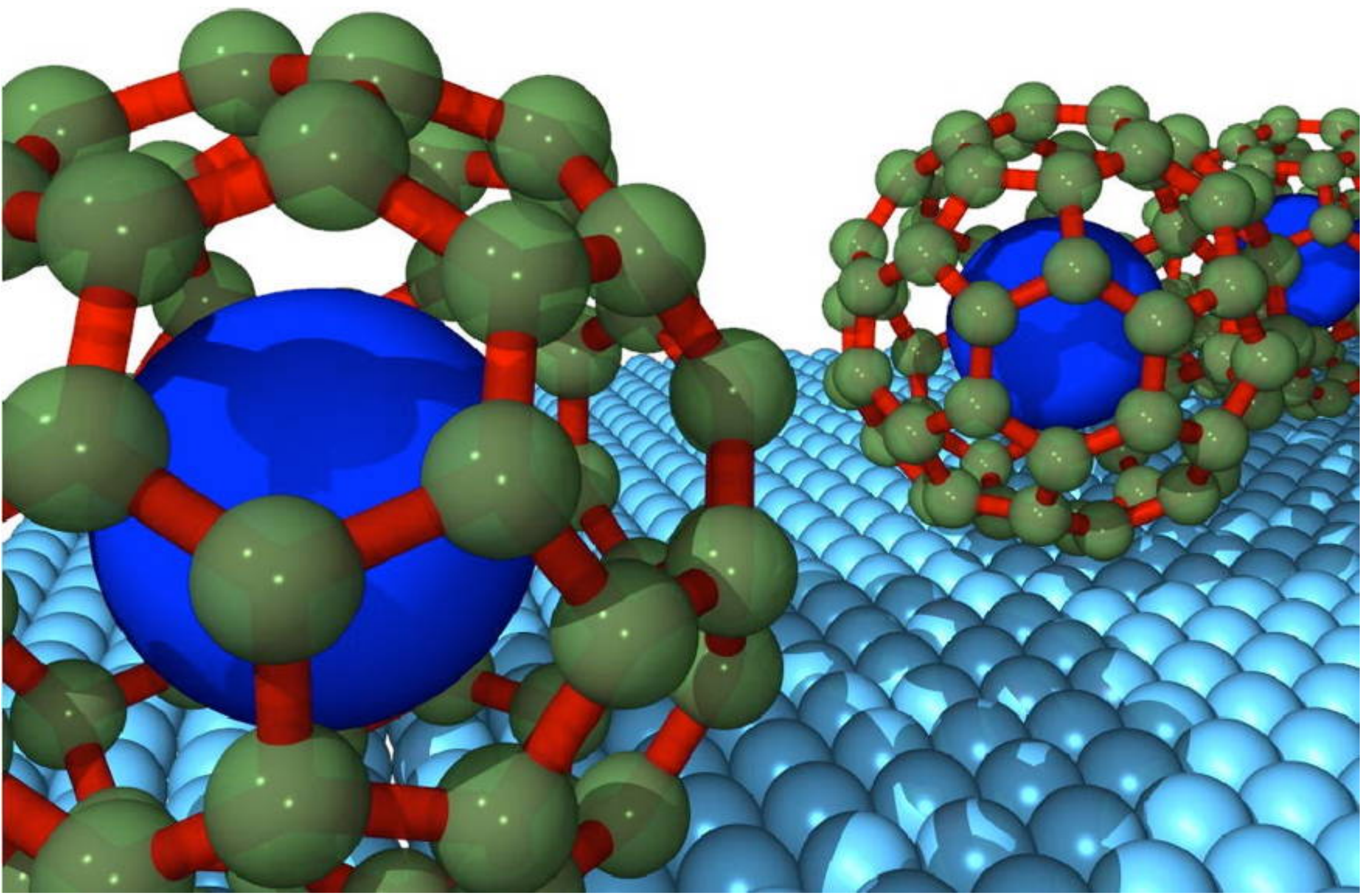
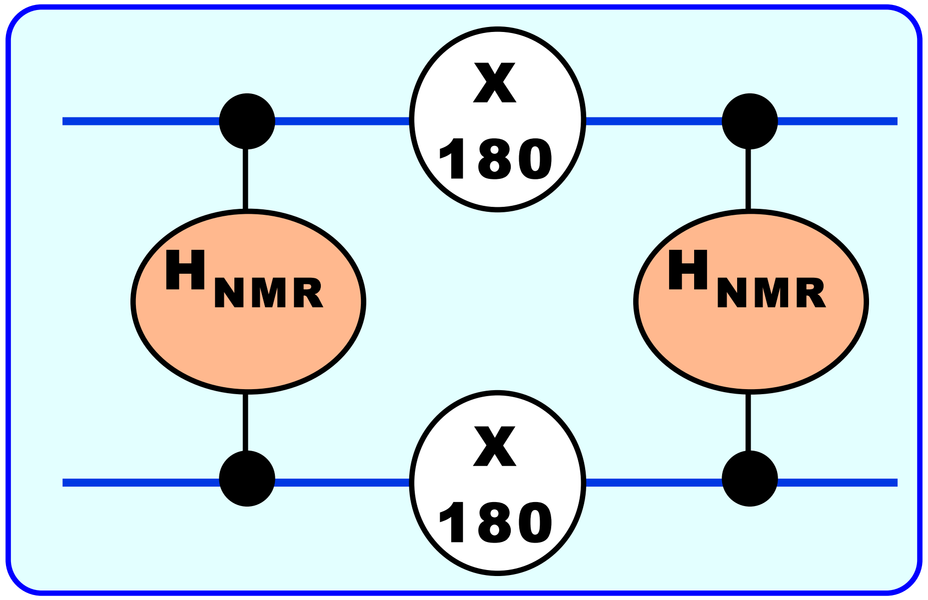
Protecting Quantum Information

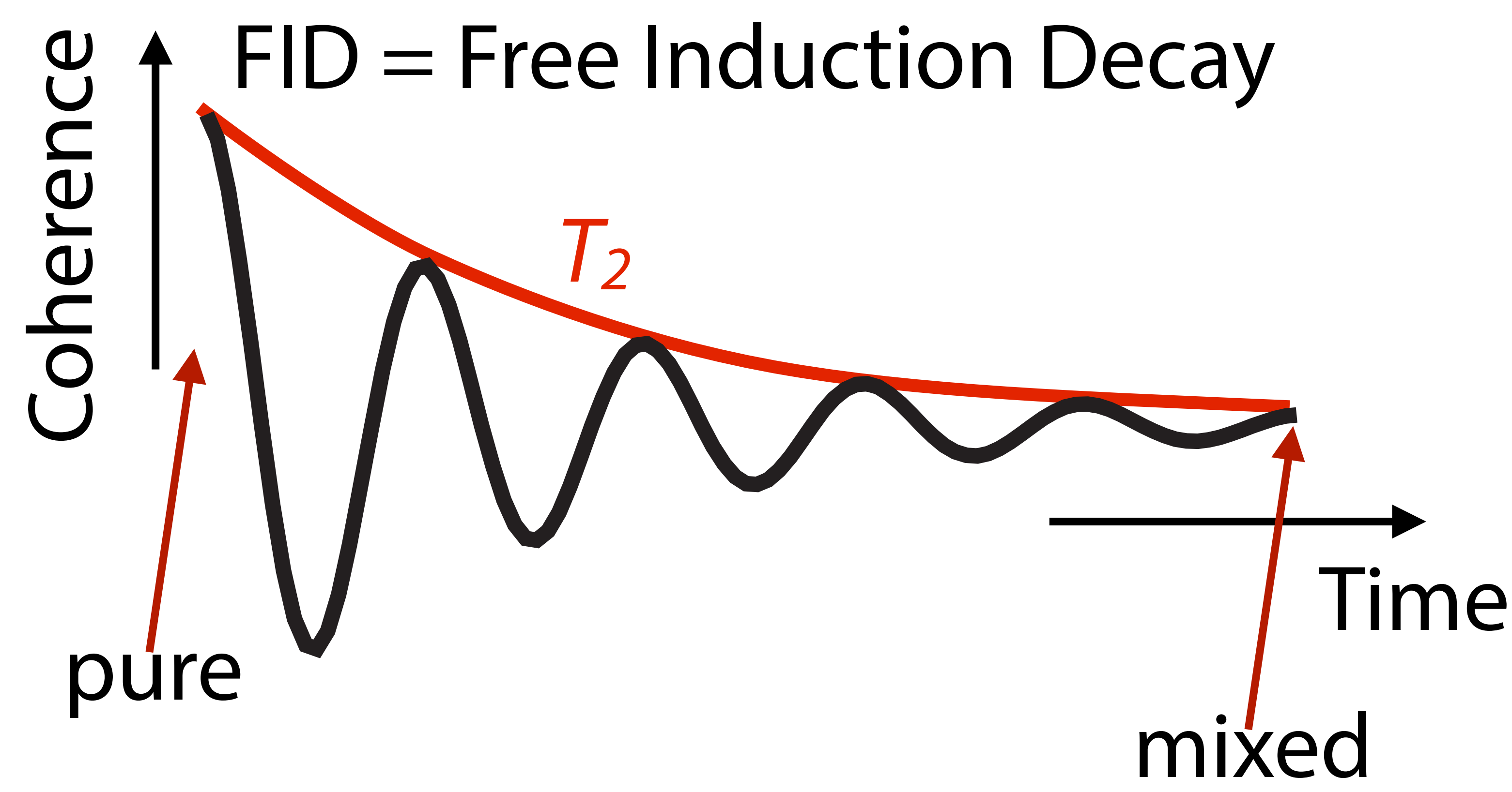


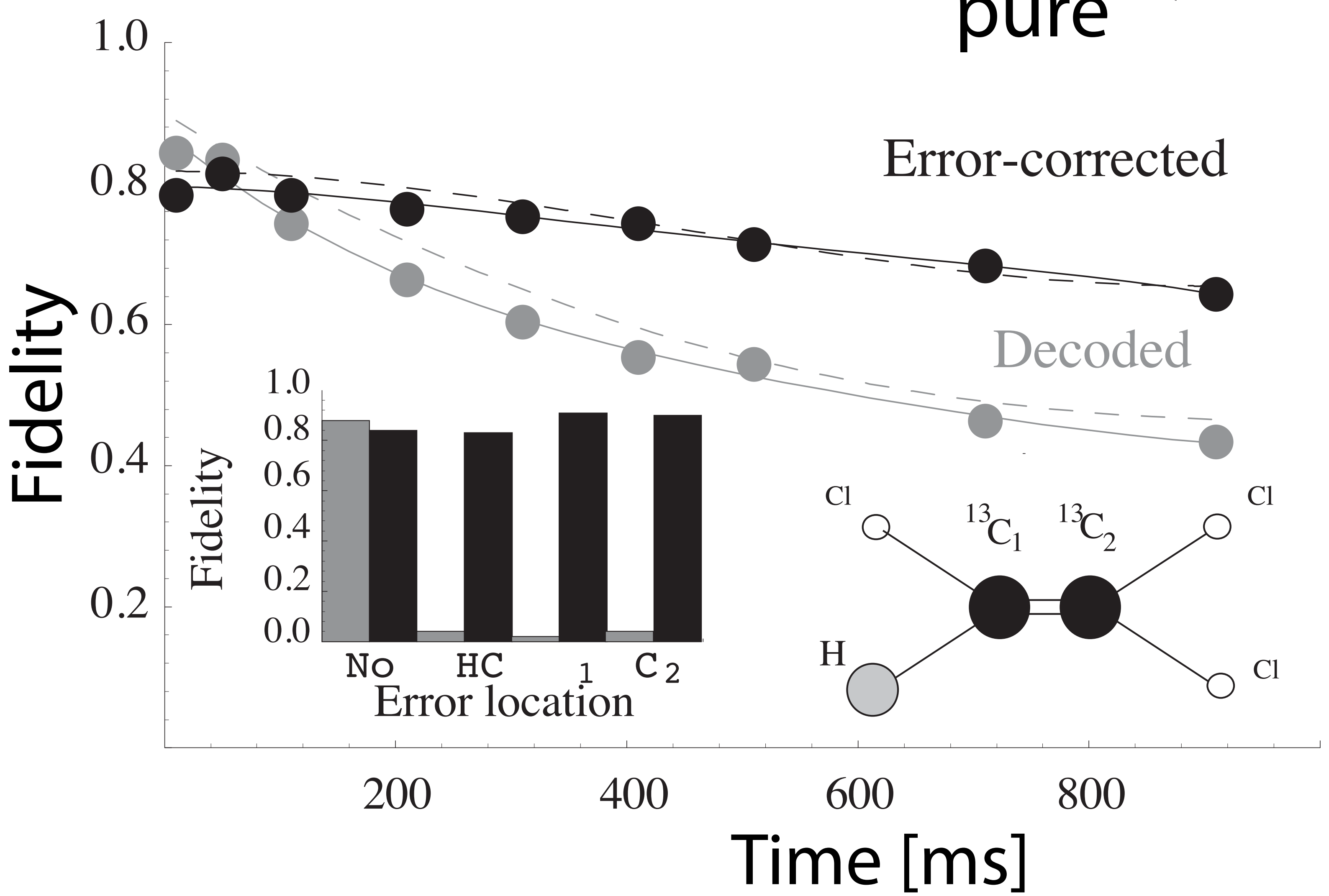


https://qnap.e3.physik.tu-dortmund.de/suter/Vorlesung/ProtectingQI.pdf

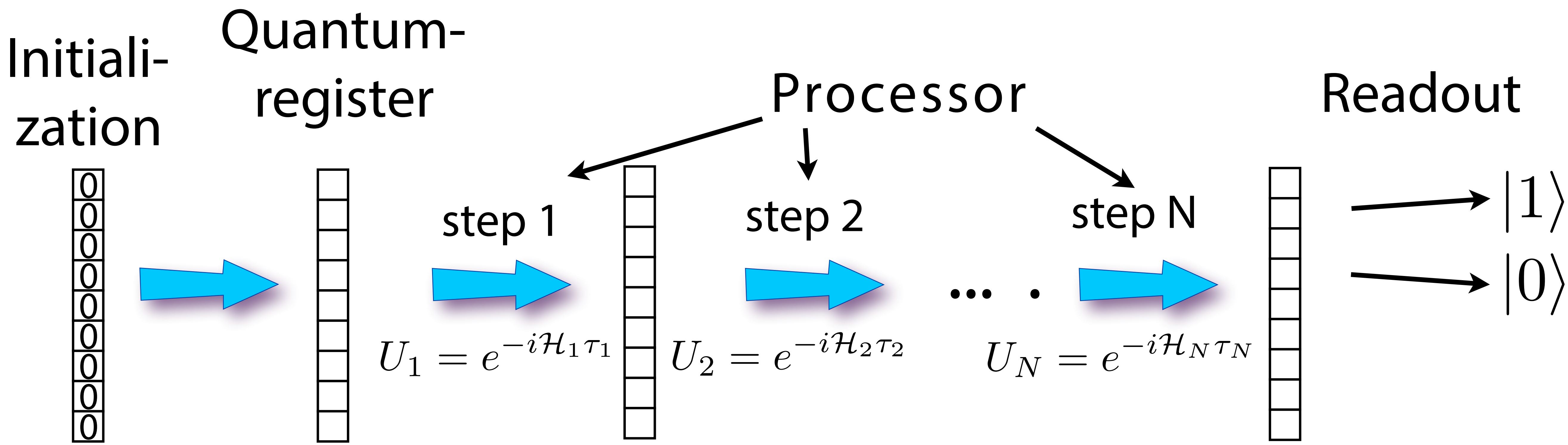
Errors and Decoherence

2+2=5





Motivation



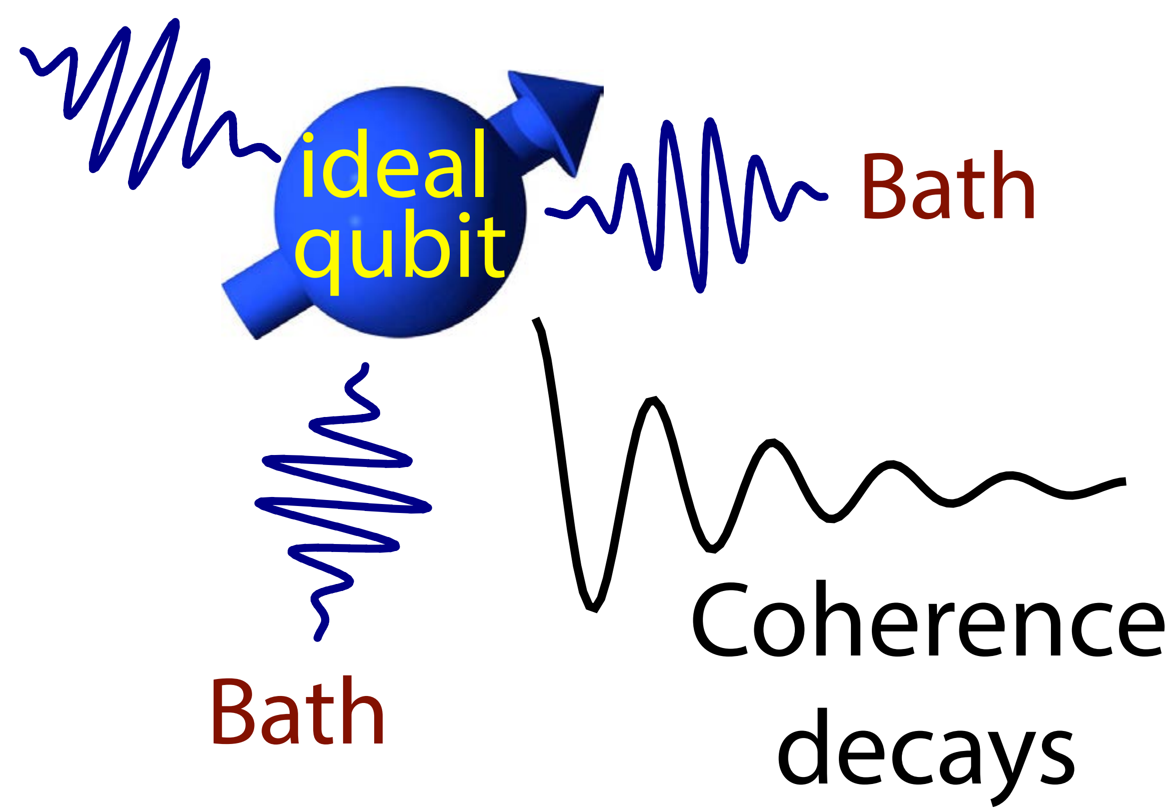
Physical systems behave differently

Sources of Errors

- Parameters of quantum register differ from the ideal ones

- Control fields have finite precision
  → errors

- Coupling to environment
  → decoherence

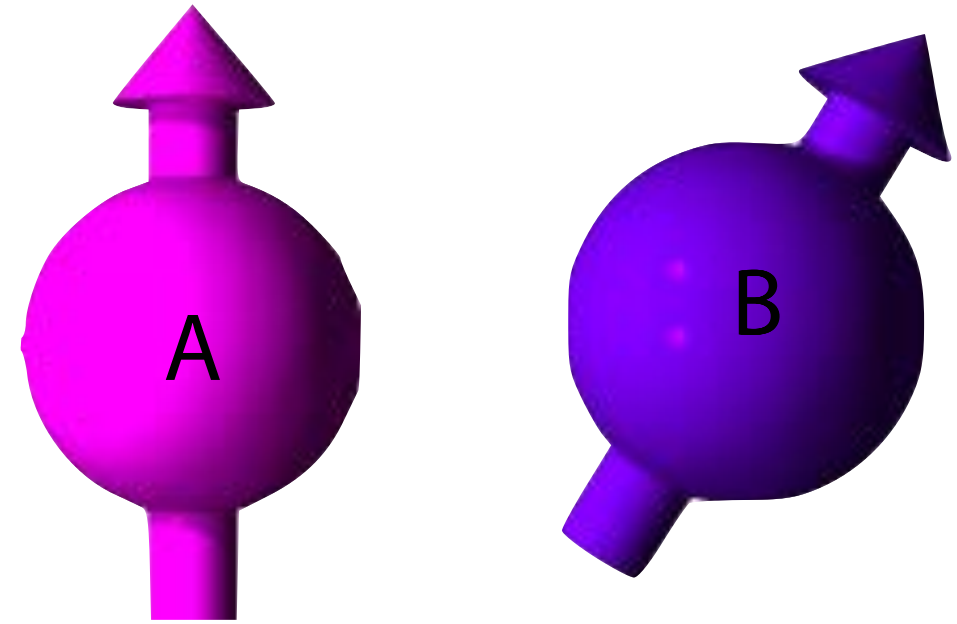


QM : Spin-Spin Model

Simpler model : 2 spins S = 1/2

Interaction Hamiltonian:

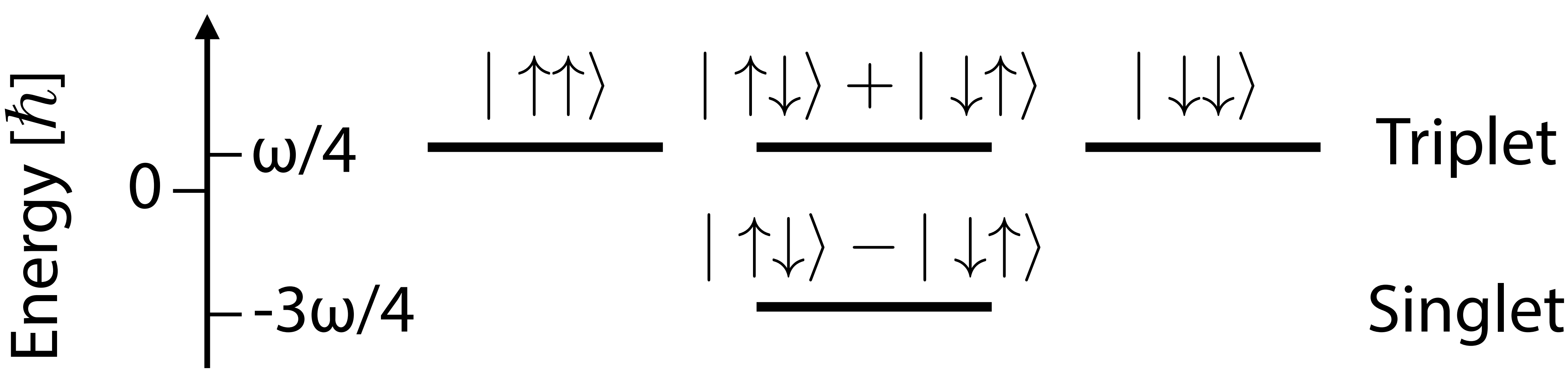
$$\mathcal{H} = \frac{\omega}{\hbar}\vec{\mathbf{S}}_A \cdot \vec{\mathbf{S}}_B$$



