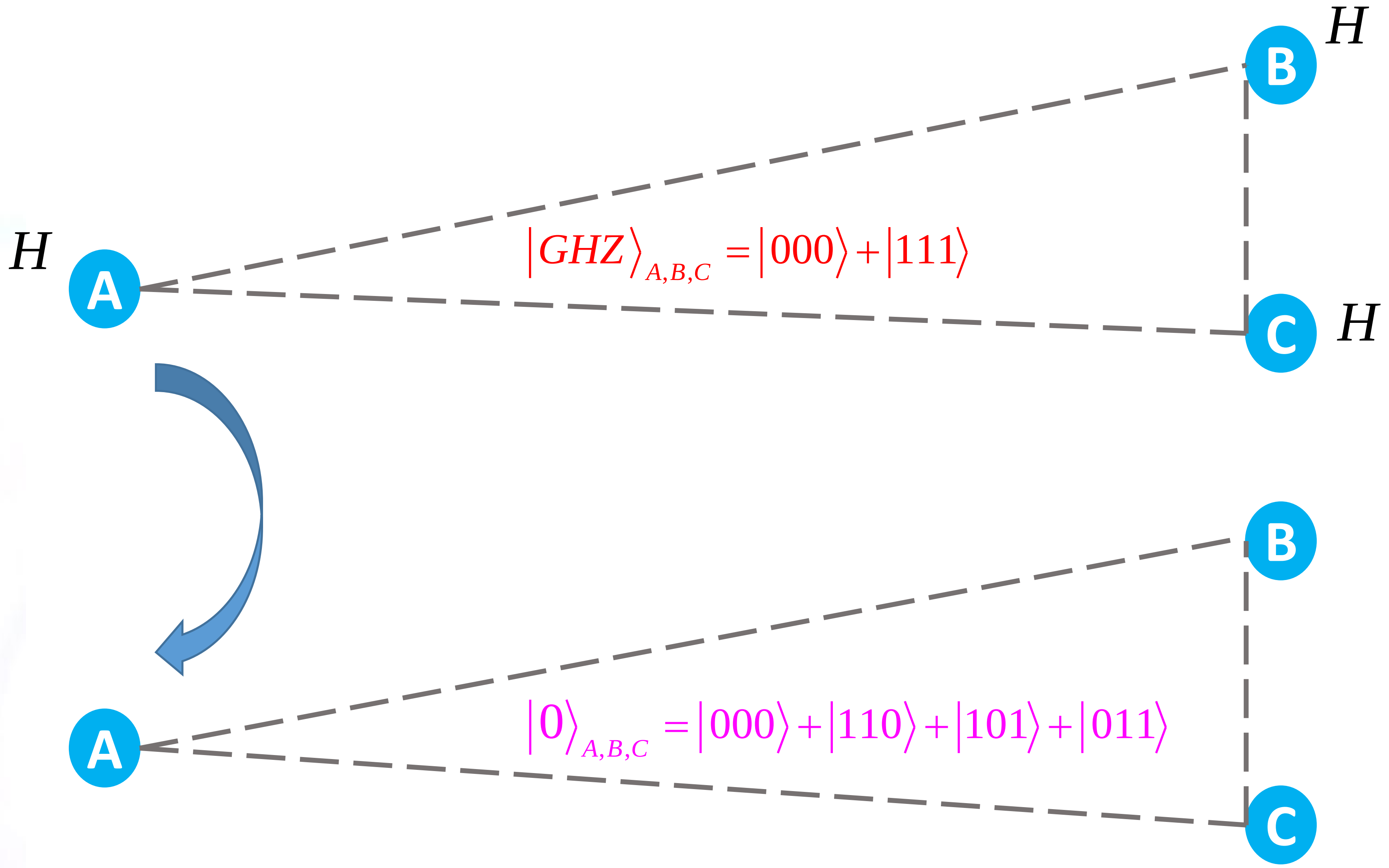
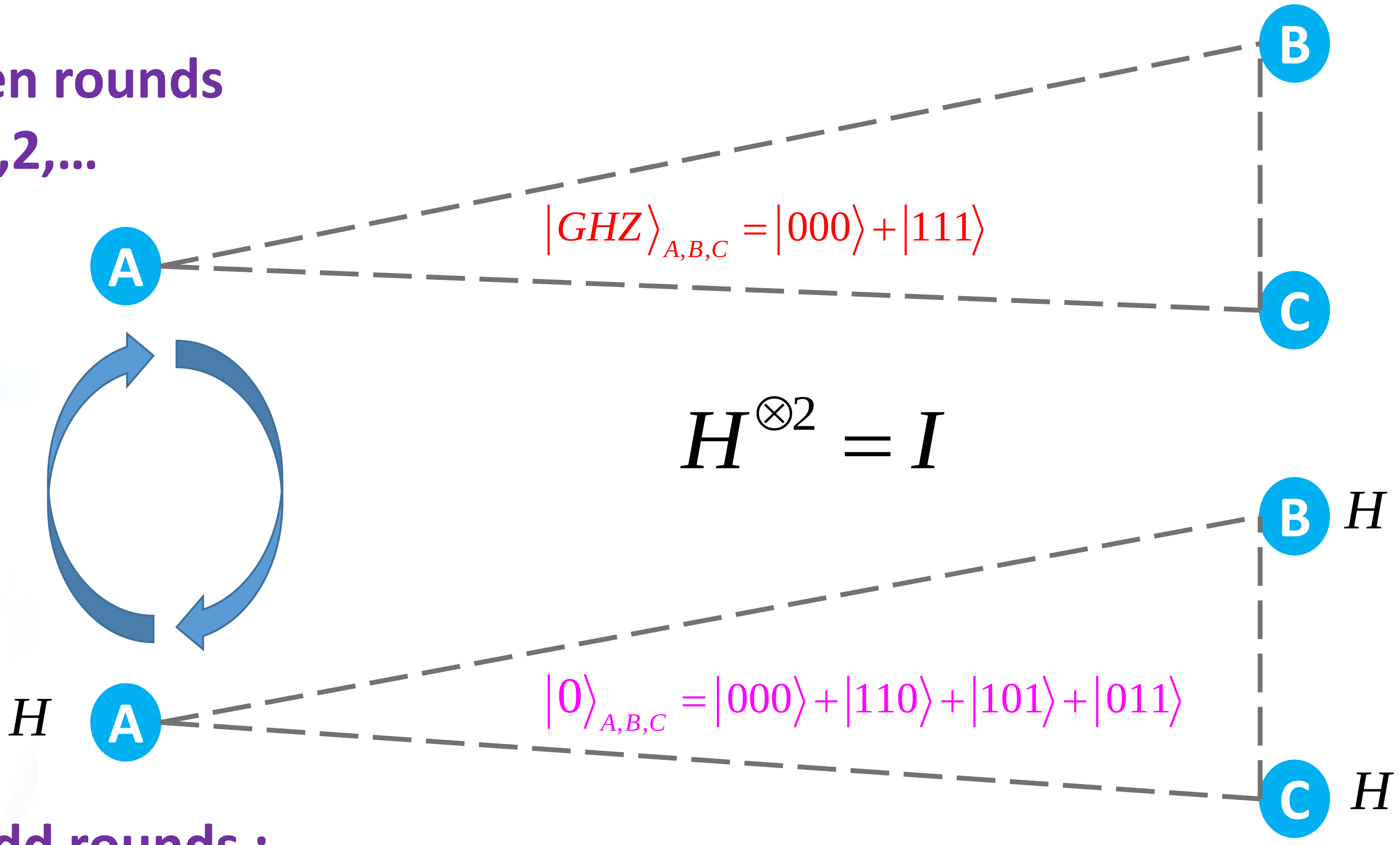