System

Environment

Eigenstates:



Entanglement and Mixing

No entanglement for $|\Psi(0)\rangle = |\uparrow\uparrow\rangle$ or $|\Psi(0)\rangle = |\downarrow\downarrow\rangle$

Maximum entanglement for $|\Psi(0)\rangle = |\uparrow\downarrow\rangle$ or $|\Psi(0)\rangle = |\downarrow\uparrow\rangle$

Time evolution $e^{i\omega t/4}|\psi(t)\rangle = \frac{1}{2}[(1+e^{i\omega t})|\uparrow\downarrow\rangle + (1-e^{i\omega t})|\downarrow\uparrow\rangle]$

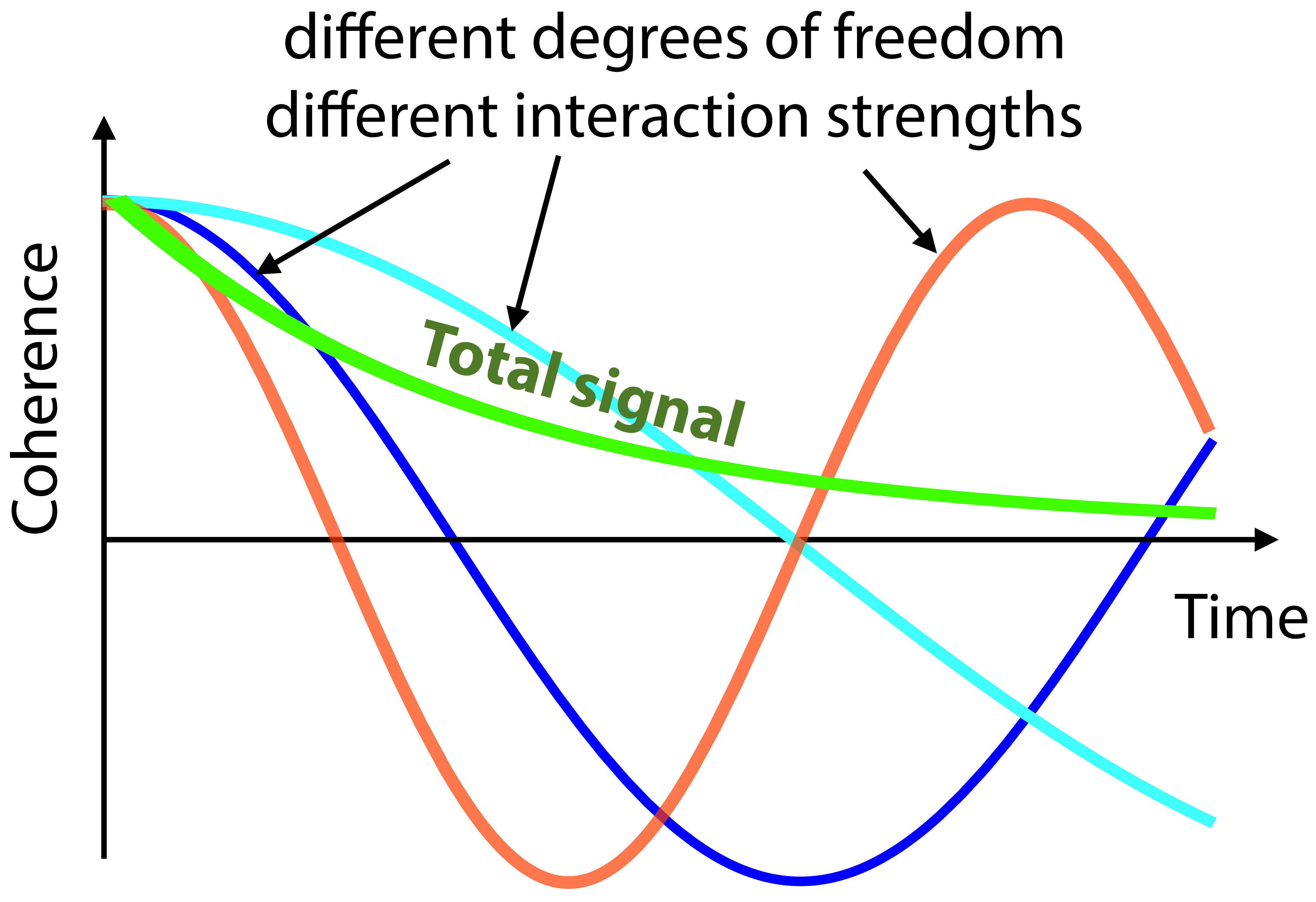
$$t = \frac{\pi}{2\omega}: \qquad |\psi(\frac{\pi}{2\omega})\rangle = e^{-i\pi/8}\frac{1}{2}[(1+i)|\uparrow\downarrow\rangle + (1-i)|\downarrow\uparrow\rangle]$$

The corresponding system density operator is

$$\rho_A(\frac{\pi}{2\omega}) = Tr_B|\psi(\frac{\pi}{2\omega})\rangle\langle\psi(\frac{\pi}{2\omega})| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Several Degrees of Freedom



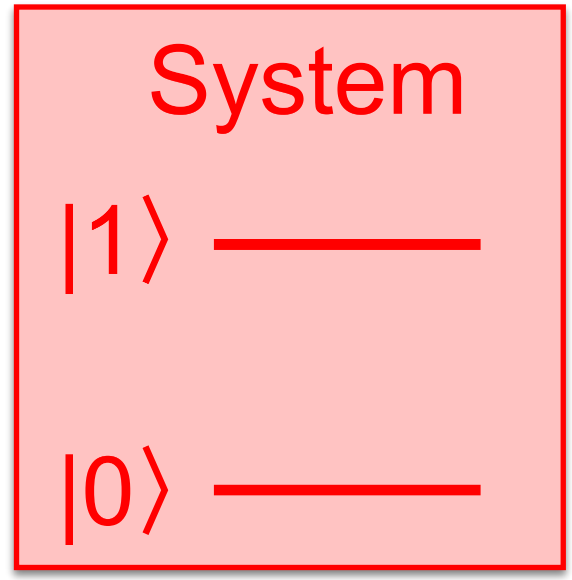
The Spin-Boson Model

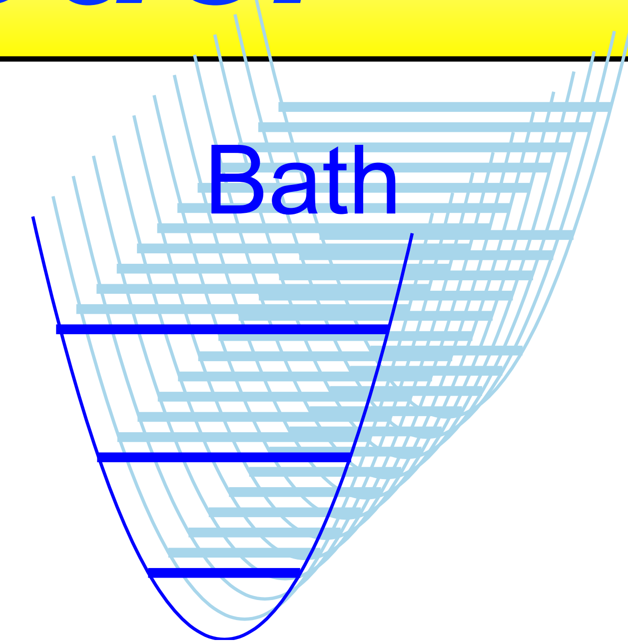
Coupling Hamiltonian
for pure dephasing:

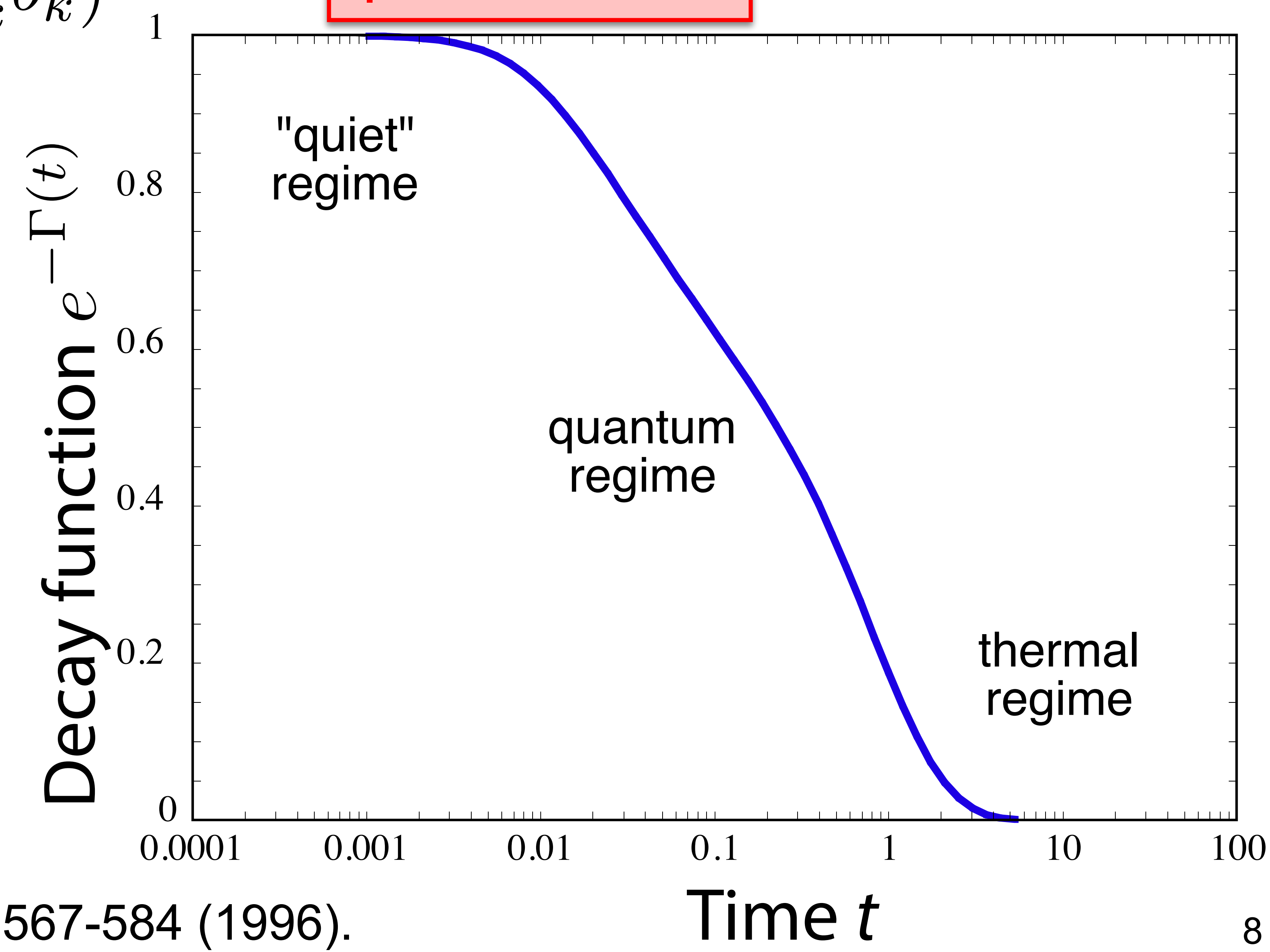
Bath

$$\mathcal{H}_I = \sum_k \sigma_z (g_k b_k^\dagger + g_k^* b_k)$$

System





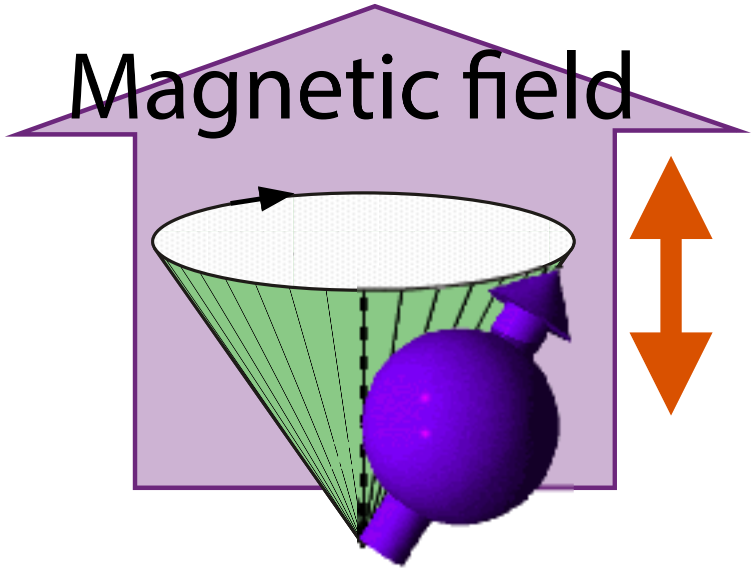


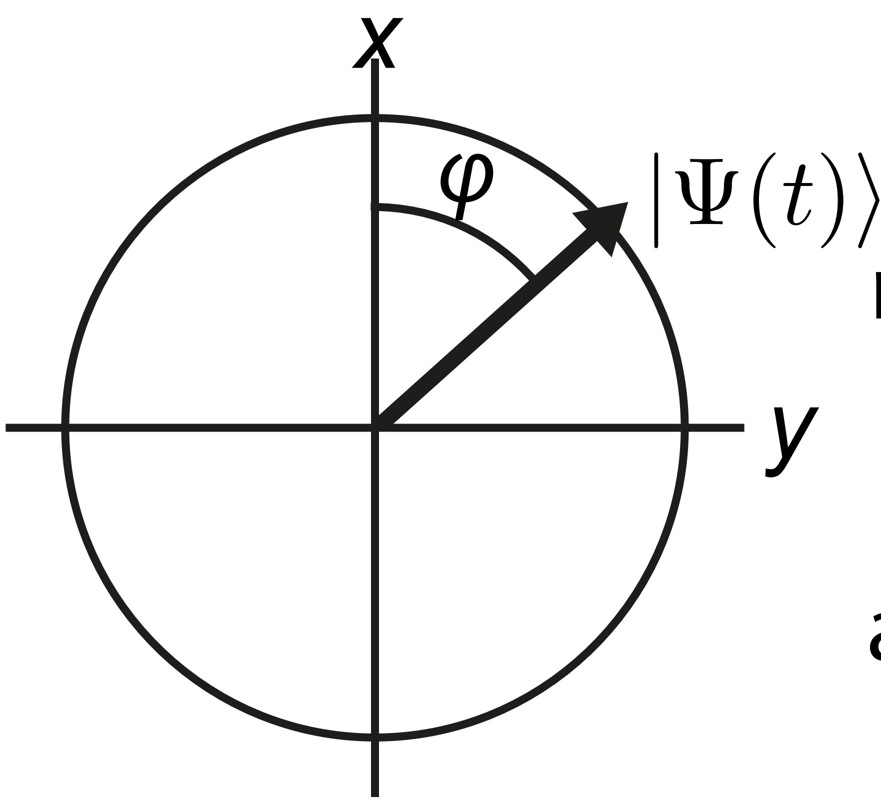
Palma et al., Proc. Roy. Soc. A 452, 567-584 (1996).

Semiclassical Description

$$|\Psi(0)\rangle = a|0\rangle + b|1\rangle$$

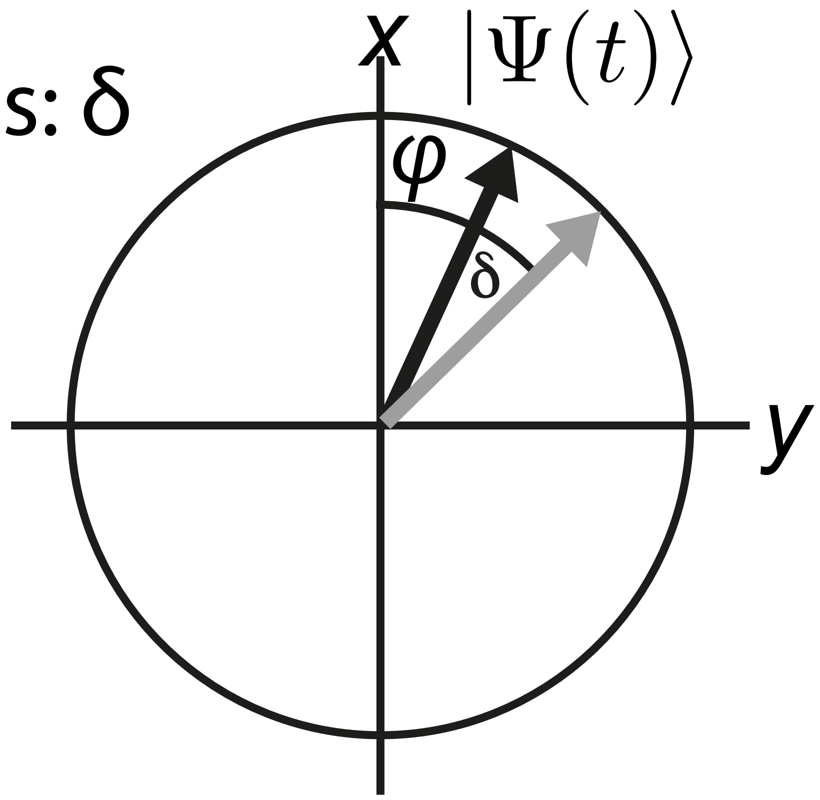
$$|\Psi(t)\rangle = a|0\rangle e^{-i\mathscr{E}_0 t/\hbar} + b|1\rangle e^{-i\mathscr{E}_1 t/\hbar}$$





relative phase: $\varphi = (\mathcal{E}_1 - \mathcal{E}_0)t/\hbar$

additional perturbations: δ



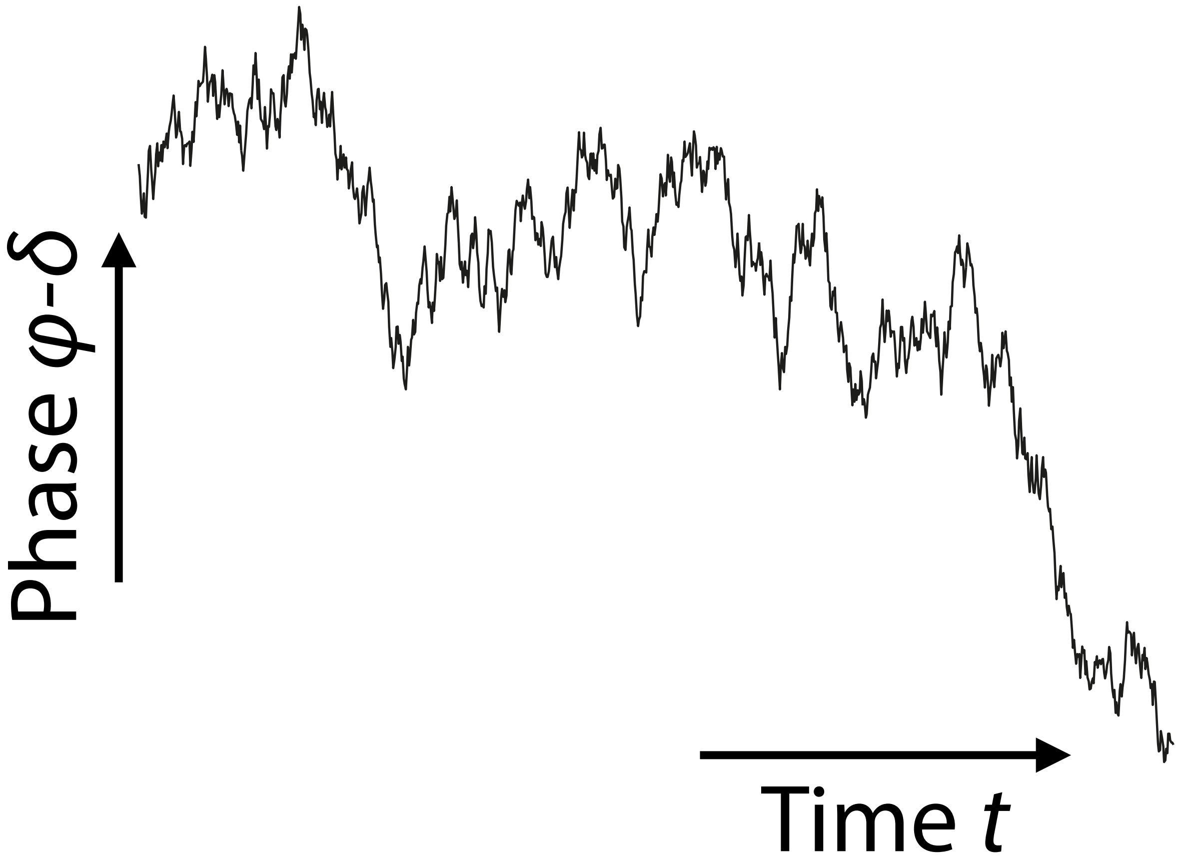
additional precession:

$$\delta(t) = \frac{1}{\hbar}\int_0^t (\delta_{\mathcal{E}_1} - \delta_{\mathcal{E}_0})dt'$$

Random Process

The coupling is in general time dependent

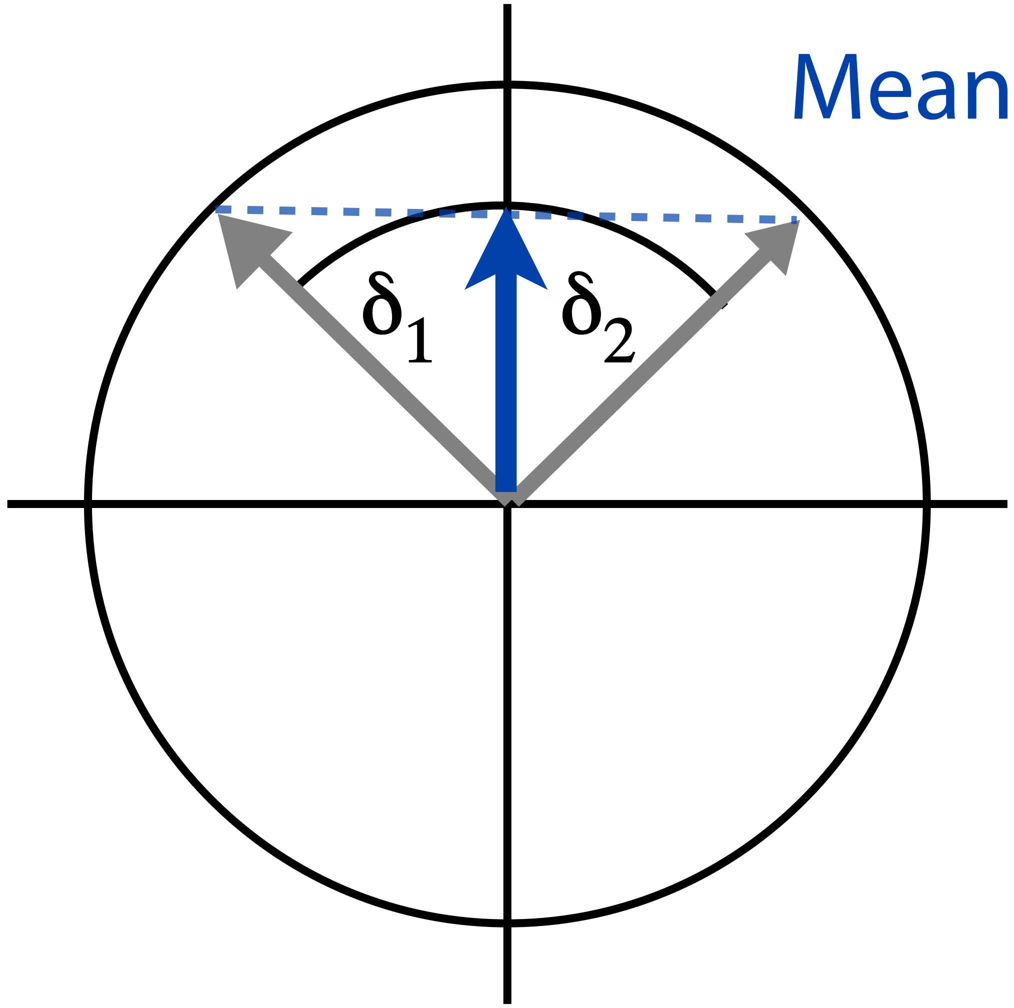
Single qubit : diffusion process



Ensemble Average

In an ensemble, different qubits have different couplings and therefore different precession angles

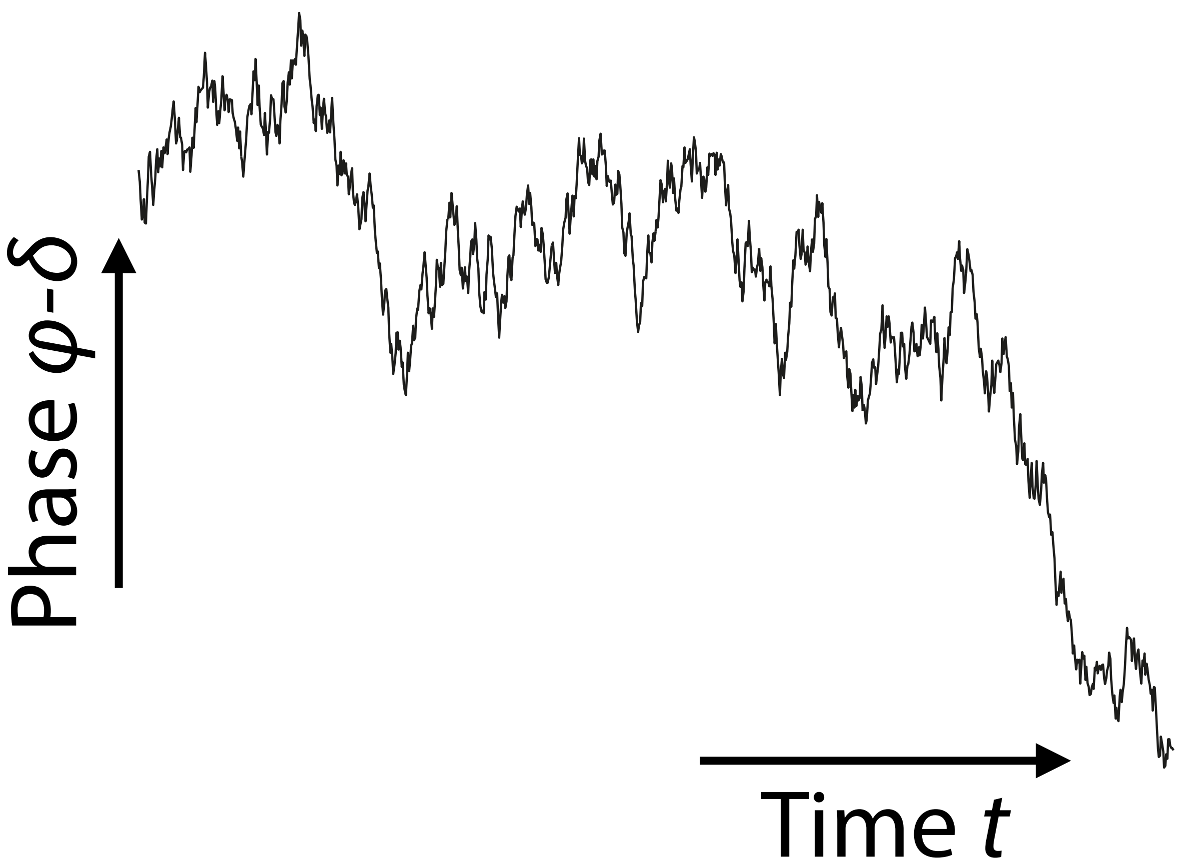
The average polarization is therefore smaller than that of the individuals



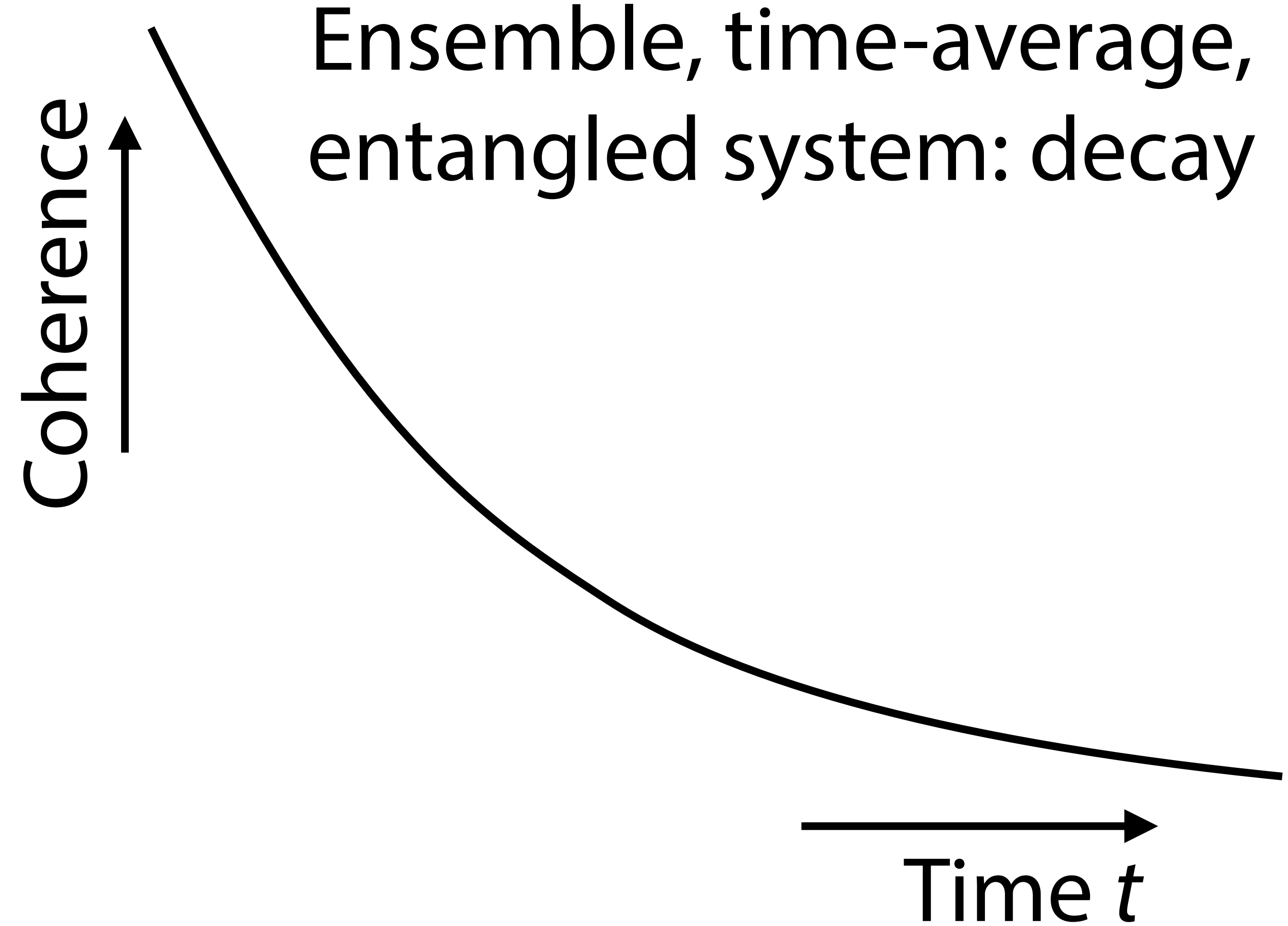
Time Dependence

The coupling is in general time dependent

Single qubit : diffusion process



Ensemble, time-average, entangled system: decay



The observed polarization therefore decays

$$\rho_{ij}(t) = \rho_{ij}(0)\, e^{-i(\mathcal{E}_i - \mathcal{E}_j)t/\hbar} e^{-t/T_2}$$

Theorem of Decoherence

Start in superposition state

$$|\psi(0)\rangle = \frac{1}{2}\left(|\uparrow\rangle + |\downarrow\rangle\right)_A \otimes \left(|\uparrow\rangle - |\downarrow\rangle\right)_B$$

If two mutually orthogonal states of the system of interest become correlated to two mutually orthogonal states of the environment, all effects of phase coherence between the two system states become lost.

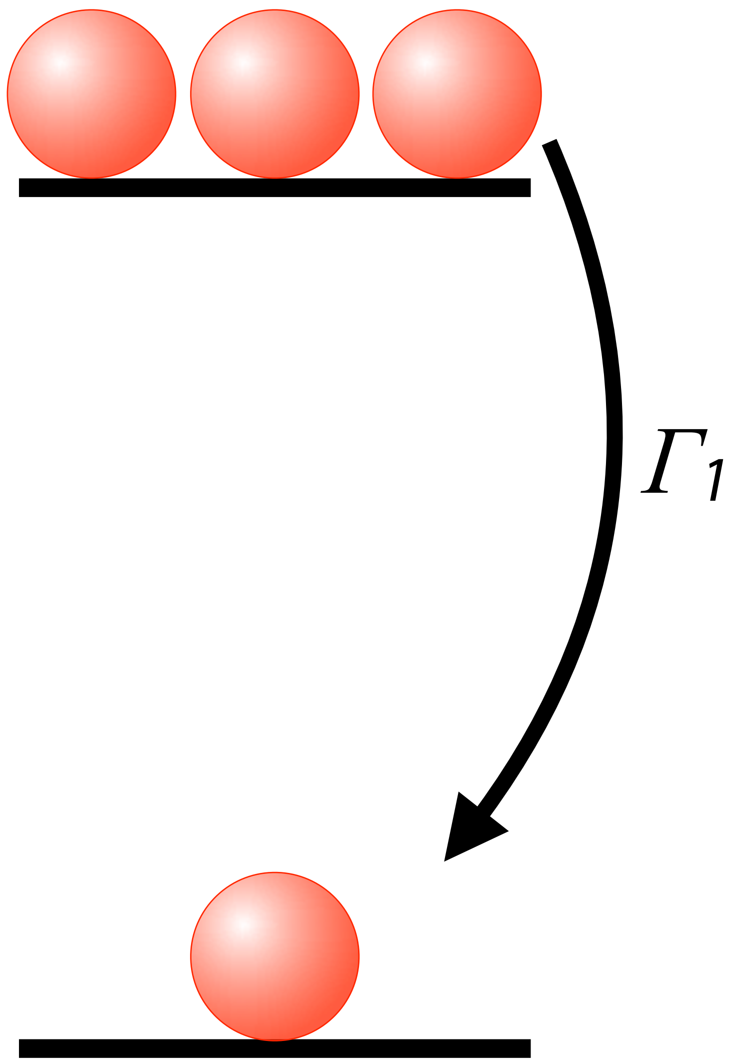
A. J. Leggett, in D. Heiss, editor, Fundamentals of Quantum Information, volume 587 of Lecture Notes in Physics, pages 3–46, Berlin, 2002. Springer Verlag.

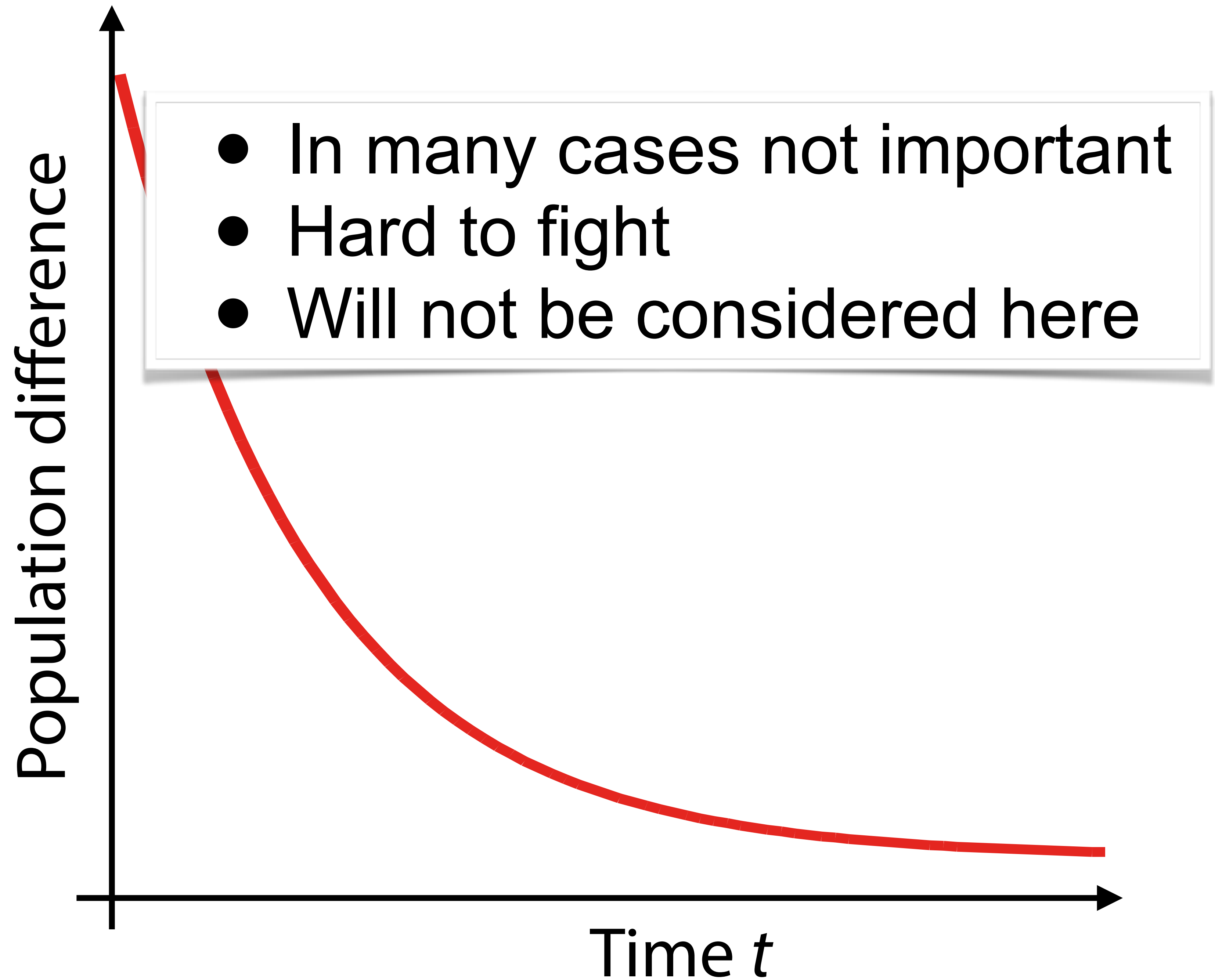
The environment has "measured" the system.