Even rounds : 0,2,...



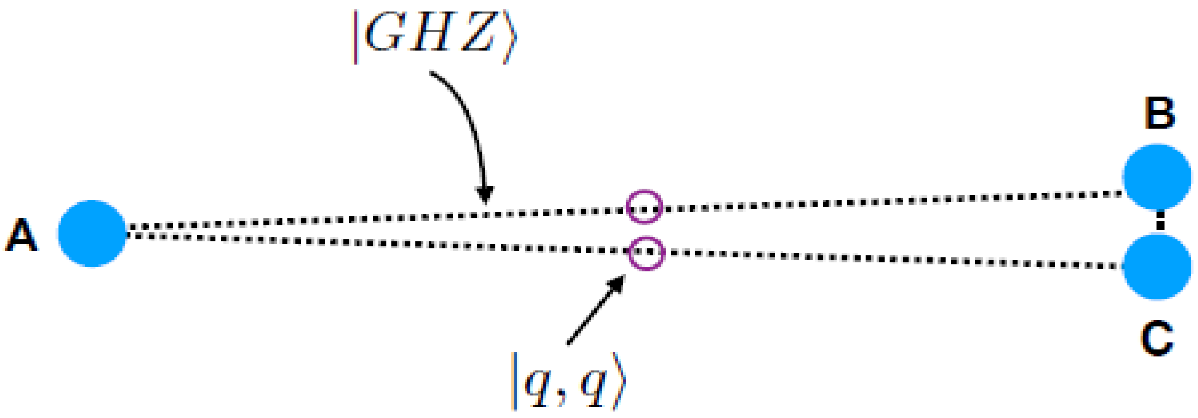
odd rounds : 1,3,...

Even rounds: 0,2,...