Relaxation of Populations

Dephasing conserves energy

Relaxation of populations does not conserve energy

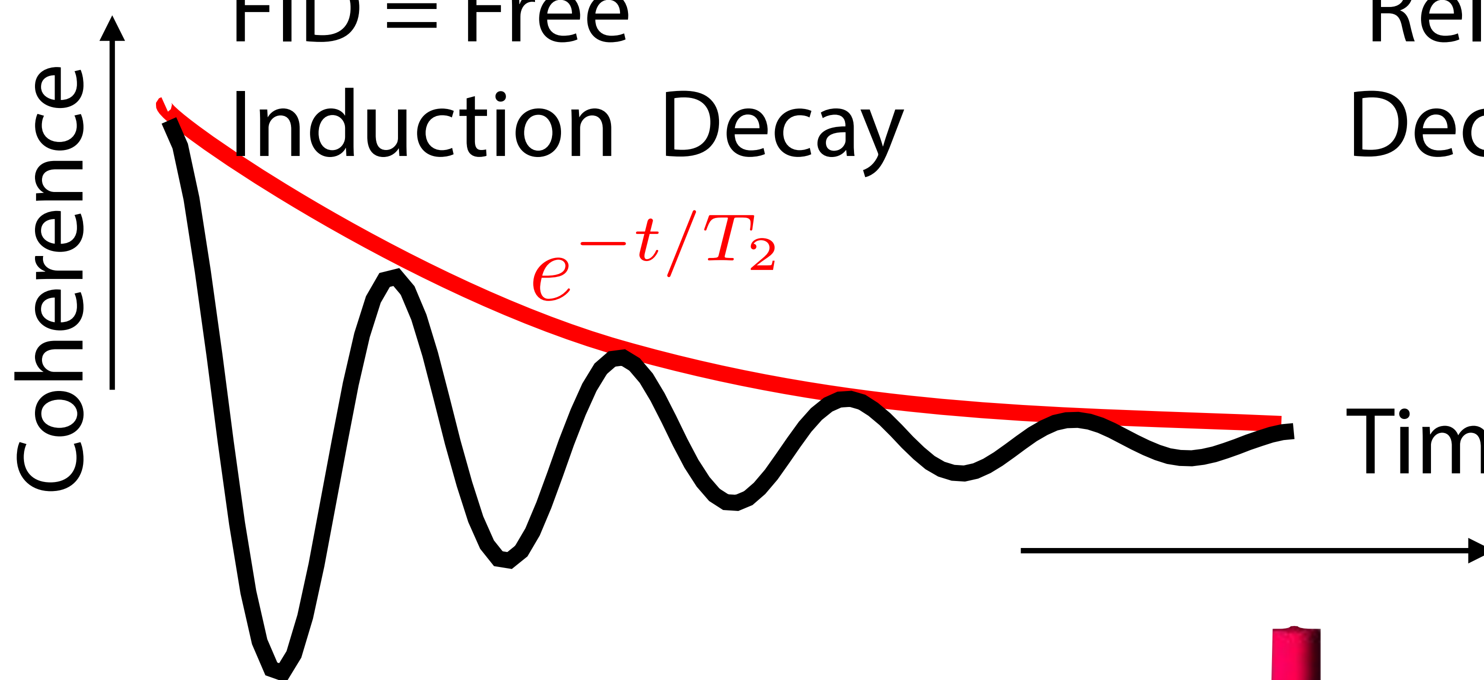




- In many cases not important
- Hard to fight
- Will not be considered here

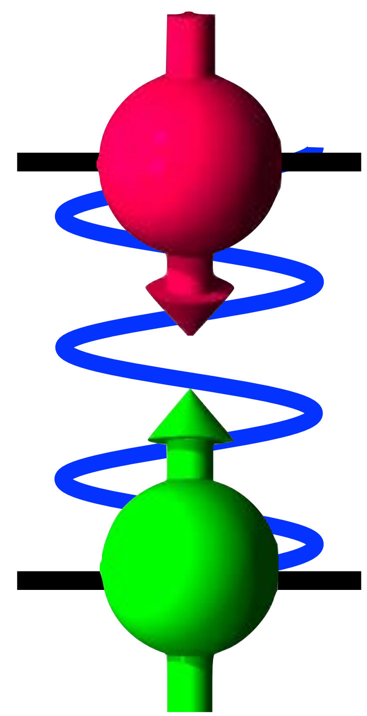
Scaling

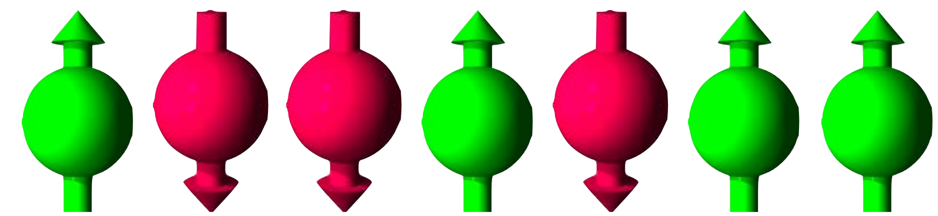
FID = Free Induction Decay



Relaxation = Decoherence

observable magnetization
=
single qubit coherence



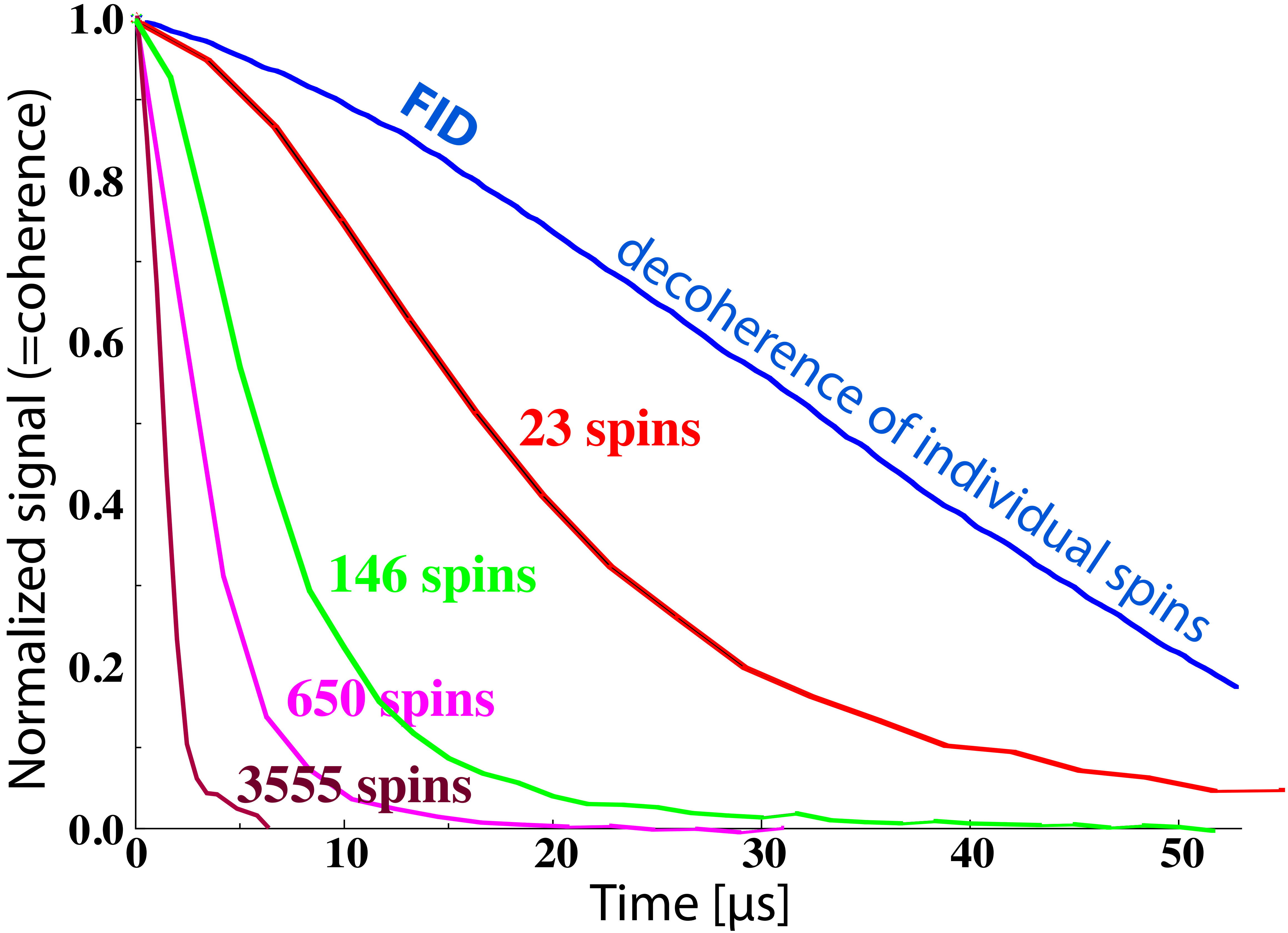


Quantum register involves coherence of many qubits

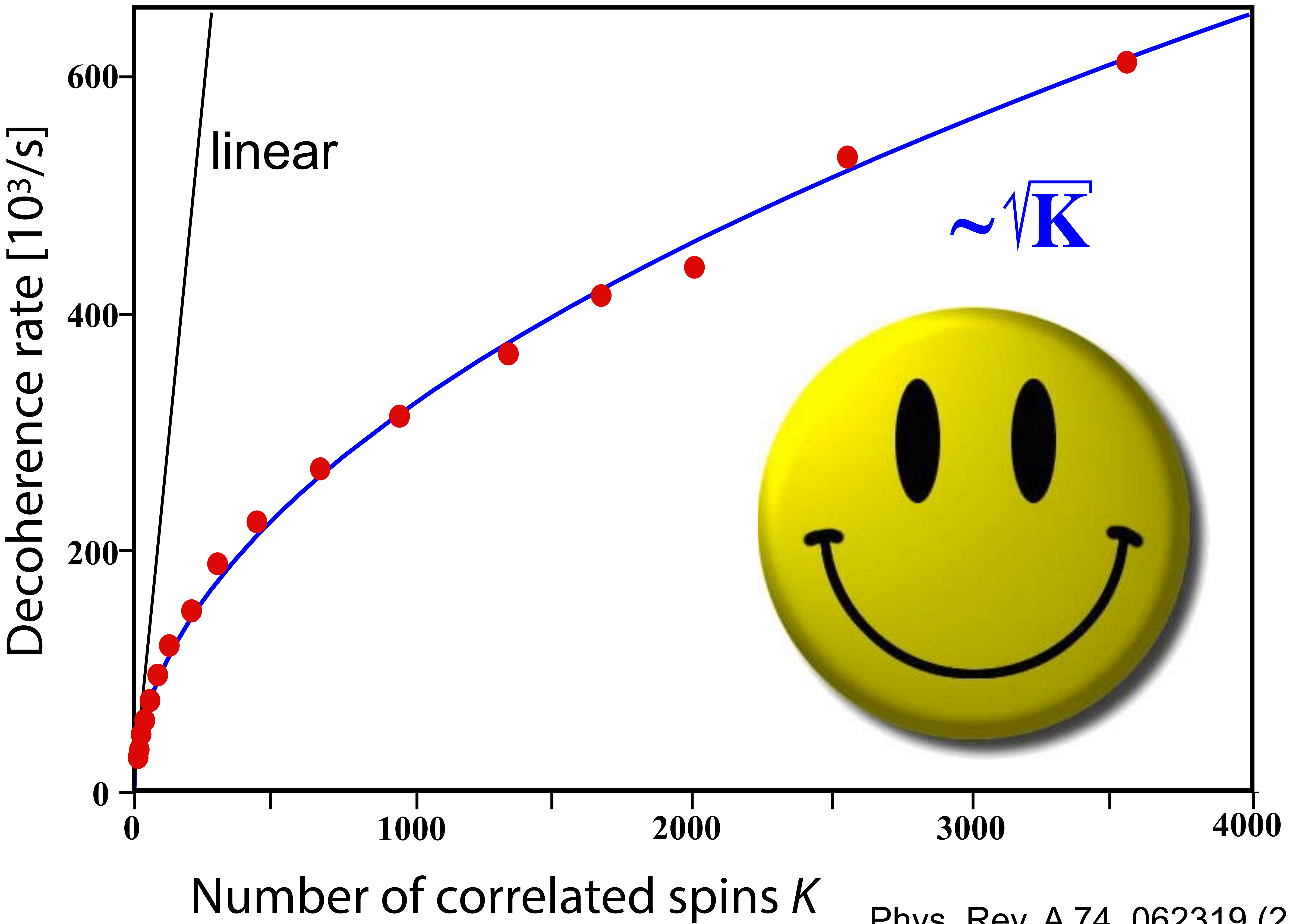
How fast will a "useful" quantum register lose information ?

Scaling of Decoherence

Multiqubit Systems

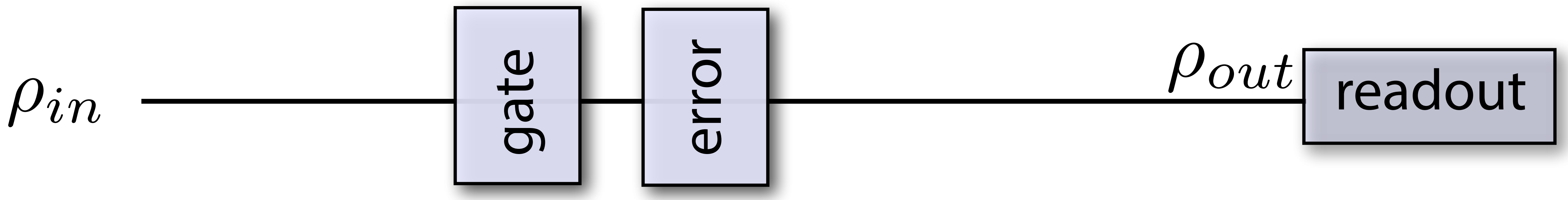


Decoherence Rates

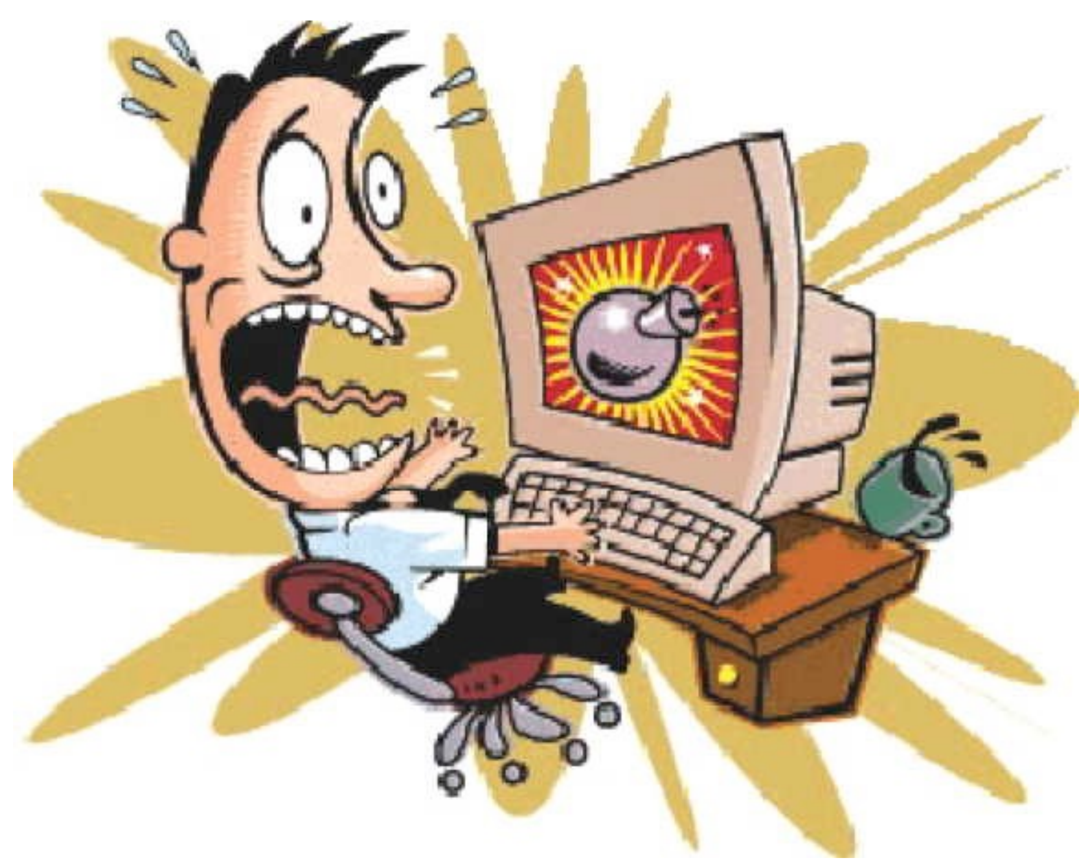


Phys. Rev. A 74, 062319 (2006)

Quantum Error Correction



Errors are hard to detect and correct in QIP



Classical Digital Information

Digital Information is inherently stable

A TTL signal is defined as "low" or L when between 0V and 0.8V with respect to the ground terminal, and "high" or H when between 2V and 5V.

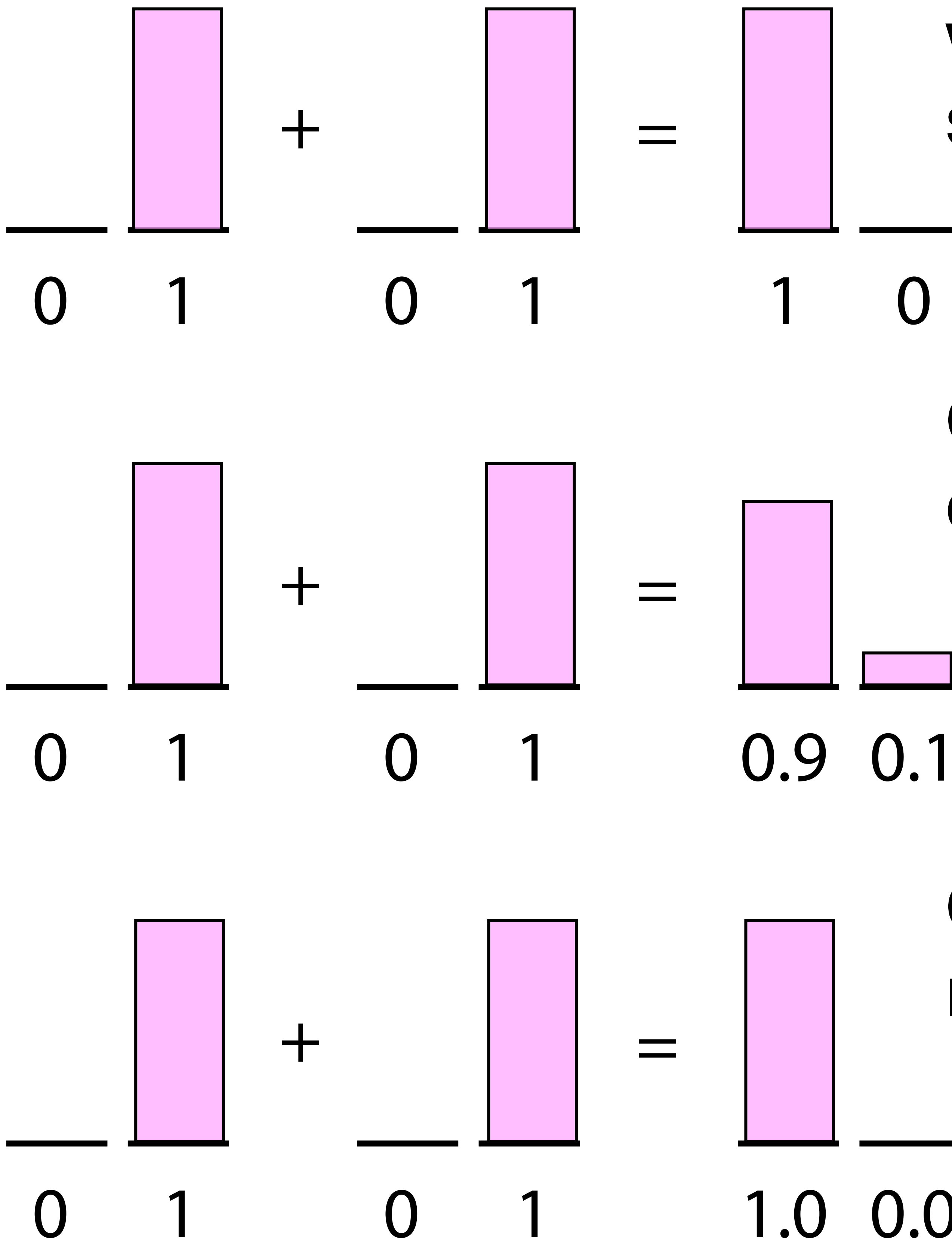
- Small voltage error does not affect information
- Only possible error : bit flip 0 ⇔ 1

Classical Error Correction

Ideal

Real

Recalibrate



works because state space is discrete

QM: state space is continuous

QM: can not do measurements

Classical Error Correction

Use redundancy, e.g. $0 \Rightarrow 0_L = 000 \qquad 1 \Rightarrow 1_L = 111$

Single bit error probability $0 < p < 1$

Probability for

0 error	$(1-p)^3$	$\sim$	$1-3p$
1 error	$3p(1-p)^2$	$\sim$	$3p$
2 errors	$3p^2(1-p)$	$\sim$	$3p^2$
3 errors	p^3	$=$	p^3

After transmission / calculation:

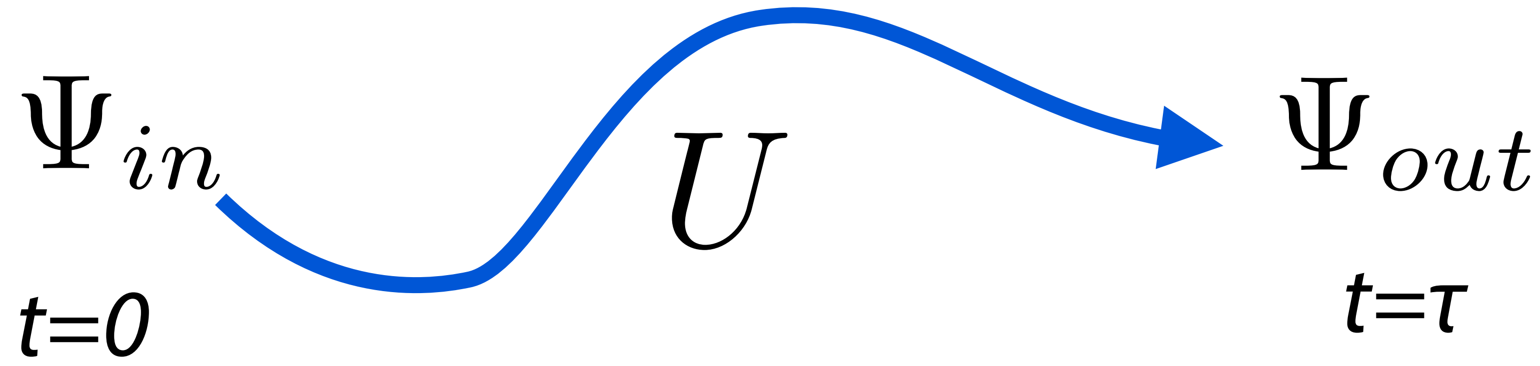
check if all bits identical; if not : flip the one that differs

$001, 010, 100 \rightarrow 000 \qquad 110, 101, 011 \rightarrow 111$

remaining error probability $\sim 3p^2$

Quantum Error Correction

Ideal:



Main difficulties:

- # possible states increases exponentially

- Cannot measure qubits during computation

- Must maintain phase coherence

Quantum vs. Classical

Quantum information "more valuable"
but more fragile

No cloning theorem

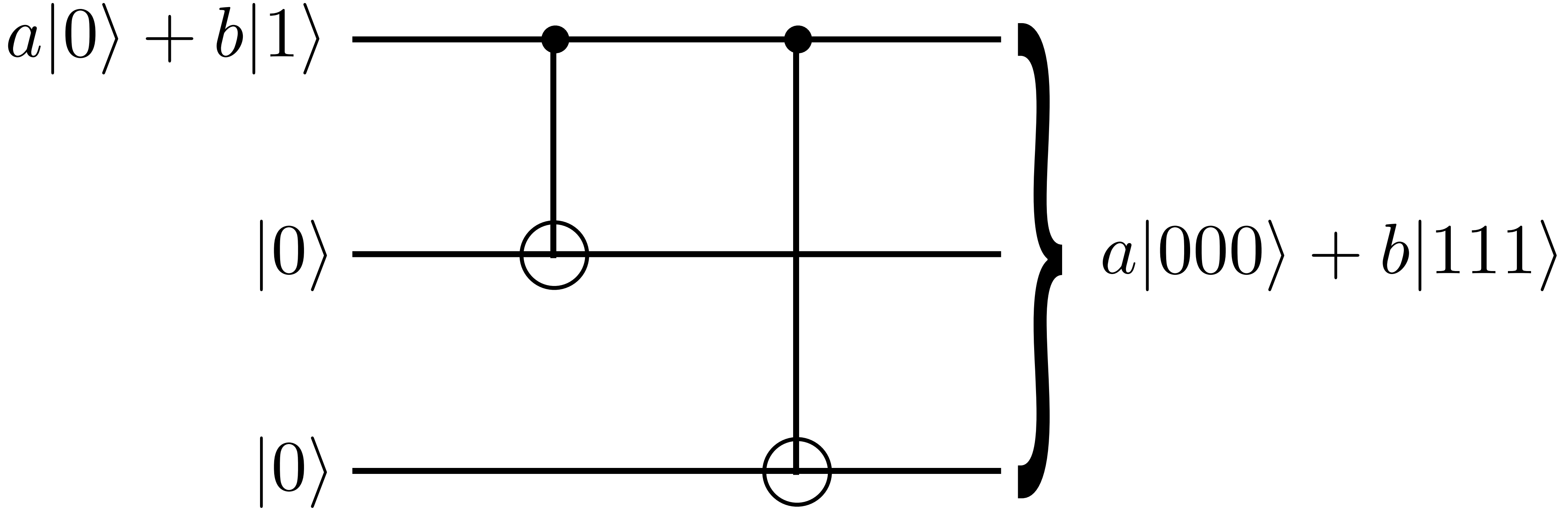
Cannot measure qubits
during calculation

$$a\,|\uparrow\rangle + b\,|\downarrow\rangle \rightarrow |\uparrow\rangle$$
$$a\,|\uparrow\rangle + b\,|\downarrow\rangle \rightarrow |\downarrow\rangle$$

yet there is hope!!

Threshold theorem

Encoding

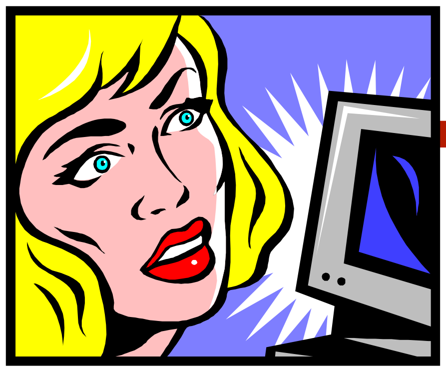


$a|0\rangle + b|1\rangle$
$|0\rangle$
$|0\rangle$
$a|000\rangle + b|111\rangle$

Single Spin-Flip Error

for Quantum Communication

Alice



Bob

$|\Psi_0\rangle = a\,|0\rangle + b\,|1\rangle$

use 2 ancilla bits in state $|0\rangle$

$|\Psi_0\rangle = a\,|000\rangle + b\,|100\rangle$

CNOT^2

$|\Psi_1\rangle = a\,|000\rangle + b\,|111\rangle$

Bit-flip on 2

$|\tilde{\Psi}_1\rangle = a\,|010\rangle + b\,|101\rangle$

Possible error probability : p

$|\Psi_1\rangle = a\,|000\rangle + b\,|111\rangle$

Perfect transmission

Detection and Correction

$|\Psi_1\rangle = a\,|000\rangle + b\,|111\rangle$

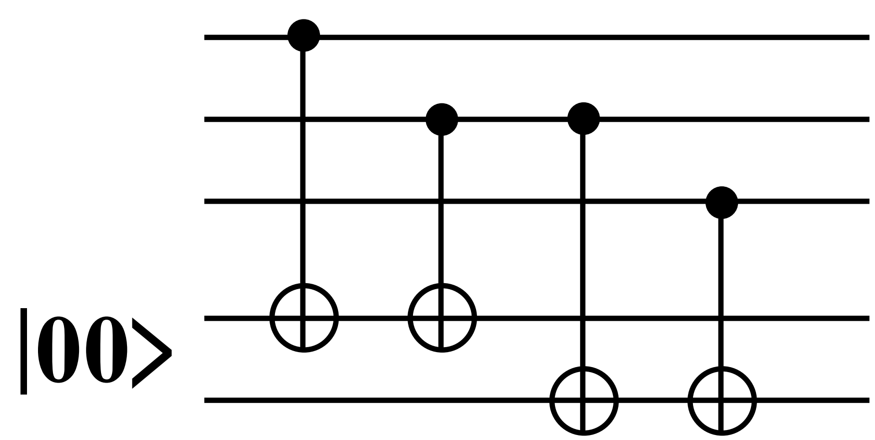
$|\tilde{\Psi}_1\rangle = a\,|010\rangle + b\,|101\rangle$

} are eigenstates of Z_1Z_2 and Z_1Z_3

	Z_1Z_2	Z_1Z_3	
$a\,|000\rangle + b\,|111\rangle$	1	1	
$a\,|100\rangle + b\,|011\rangle$	-1	-1	flip 1 : X_1
$a\,|010\rangle + b\,|101\rangle$	-1	1	flip 2 : X_2
$a\,|001\rangle + b\,|110\rangle$	1	-1	flip 3 : X_3

Alternative error detection:

use 2 ancilla qubits in $|00\rangle$ state



remaining error probability ~$3p^2$

Arbitrary Single Qubit Errors

9-bit code (see above):

P.W. Shor, 'Scheme for reducing decoherence in quantum computer memory', Phys. Rev. A 52, R2493 (1995).

Other encoding schemes

7-bit code:

A.M. Steane, 'Error Correcting Codes in Quantum Theory', Phys. Rev. Lett. 77, 793 (1996).

5-bit code:

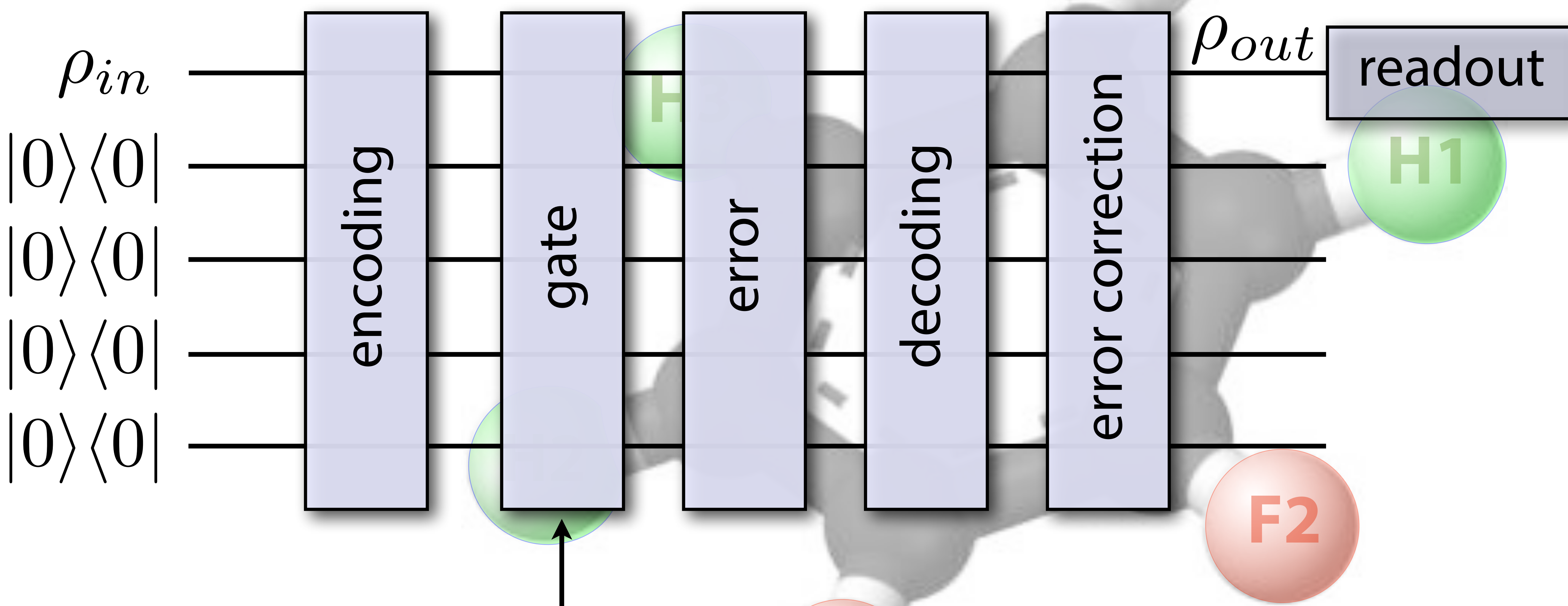
R. Laflamme, C. Miquel, J.P. Paz, and W.H. Zurek, 'Perfect Quantum Error Correcting Code', Phys. Rev. Lett. 77, 198 (1996).

C.H. Bennett, D.P. DiVincenzo, J.A. Smolin, and W.K. Wootters, 'Mixed-state entanglement and quantum error correction', Phys. Rev. A 54, 3824 (1996).

Quantum Error Correction

QEC for quantum computing

Is QEC compatible with processing ?

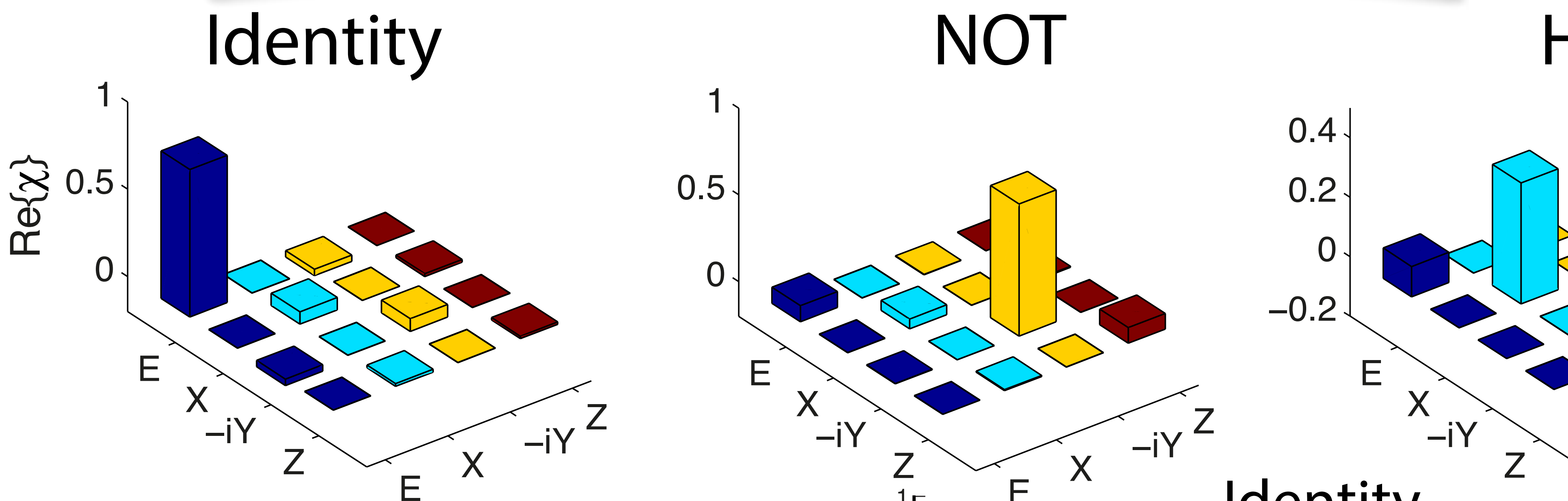


must be applied to logical states

5-qubit gate in the space of physical qubits

Quantum Error Correction

Process tomography of encoded gates

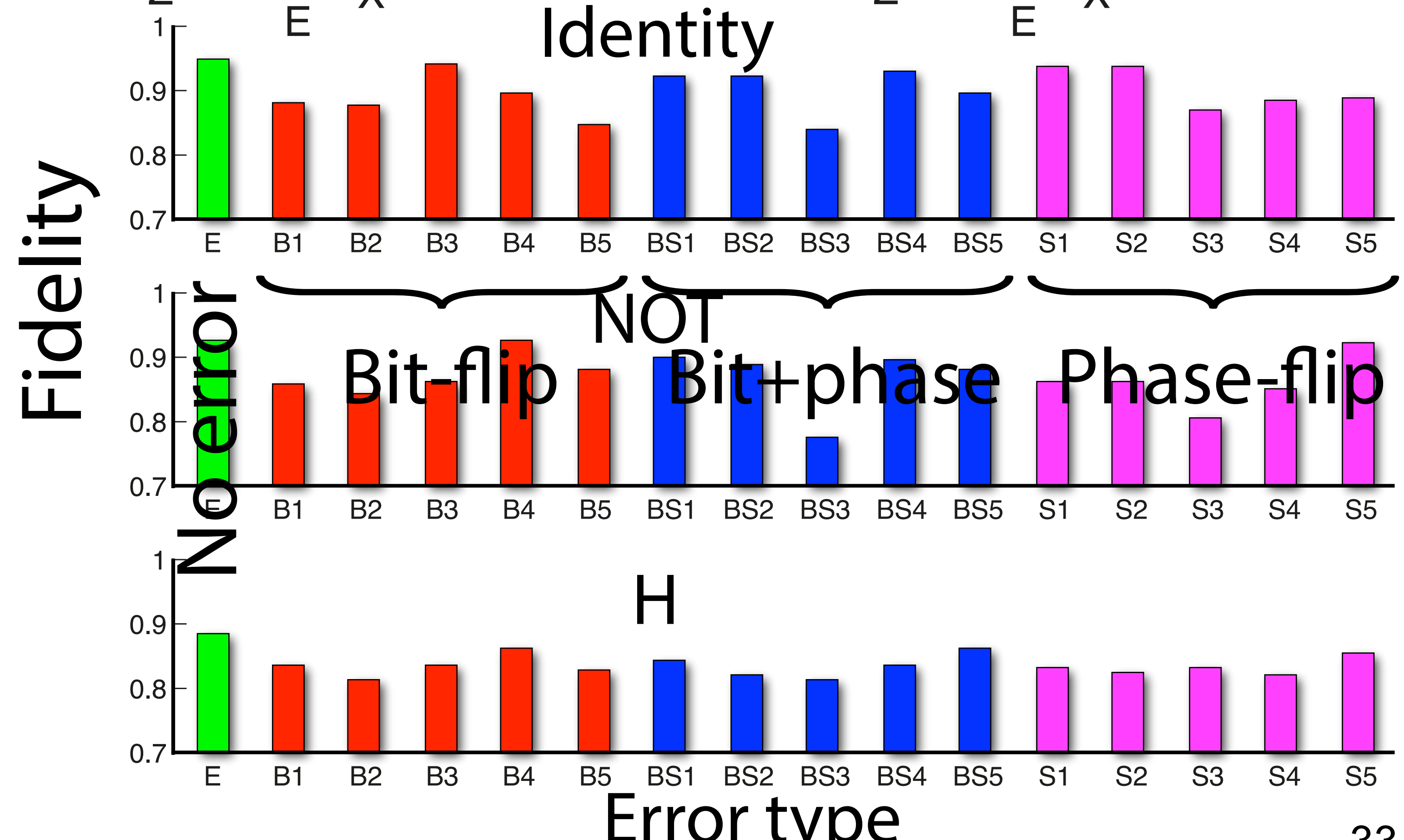


Individual results:

16 possible outcomes:

NoErr

3·5 = 15 different errors



Phys. Rev. Lett.109:100503 (2012).

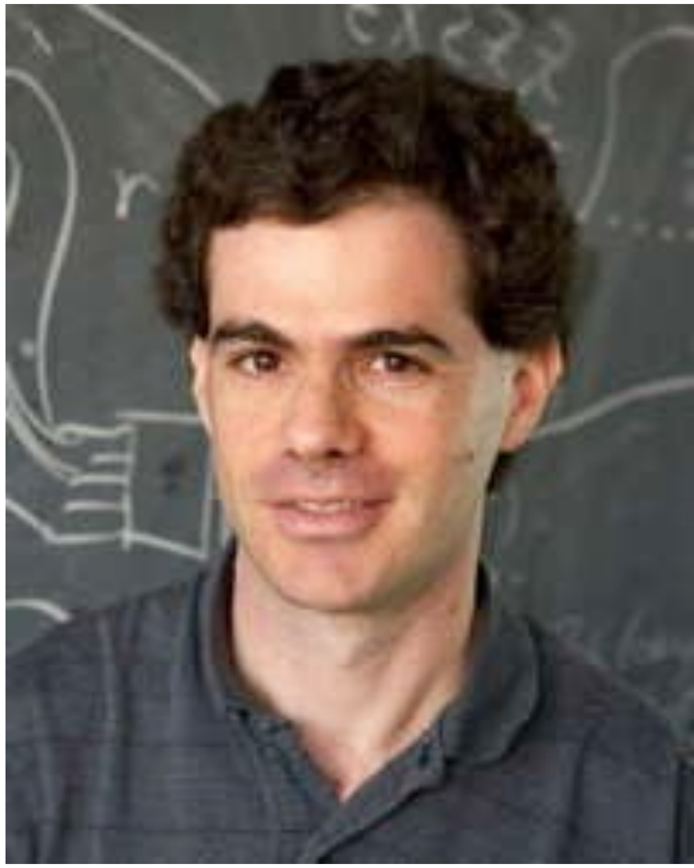
Quantum Error Correction Sonnet

We cannot clone, perforce; instead, we split
Coherence to protect it from that wrong
That would destroy our valued quantum bit
And make our computation take too long.

Correct a flip and phase - that will suffice.
If in our code another error's bred,
We simply measure it, then God plays dice,
Collapsing it to X or Y or Zed.

We start with noisy seven, nine, or five
And end with perfect one. To better spot
Those flaws we must avoid, we first must strive
To find which ones commute and which do not.

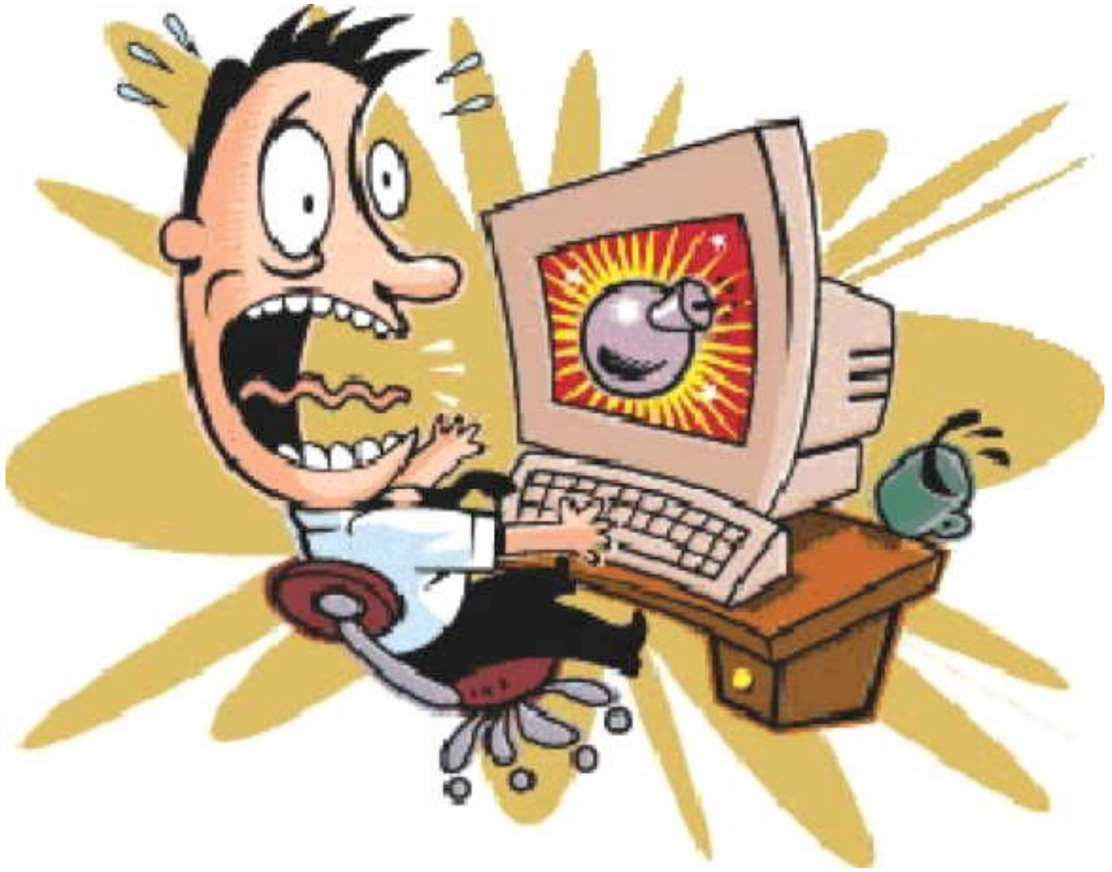
With group and eigenstate, we've learned to fix
Your quantum errors with our quantum tricks.



Daniel Gottesman

Threshold Theorem

QEC can detect and correct certain errors.



It requires additional resources and thus introduces additional error sources.

... but ...

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

Threshold Theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

J. Preskill, 'Reliable quantum computers', Proc. R. Soc. Lond. A 454, 385 (1998).

E. Knill. Quantum computing with realistically noisy devices. Nature 434, 39 (2005).

P. Aliferis, D. Gottesman, and J. Preskill. Accuracy threshold for postselected quantum computation. Quantum Information and Computation 8, 181 (2008).

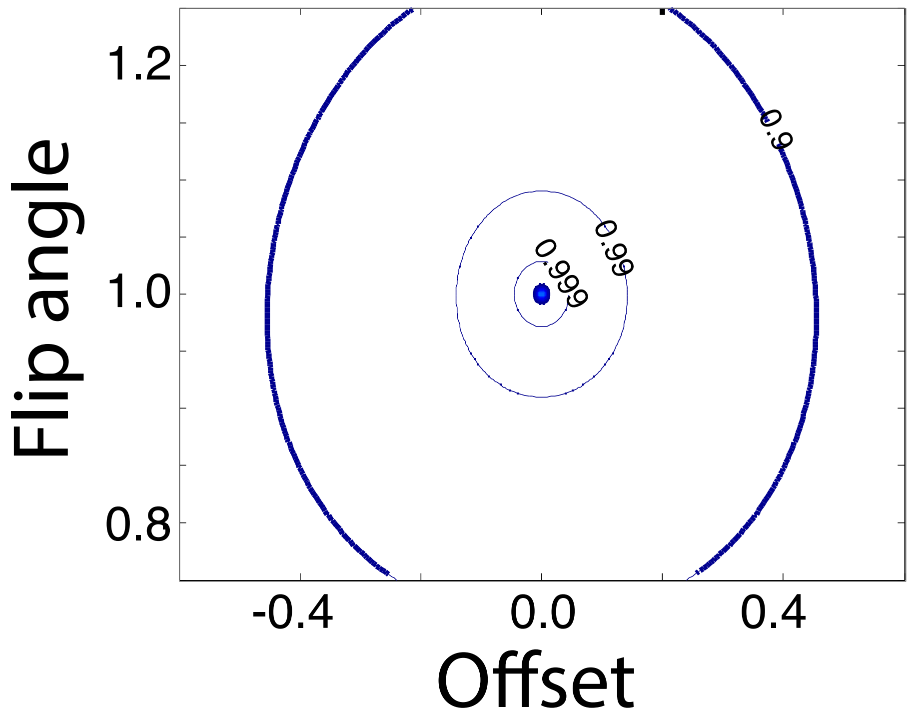
Fighting Errors

Threshold must be reached: 10^{-2} .. 10^{-4}

- Optimize the classical apparatus that controls the quantum system to make the gate operations as perfect as possible.
- Design gate operations in such a way that errors in experimental parameters tend to cancel rather than amplify.
- Use error correction schemes.
- Store the information in areas of the Hilbert space that are least affected by the interaction between the system and its environment.
- Use active schemes for decoupling the system from the environment, such as dynamical decoupling.