1) Alice encodes $|q\rangle \longrightarrow |q,q\rangle_{1,2}$

2) Then by appling $C_{A,1} \otimes C_{A,2}$ Entangles the $|q,q\rangle_{1,2}$ To the carrier :

$$|\Psi_{even}\rangle_{ABC,1,2} = |000\rangle|q,q\rangle + |111\rangle|\overline{q},\overline{q}\rangle$$



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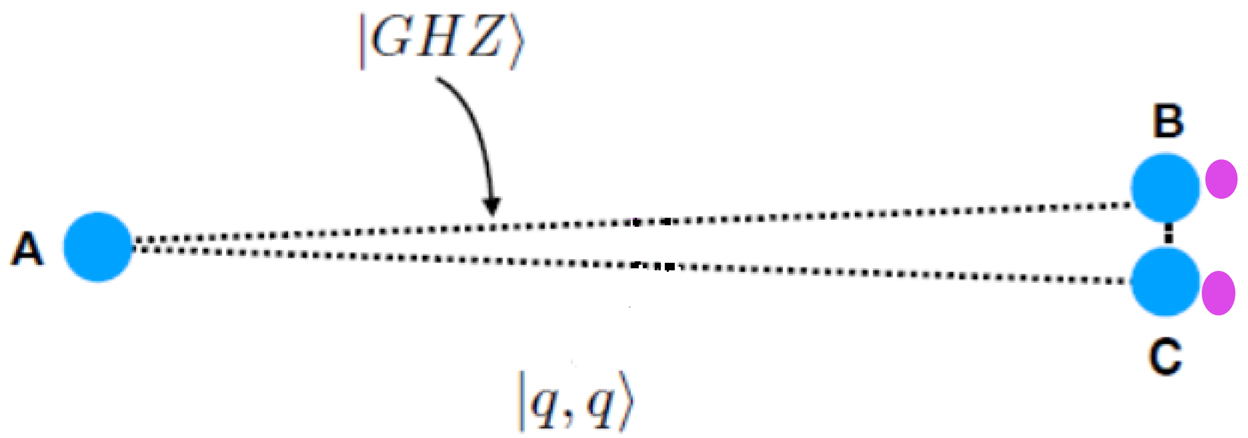
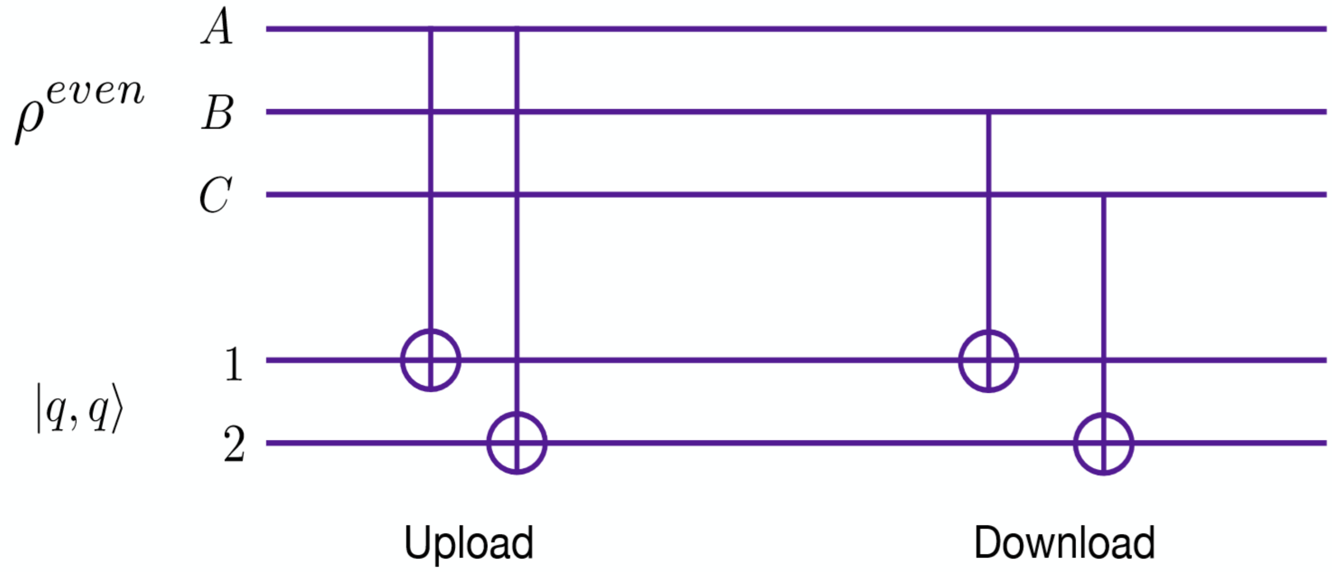
3) At the other end ,Bob and Charlie disentangle the state $|q,q\rangle_{1,2}$ By operations :