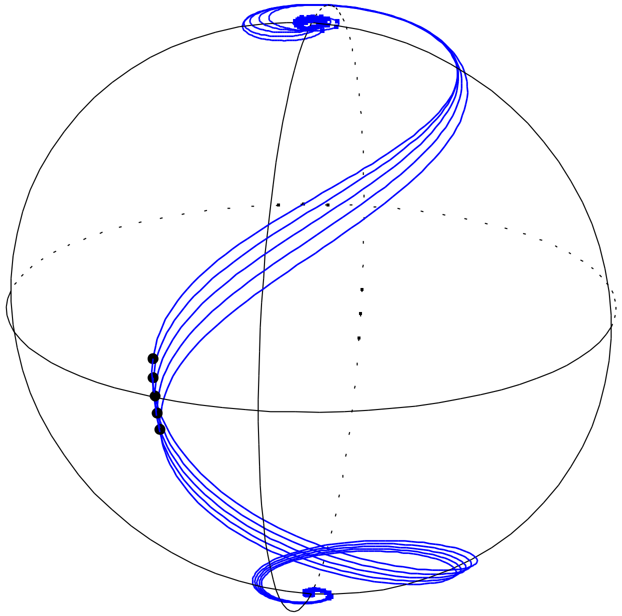
All schemes must be combined!

Counter-Strategy

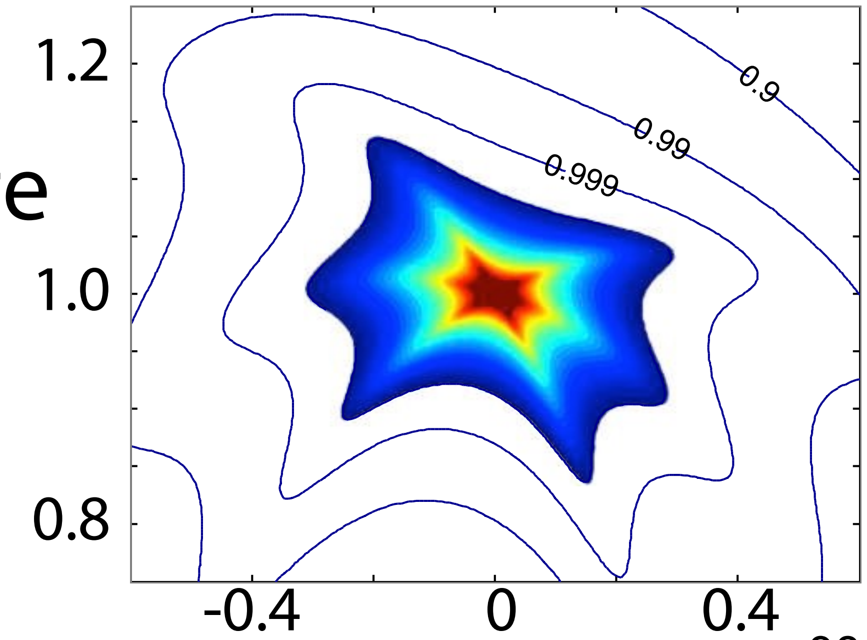


Design gate operations such that errors in experimental parameters cancel rather than amplify each other

hyperbolic secant pulse

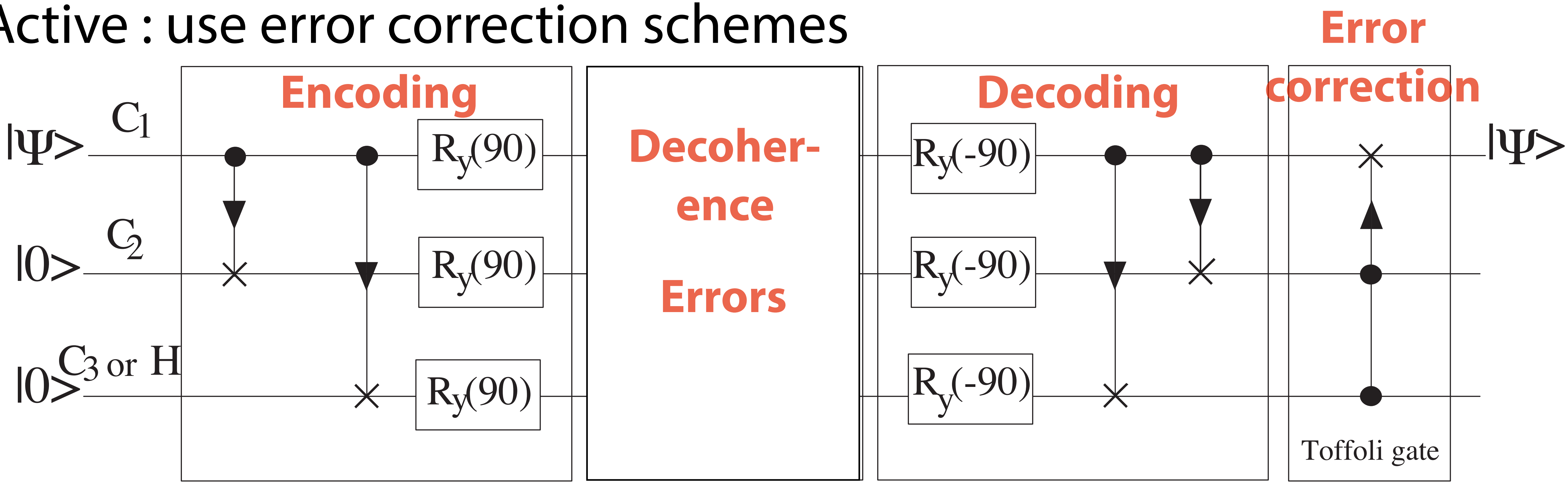


Composite pulse

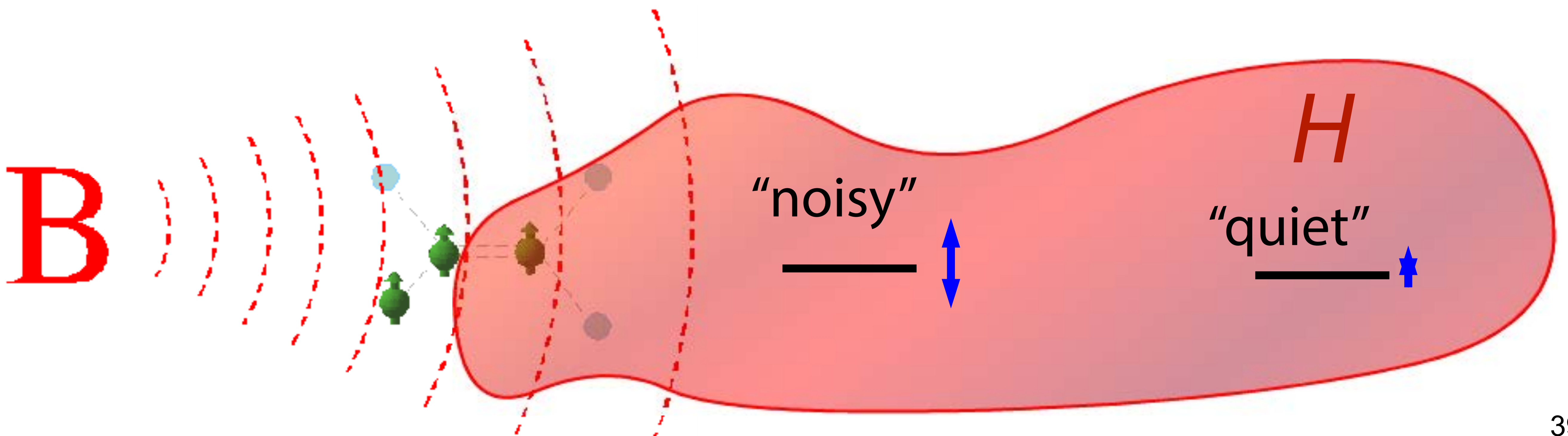


Active and Passive

Active : use error correction schemes



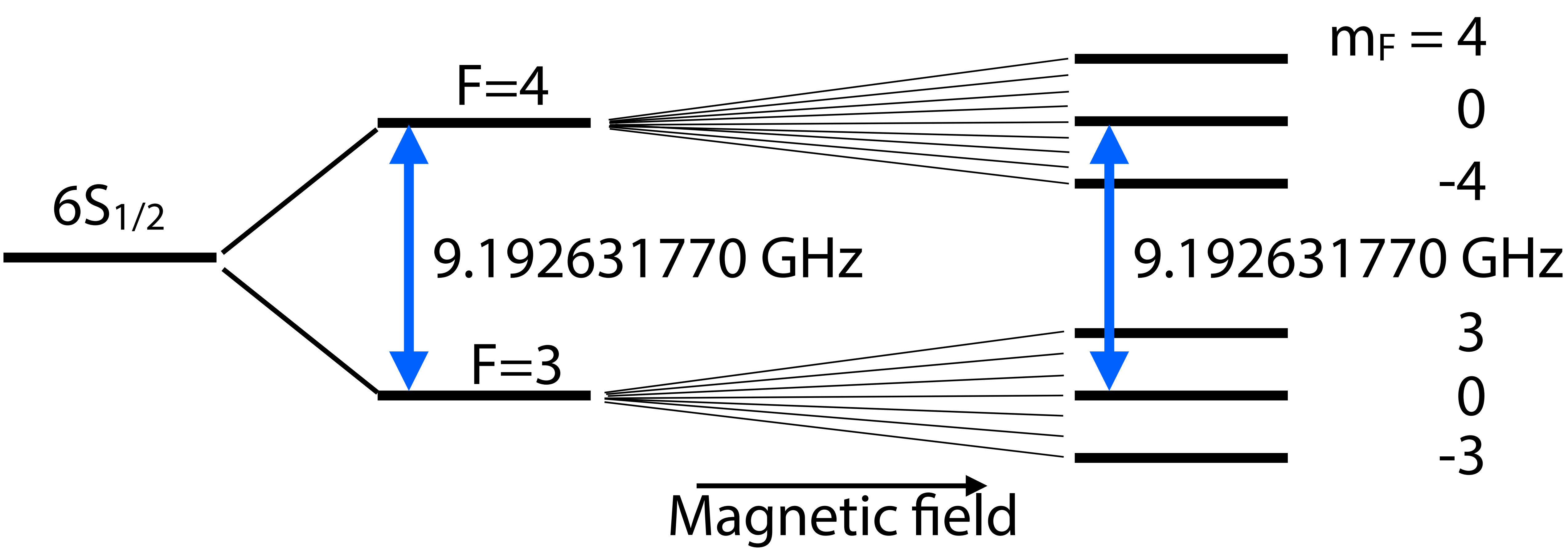
Passive : store information in "quiet" parts of Hilbert space



Clock Transitions

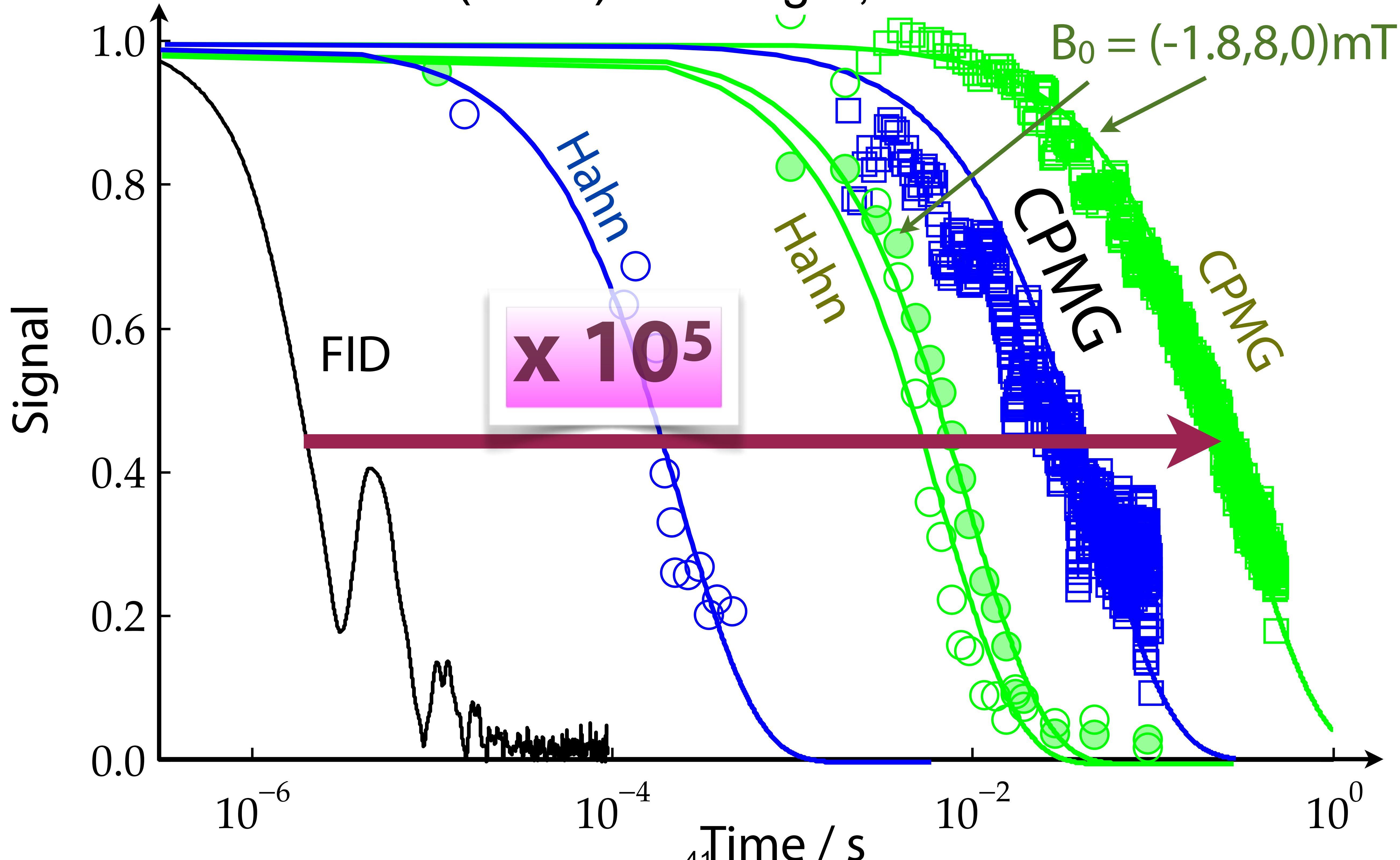
Definition

1 second = 9192631770 periods of the
^{133}Cs, F = 3, m_F = 0 ↔ F = 4, m_F = 0 hyperfine transition