$$C_{B,1} \,\&\, C_{C,2}$$



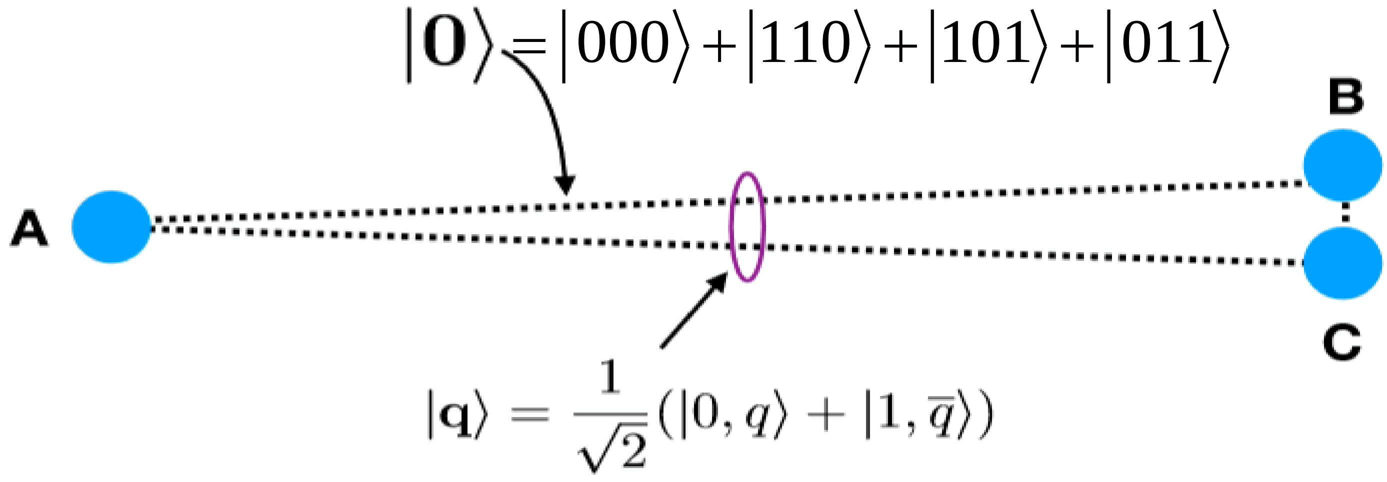


circuit form of the operations in even rounds

odd rounds: 1,3,…

1) Alice encodes $|q\rangle \longrightarrow |q\rangle_{1,2} = \frac{1}{\sqrt{2}}\left(|0,q\rangle + |1,\bar{q}\rangle\right)$

2) Then Alice by applying C_{A1} entangles this state to the carrier



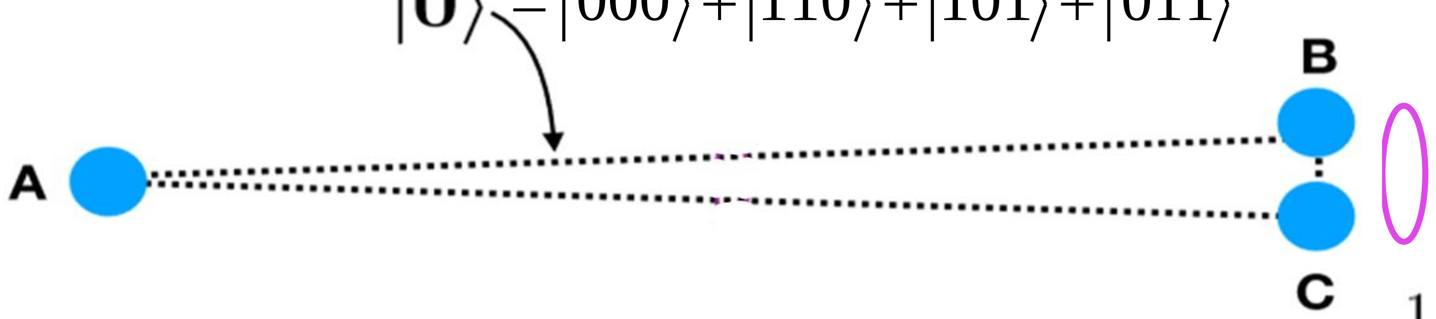
odd rounds: 1,3,…

1) Alice encodes $|q\rangle \longrightarrow |q\rangle_{1,2} = \frac{1}{\sqrt{2}}\left(|0,q\rangle + |1,\bar{q}\rangle\right)$