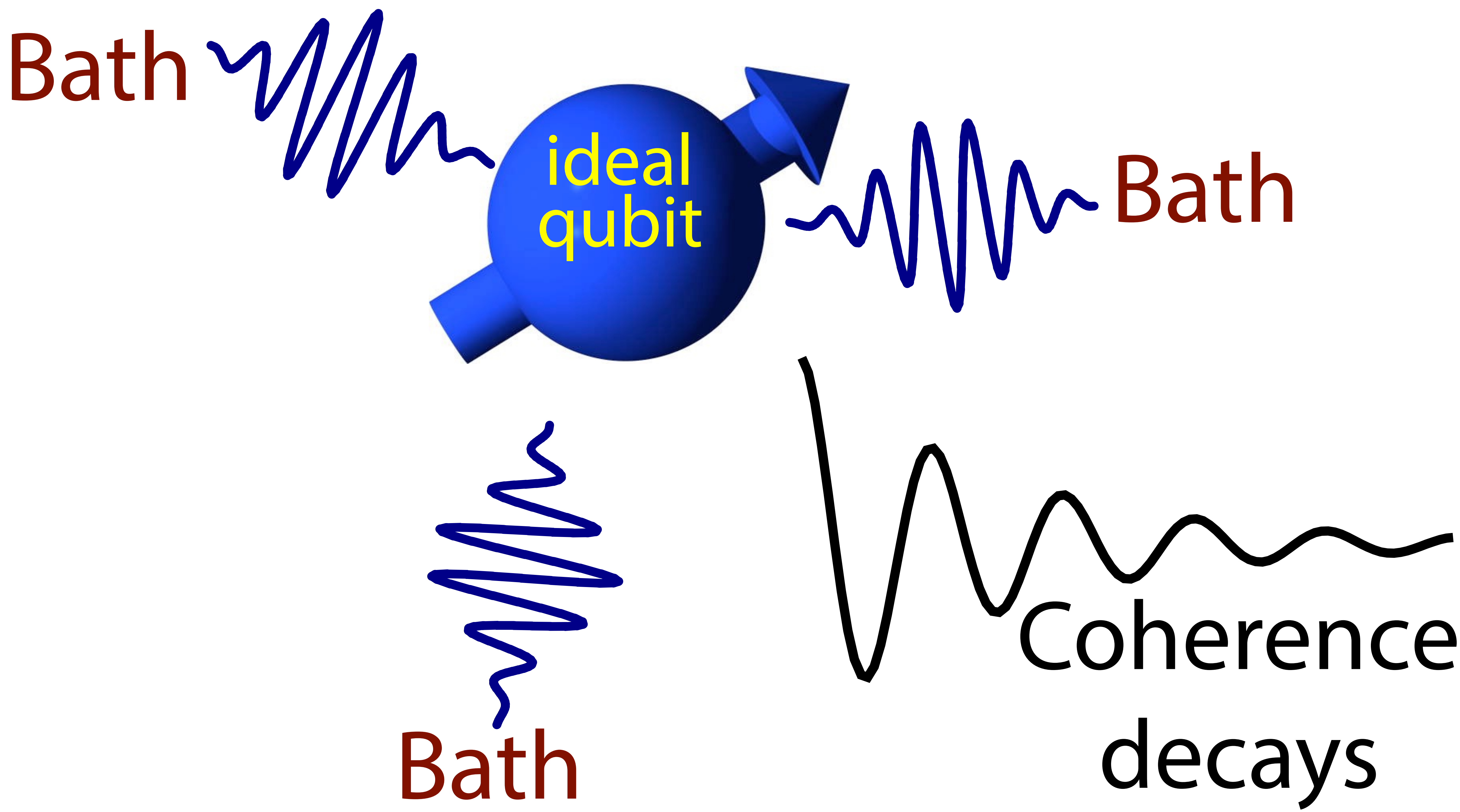
Keeping a Photon Alive

store it in $Pr^{3+}:La_2(WO_4)_3$ + longer; current record : 6 hours

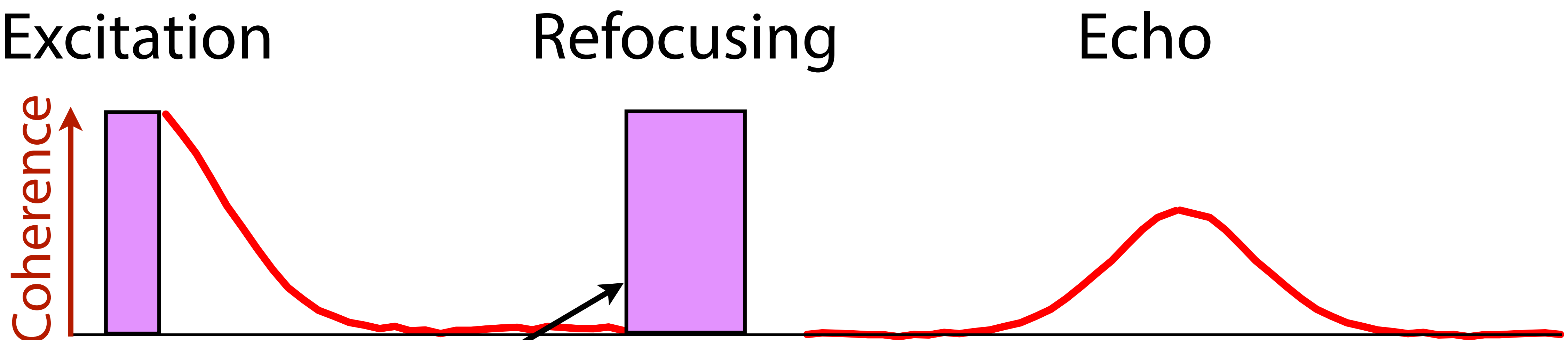


Fighting Decoherence

Decoherence a.k.a. Relaxation



Refocusing



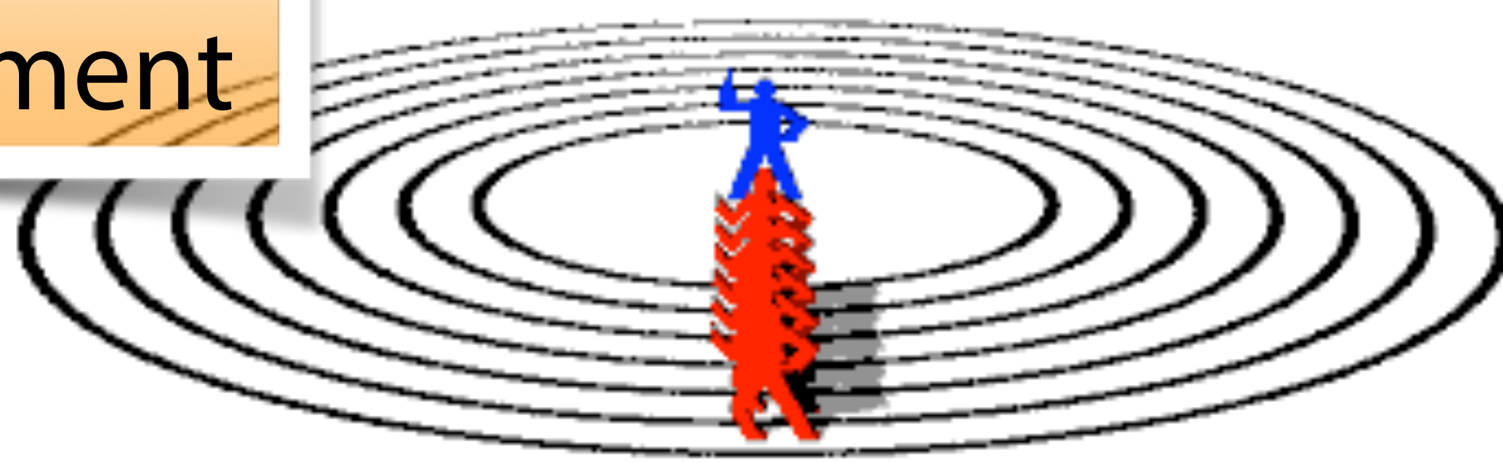
inverts H_{SE}

PHYSICAL REVIEW VOLUME 80, NUMBER 4 NOVEMBER 15, 1950

Spin Echoes*†

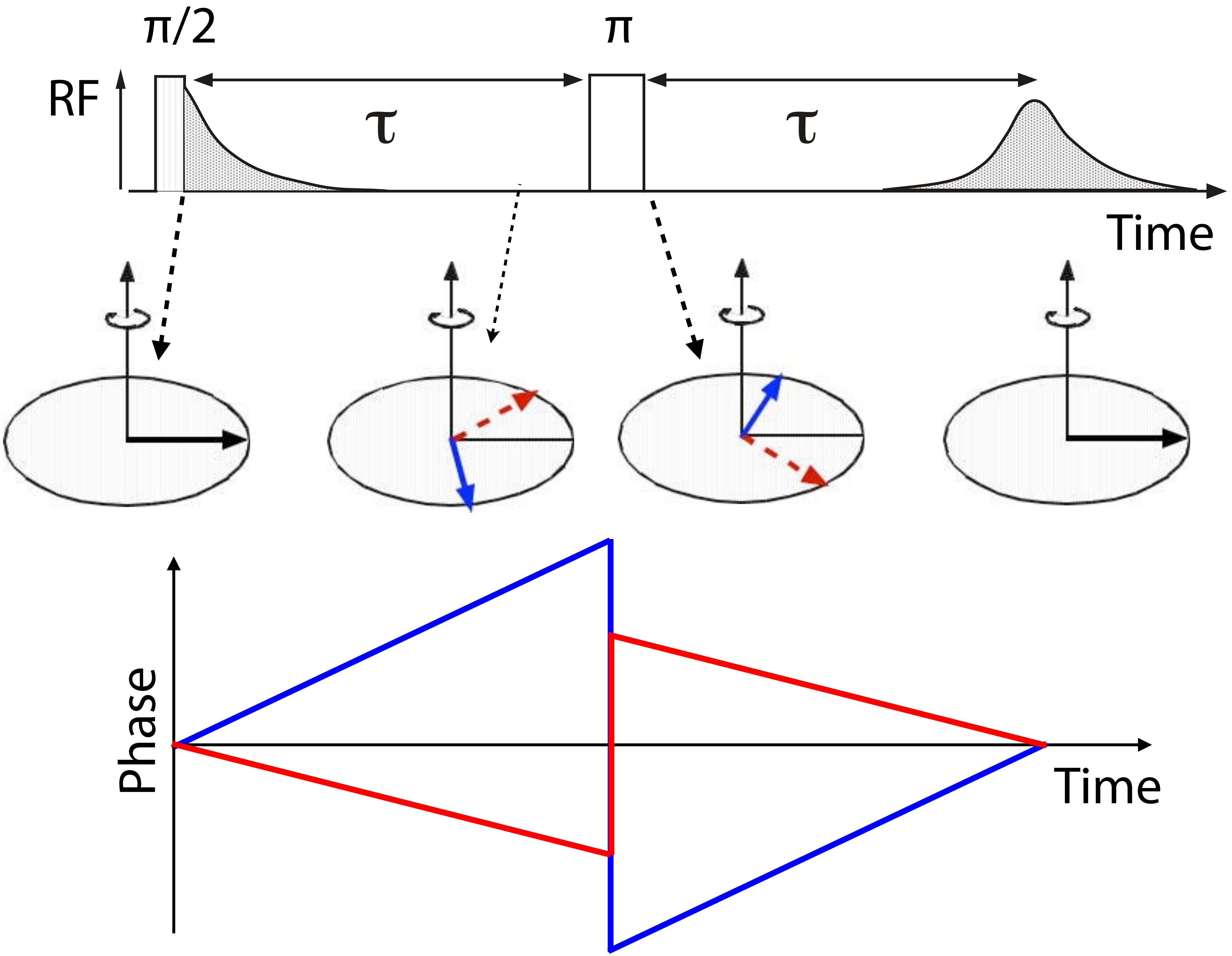
E. L. HAHN‡
Physics Department, University of Illinois, Urbana, Illinois
(Received May 22, 1950)

Can be applied to any qubit in a dephasing environment

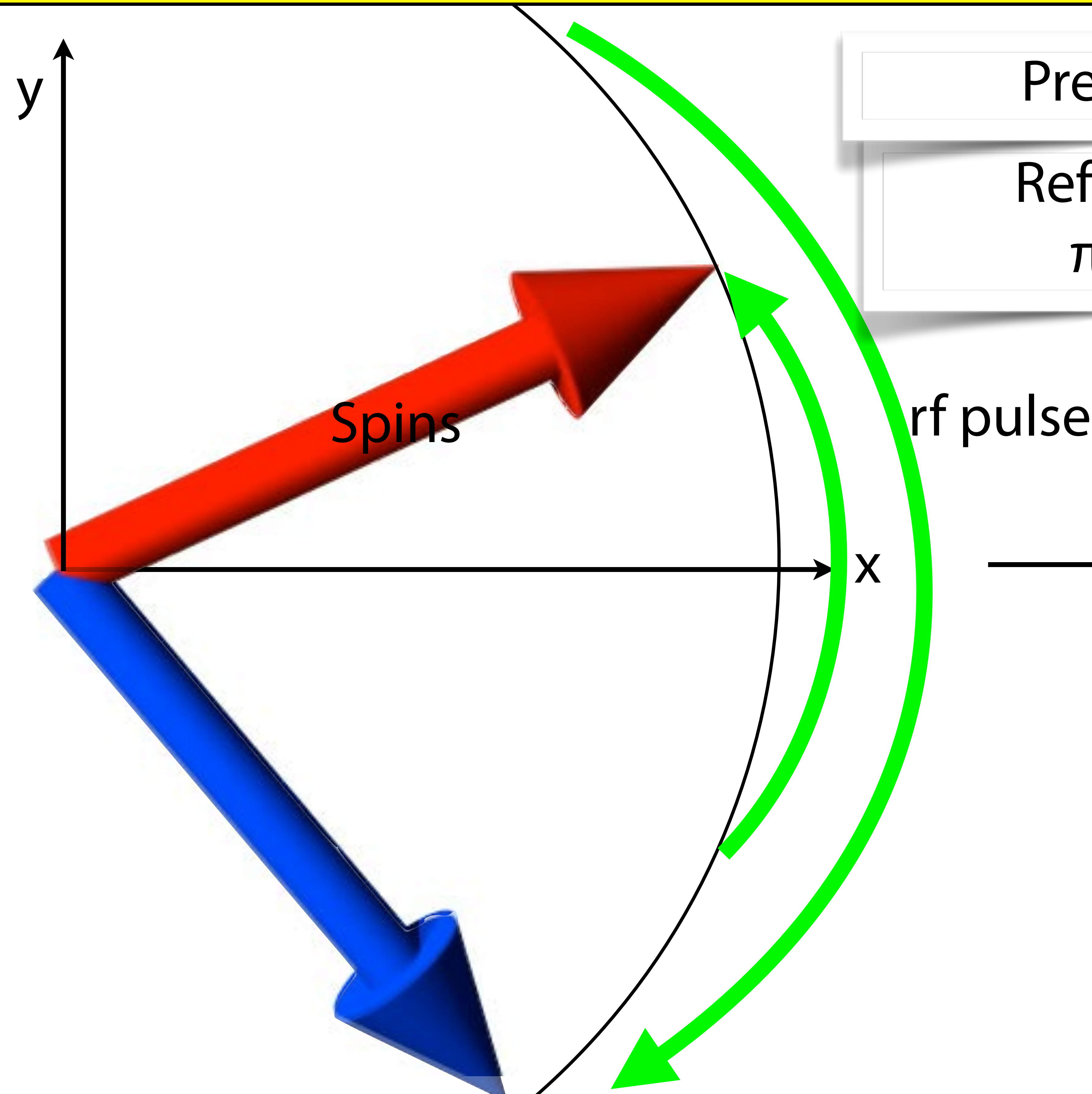


Other examples: photon echoes, charge qubits, ...