2) Then Alice by aplying C_{A1} entangles this state to the carrier

3) Bob and Charlie disentangle the state from the carrier by aplying $C_{B,1}C_{C,2}$

$$|\mathbf{O}\rangle = |000\rangle + |110\rangle + |101\rangle + |011\rangle$$



$$|\mathbf{q}\rangle = \frac{1}{\sqrt{2}}(|0,q\rangle + |1,\bar{q}\rangle)$$

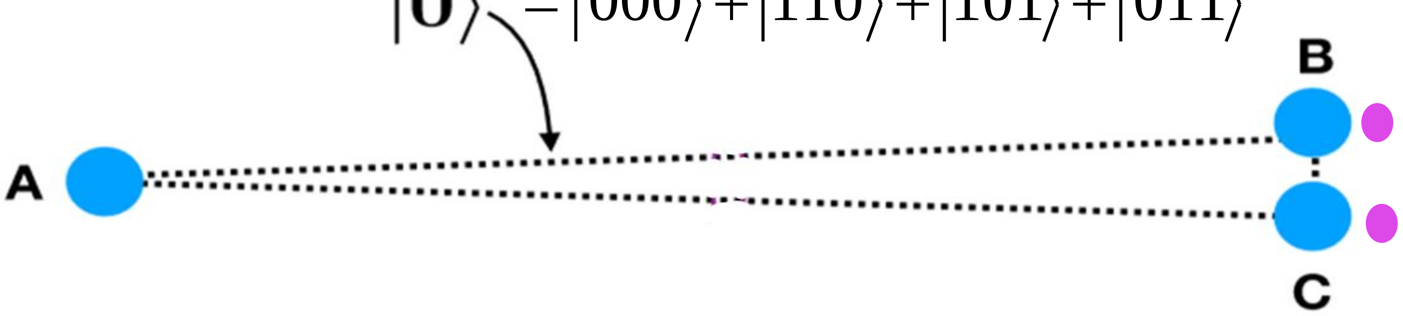
odd rounds: 1,3,…

1) Alice encodes $|q\rangle \longrightarrow |q\rangle_{1,2} = \frac{1}{\sqrt{2}}\left(|0,q\rangle + |1,\overline{q}\rangle\right)$

2) Then Alice by aplying C_{A1} entangles this state to the carrier

3) Bob and Charlie disentangle the state from the carrier by aplying $C_{B,1}C_{C,2}$

$$|\mathbf{0}\rangle = |000\rangle + |110\rangle + |101\rangle + |011\rangle$$



$$C_{1,2}$$

$$|\mathbf{q}\rangle = \frac{1}{\sqrt{2}}(|0,q\rangle + |1,\overline{q}\rangle)$$

The basic question :

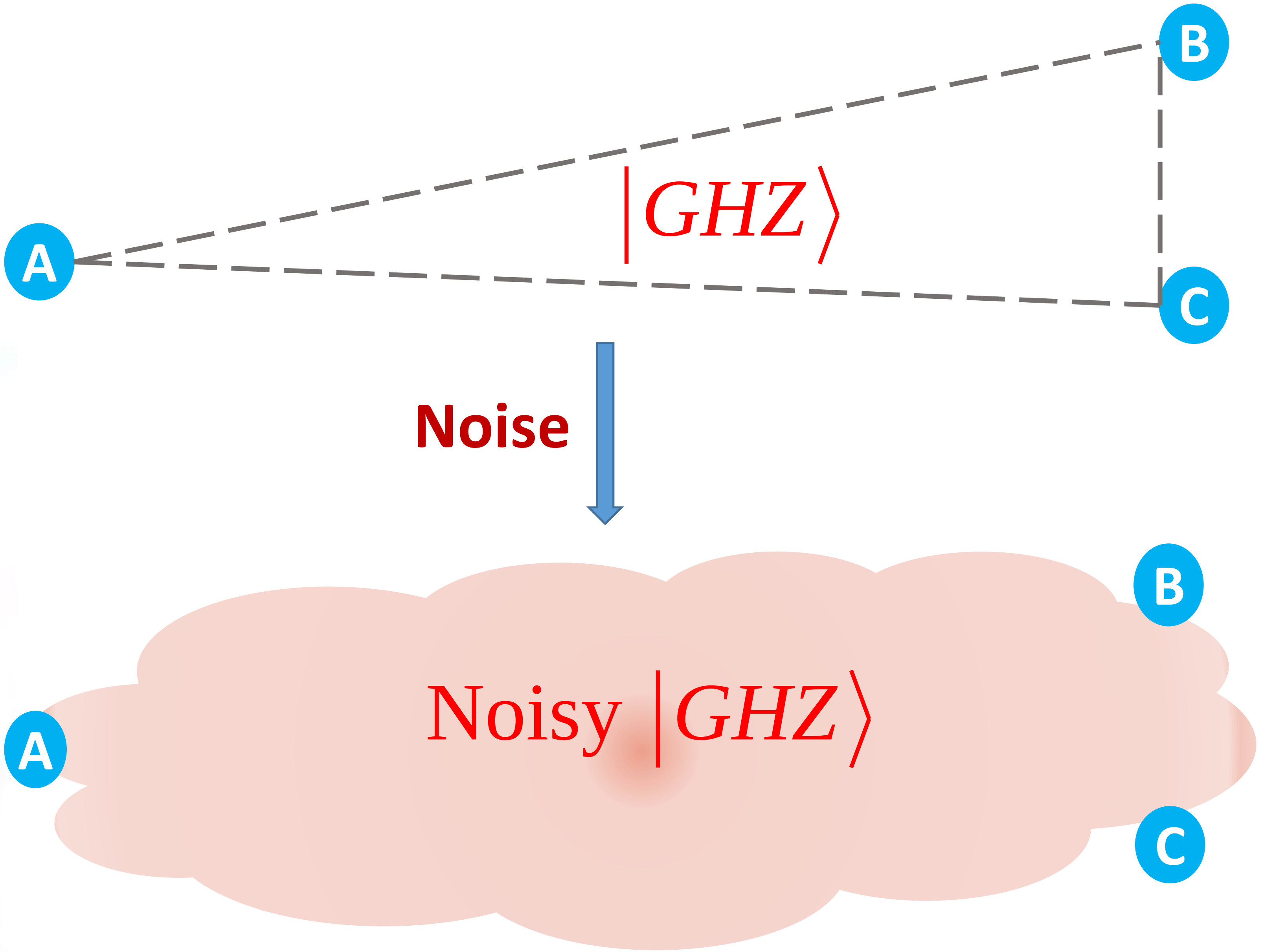
…. Is this protocol robust against the noise?

Entangled States as Robust and Re-usable Carriers of Information

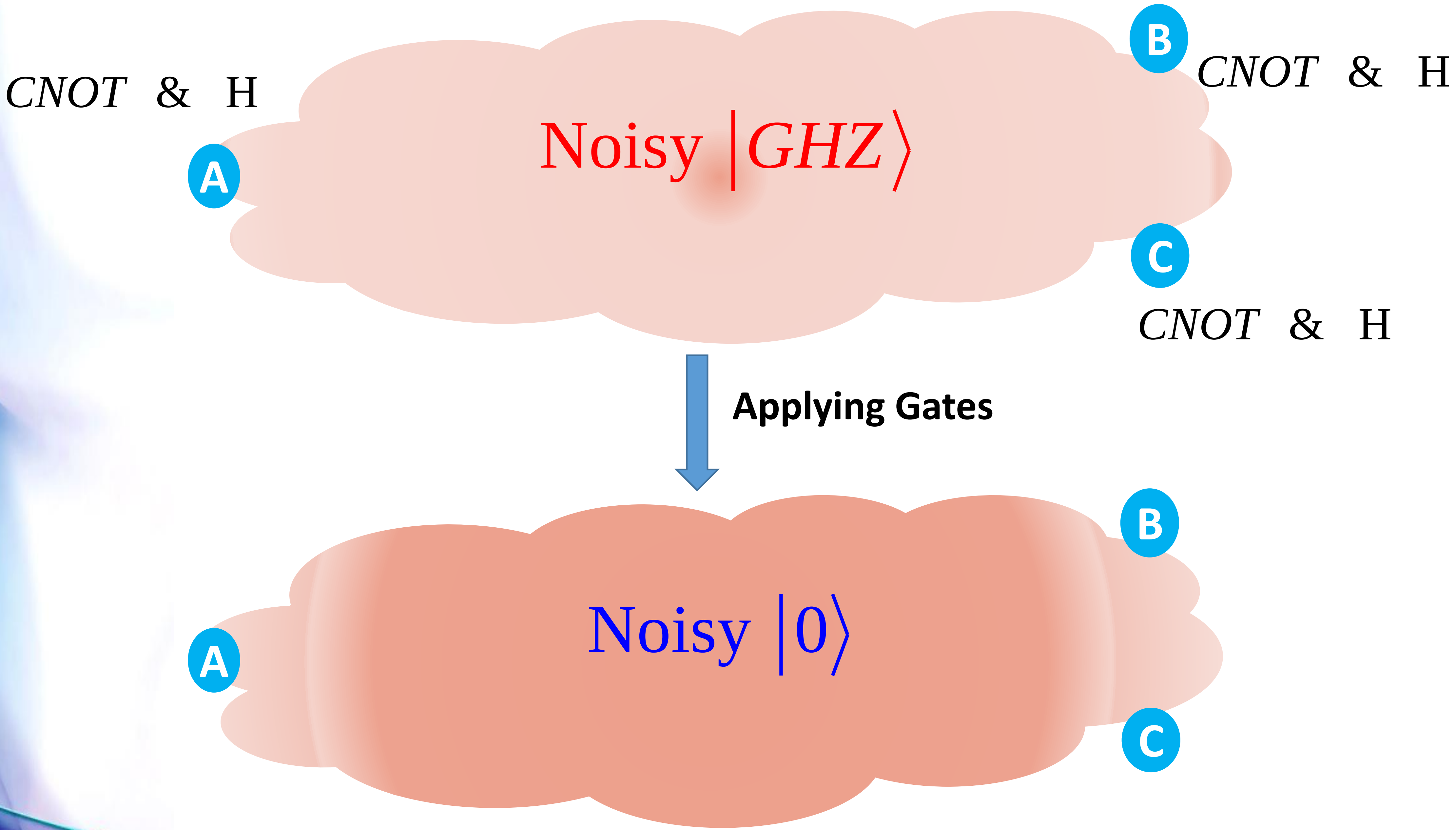
Shima Emamipanah, Marzieh Asoudeh, and Vahid Karimipour

arXiv:1905.08479v1 [quant-ph] 21 May 2019

Effect of the noise :



Applying the gates (Hadamard, CNOT,….)may makes the situation worse



Effect of de-phasing noise :

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{e^{i\theta\sigma_z}} e^{i\theta z}\alpha|0\rangle + e^{-i\theta z}\beta|1\rangle$$

$$\rho_{final} = \int e^{i\theta\sigma_z}|\psi\rangle\langle\psi|e^{-i\theta\sigma_z}Q(\theta)d\theta = \begin{pmatrix} \alpha^2 & 0 \\ 0 & \beta^2 \end{pmatrix}$$

Effect of de-phasing noise :

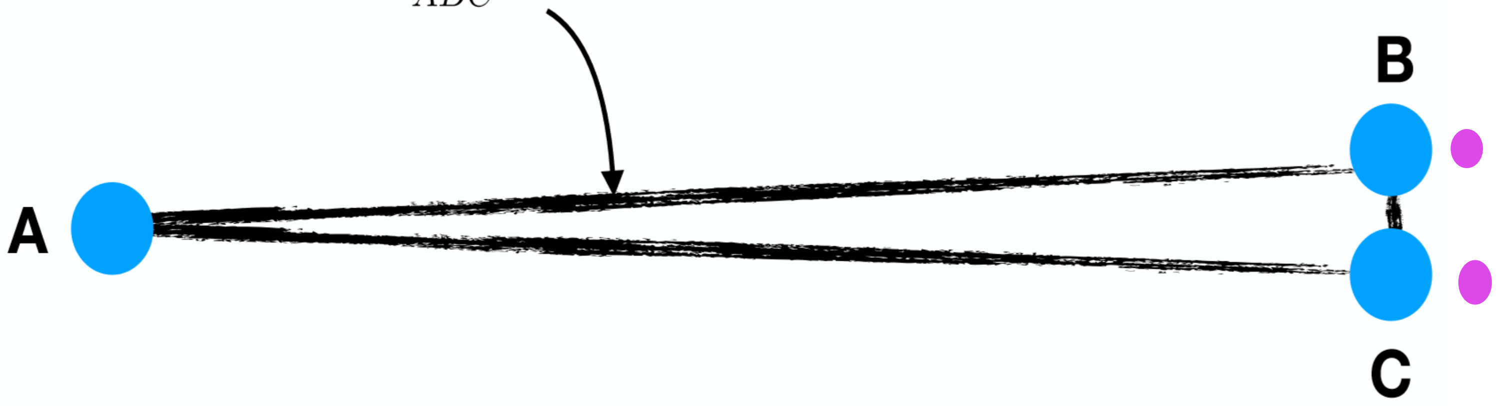
Even rounds:

0,2,...

$$\rho^{even} = (1-2p)|GHZ\rangle\langle GHZ| + p|000\rangle\langle 000| + p|111\rangle\langle 111|$$

$$|GHZ'\rangle = |000\rangle - |111\rangle$$

$$(1-p)|GHZ\rangle\langle GHZ|_{ABC} + p|GHZ'\rangle\langle GHZ'|_{ABC}$$

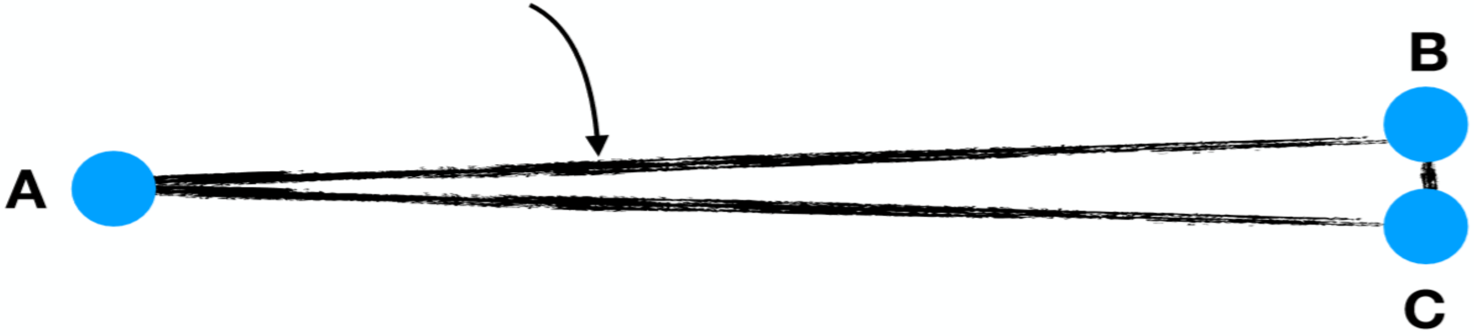


$$H_A \otimes H_B \otimes H_C$$

In the even rounds de-phasing noise has no detrimental effect on the scheme

odd rounds: 1,3,…

$$(1-p)|\mathbf{0}\rangle_{ABC}\langle\mathbf{0}| + p|\mathbf{1}\rangle_{ABC}\langle\mathbf{1}|$$



$$|1\rangle_{ABC} = |001\rangle + |010\rangle + |100\rangle + |111\rangle$$

To analyze the performance of this carrier, we need only study the carrier $|1\rangle_{ABC}$

$$\Omega^{odd} = C_{C,2} C_{B,1} C_{A,1}$$

$$|1\rangle_{ABC} |q\rangle_{1,2} \rightarrow |1\rangle_{ABC} |\bar{q}\rangle_{1,2}.$$

Probability of Error

In even rounds :

The message state is delivered without error

In odd rounds :

The message state is delivered with probability $(1-p)$ without error

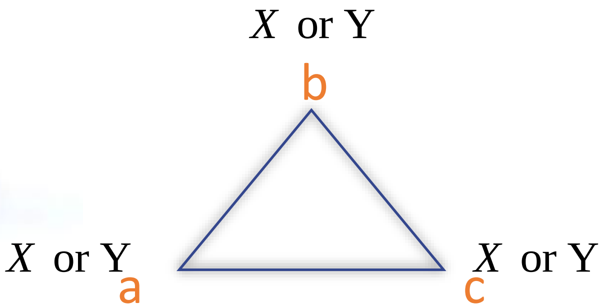
The message state is delivered with probability p with error

The important point is that the sequence of operations do not change the carriers anymore and the Hadamard operations at the end of each round turn the carriers into each other.

Thanks for your attention

QSS With Entanglement

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0,0,0\rangle + |1,1,1\rangle\right)$$



step 1:
random measures of qubit in X or Y bases

step 2:
Public announcement of the bases
(not the results)

Valid rounds:

- X X X
- Y X Y
- X Y Y
- Y Y X

M.Hillery, V.Buzek and A.Berthiaume Phys.Rev.A 59,1829(1999)

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0,0,0\rangle + |1,1,1\rangle\right)$$

X_a = result of a, in the X basis

Y_a = result of a, in the Y basis

In valid rounds there is perfect correlations between the random results:

$$X_a X_b X_c = 1$$

$$Y_a X_b Y_c = -1$$

$$X_a Y_b Y_c = -1$$

$$Y_a Y_b X_c = -1$$

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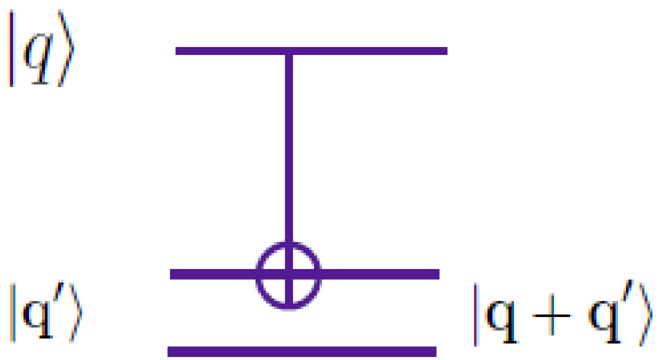
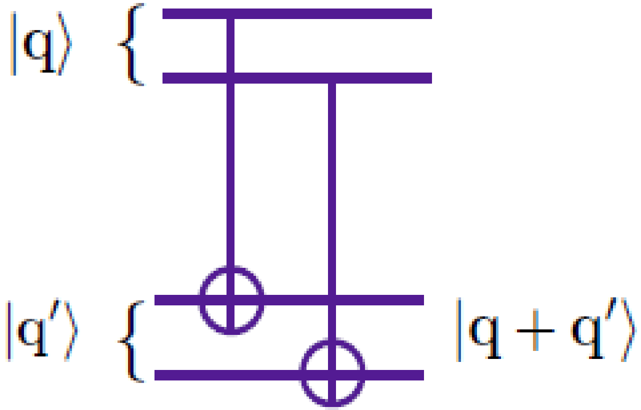


$|q\rangle$
$|q'\rangle$
$|q + q'\rangle$



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