Hahn Echo

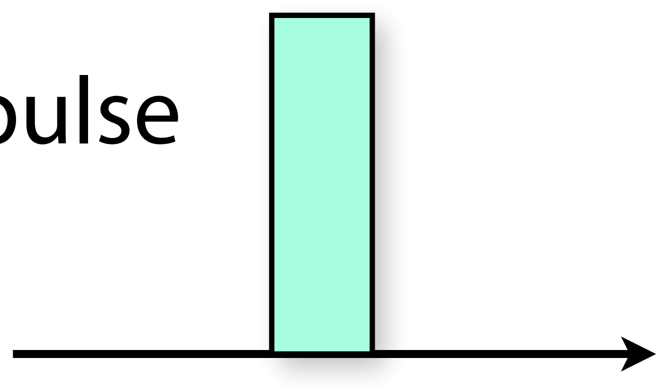


Dephasing / Rephasing



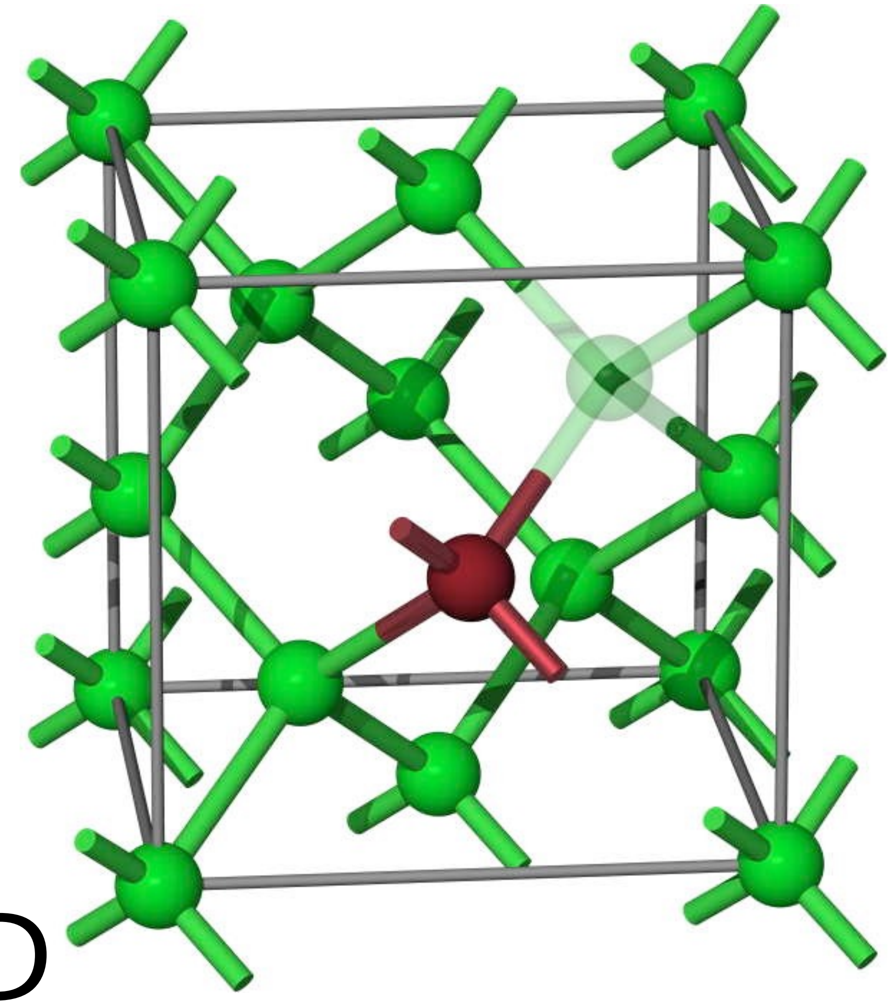
Precession

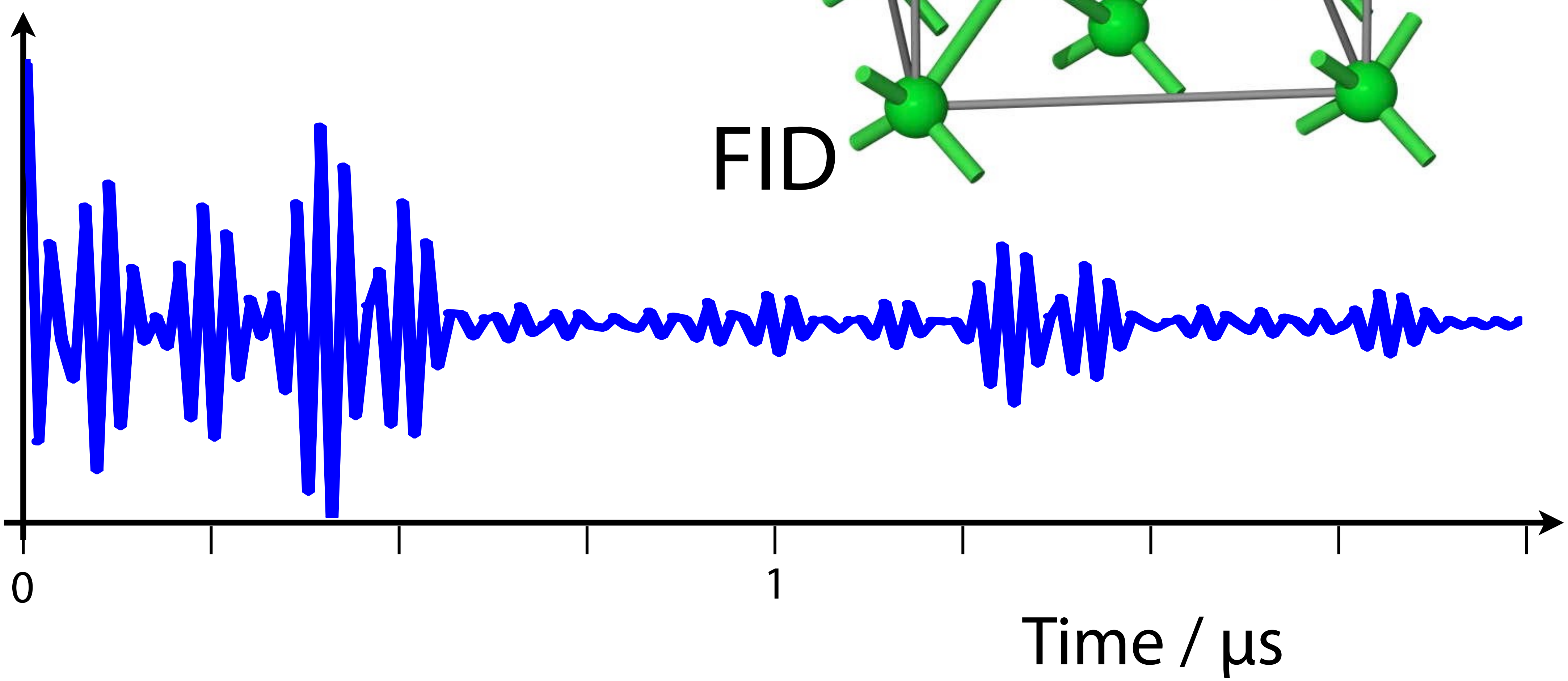
Refocussing:
π_x pulse



Single Spin Hahn Echo

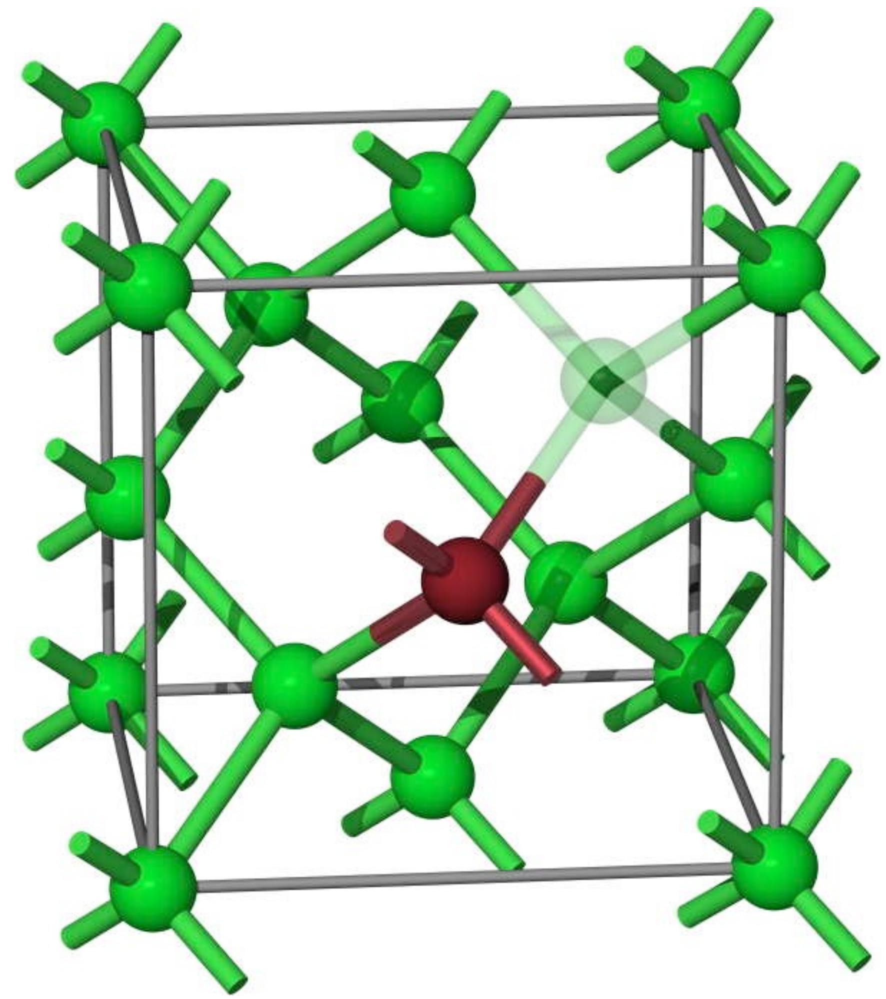
Diamond NV center

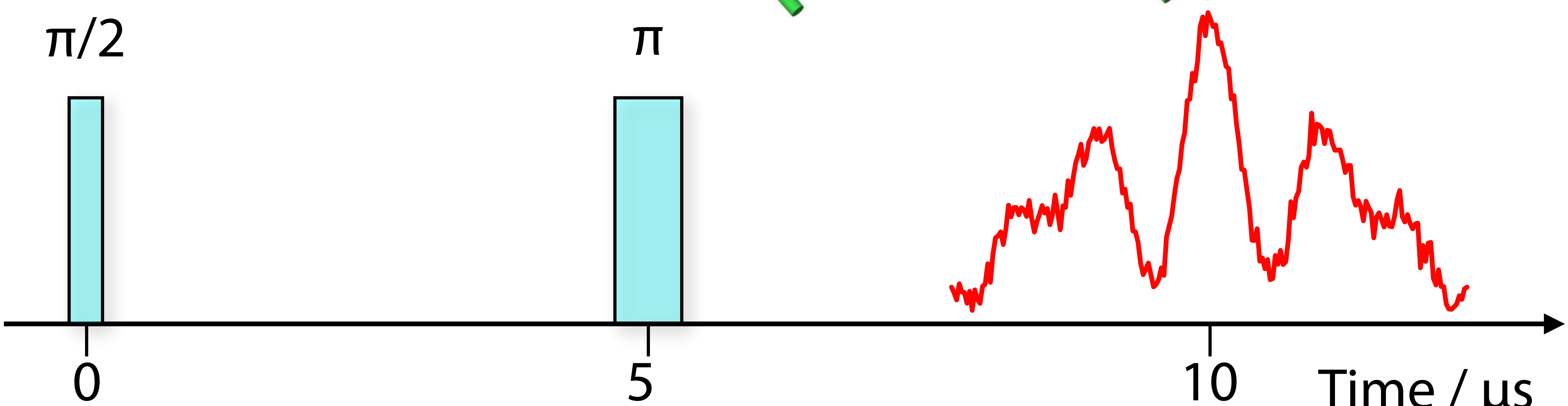




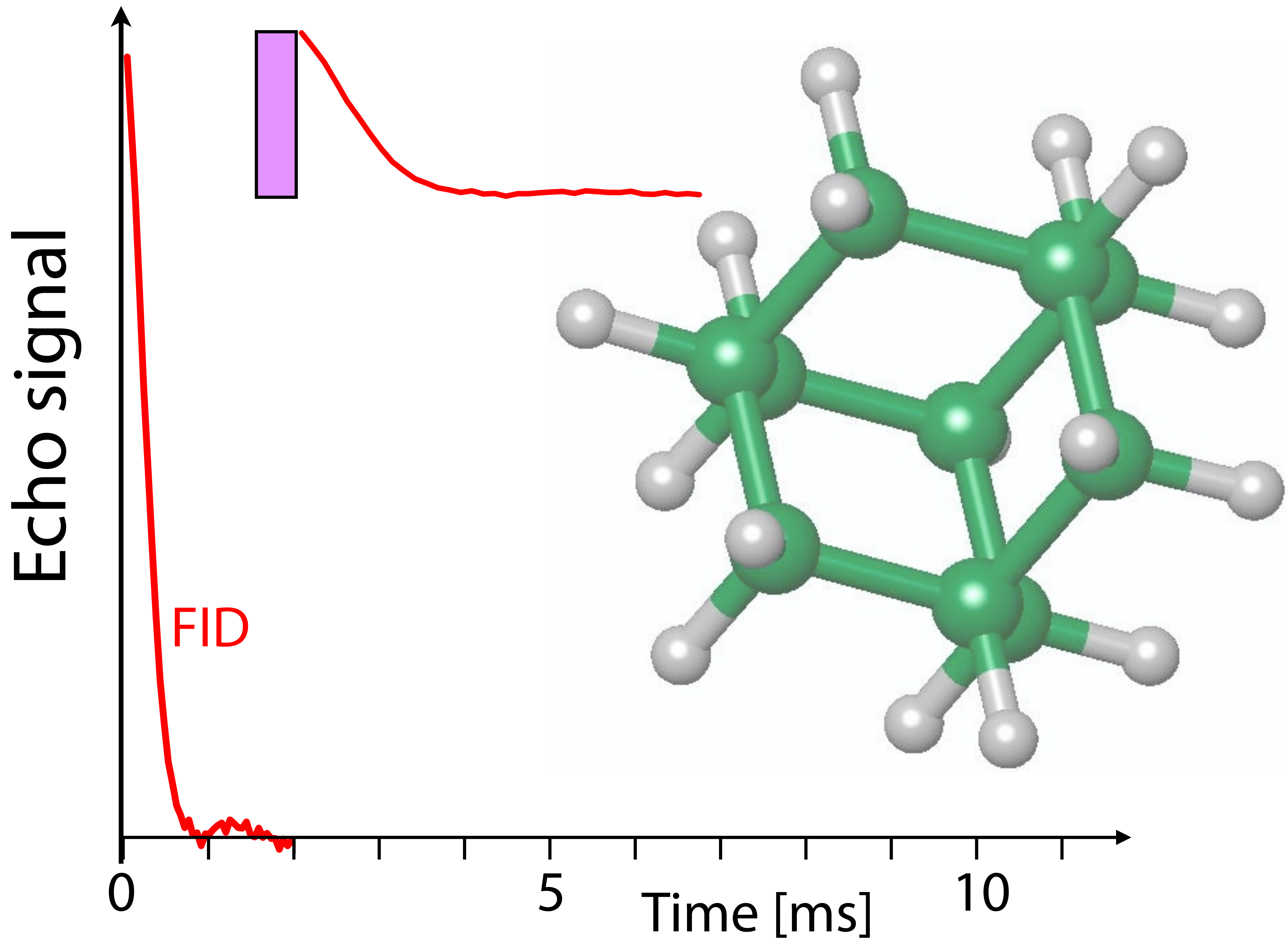
Single Spin Hahn Echo

Diamond NV center

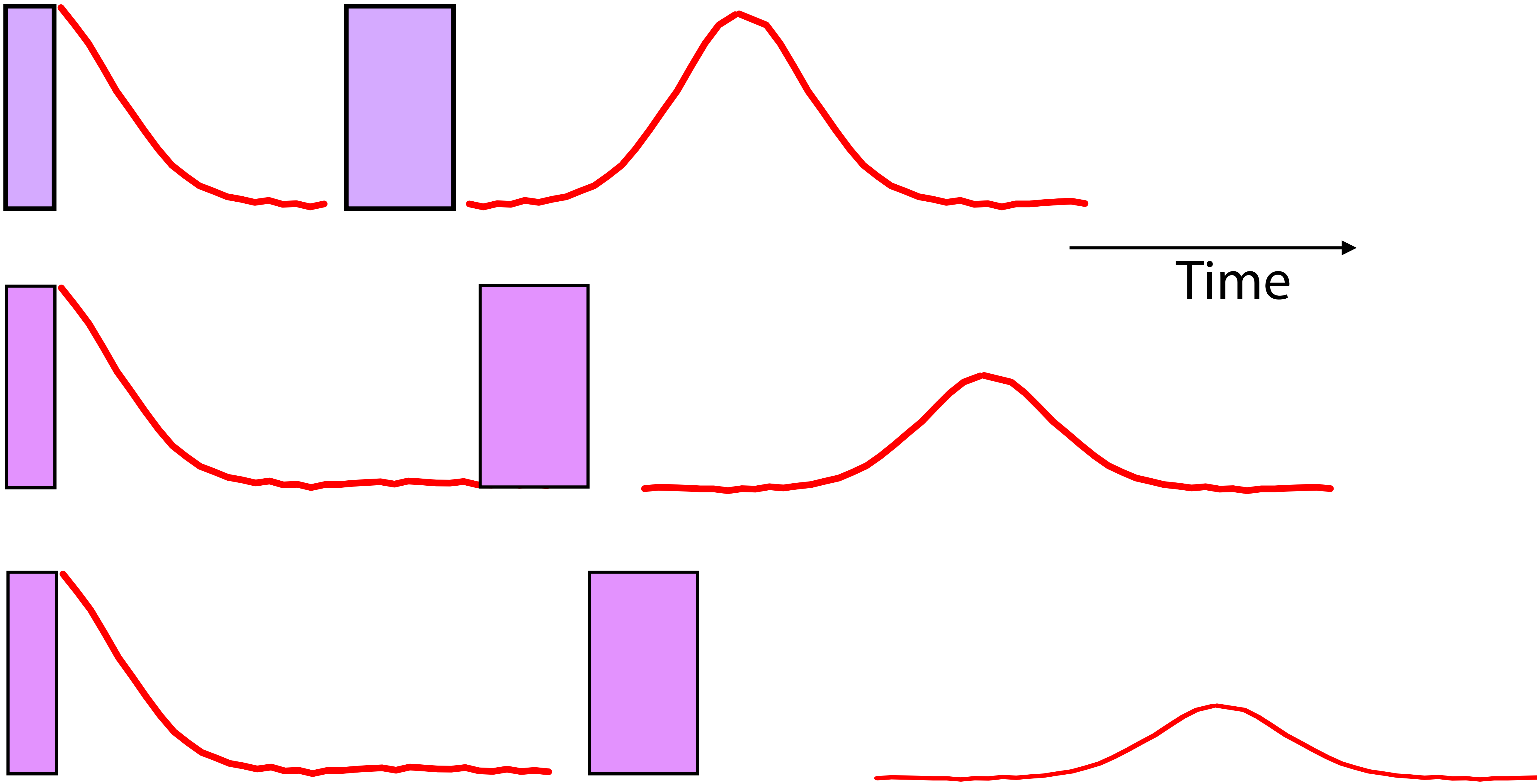




Nuclear Spin Qubits

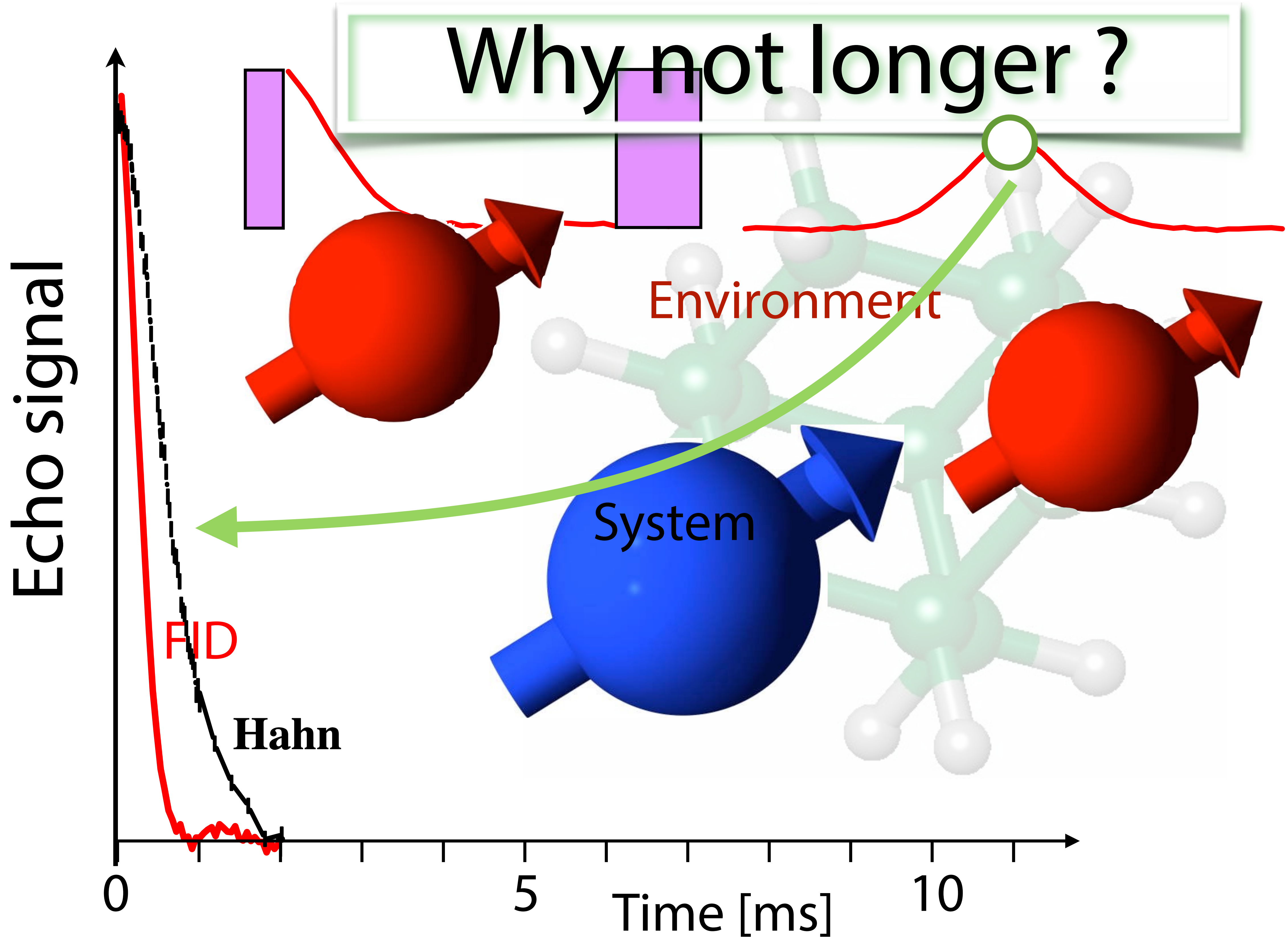


Reversing Dephasing

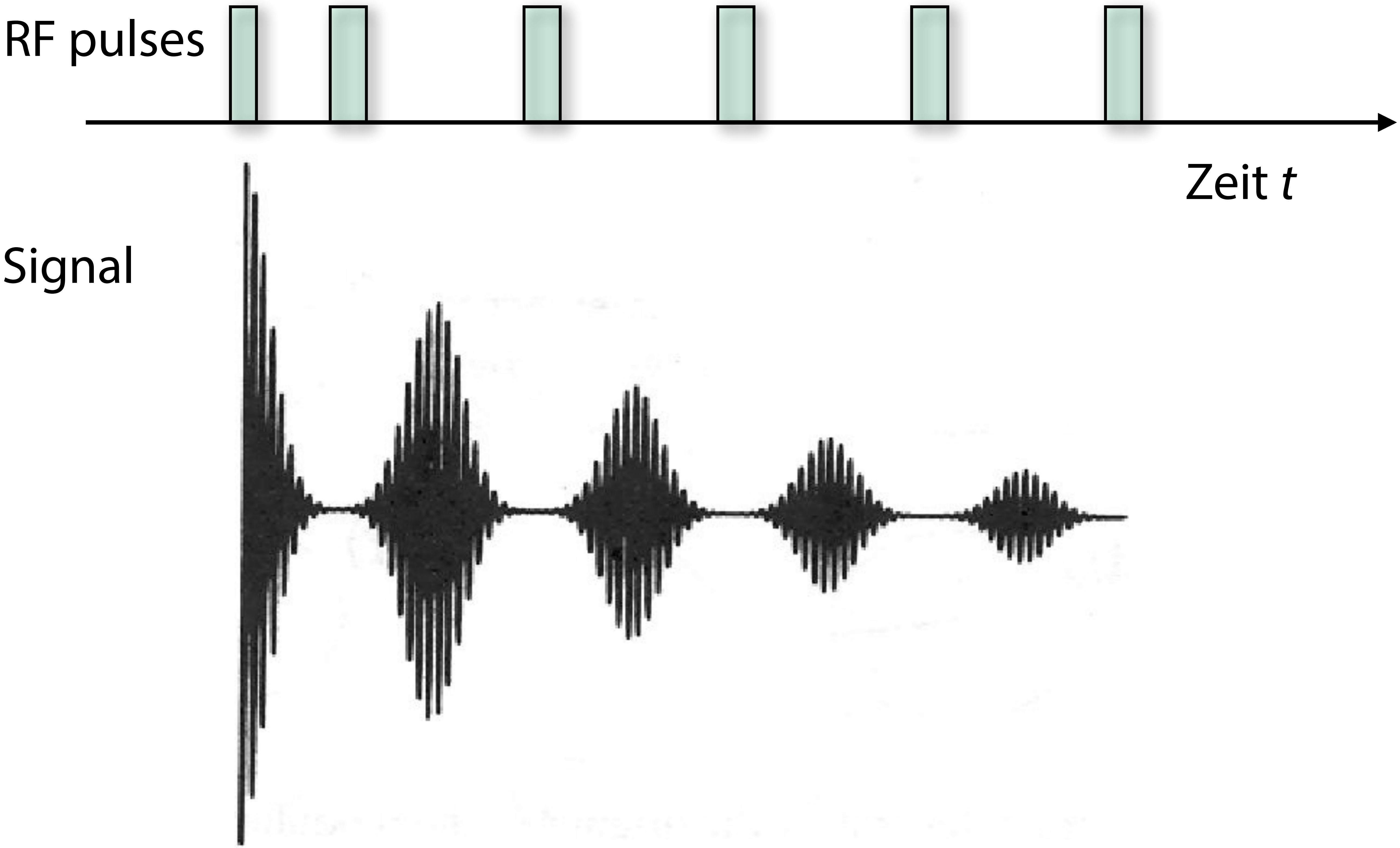


Effectiveness decreases with time

Echo

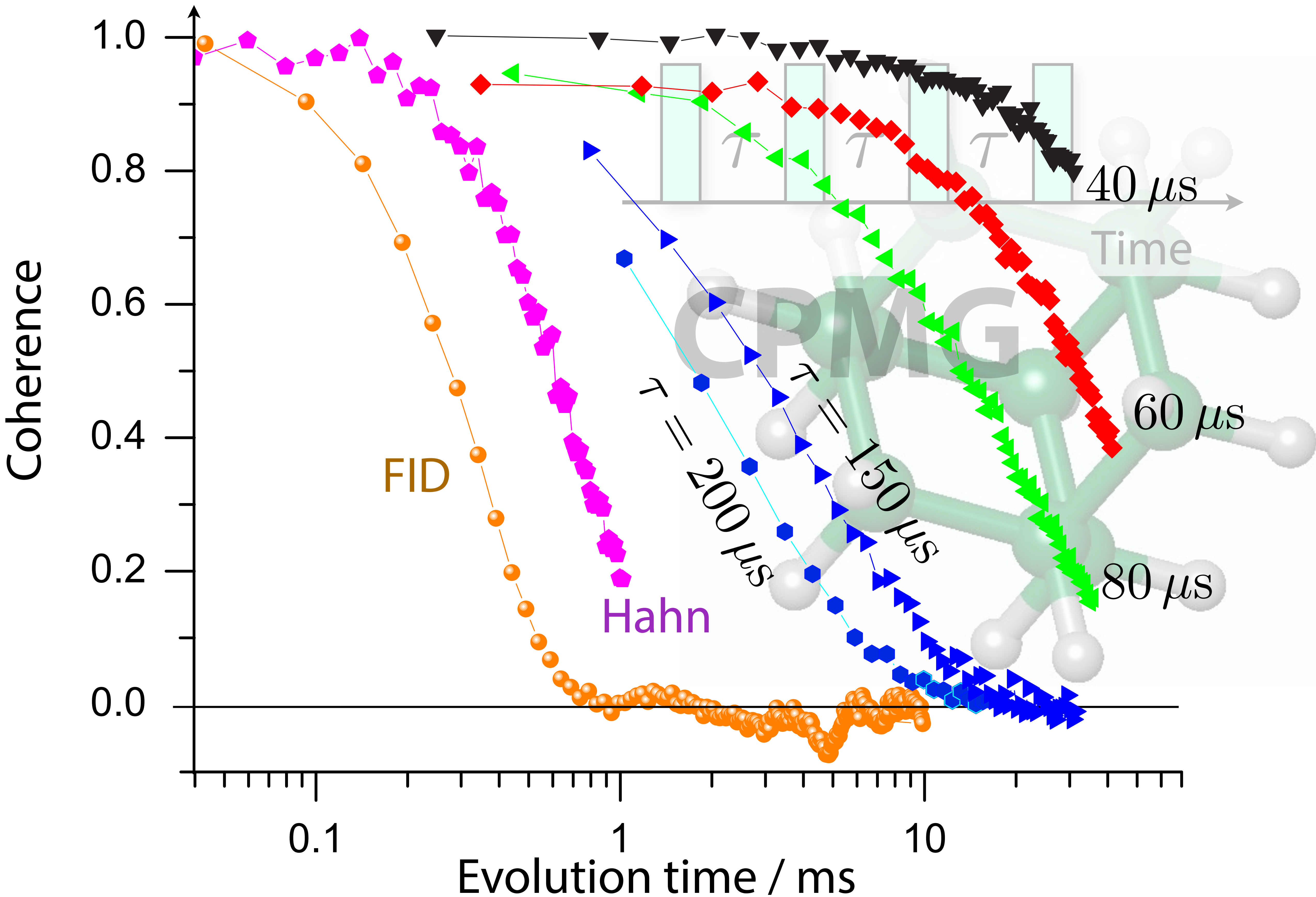

Why not longer ?
Environment
System
Echo signal
FID
Hahn
0
5
10
Time [ms]

Decoupling Sequence

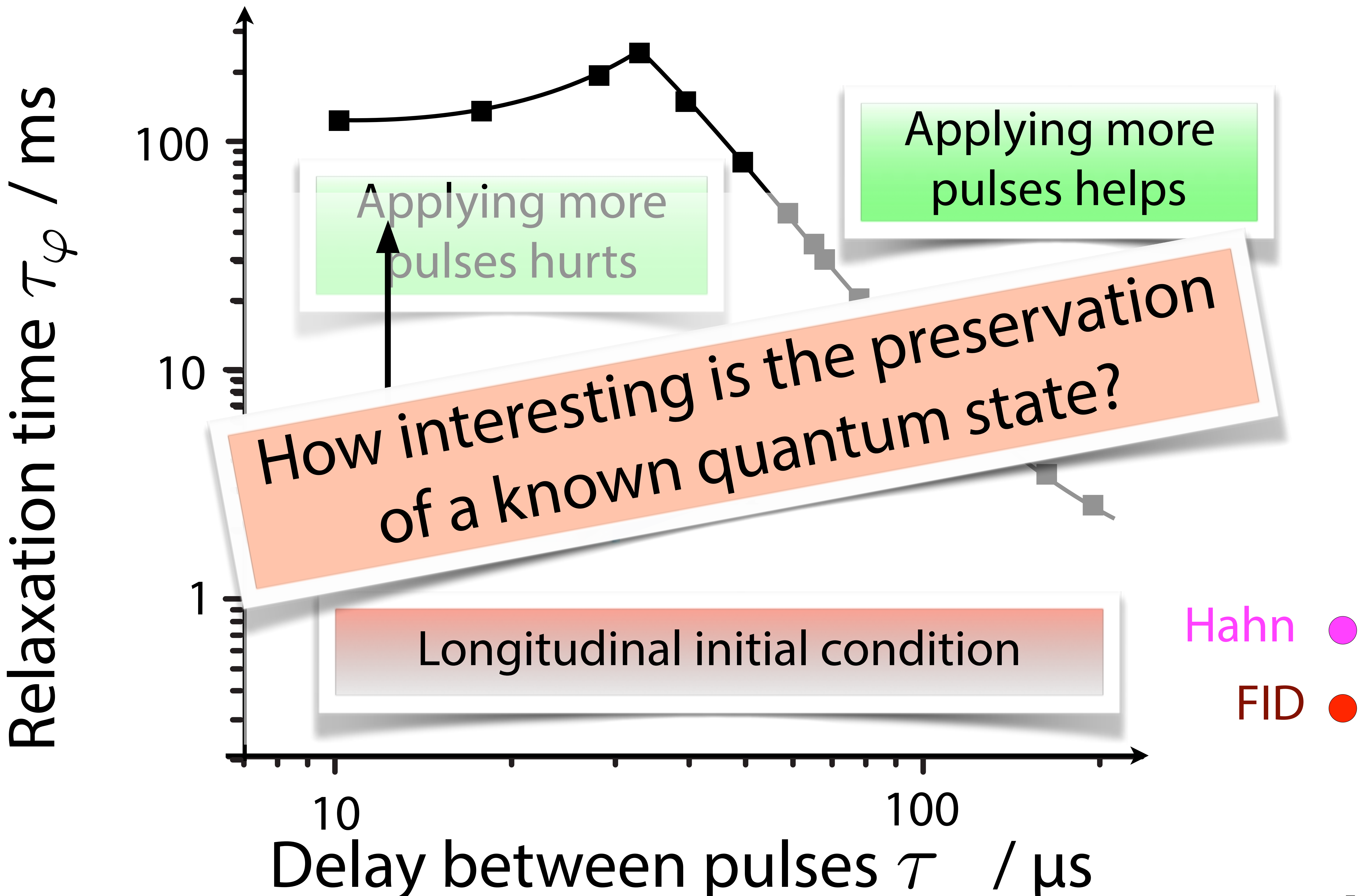


H. Carr and E. Purcell. Phys. Rev. 94, 630 (1954).

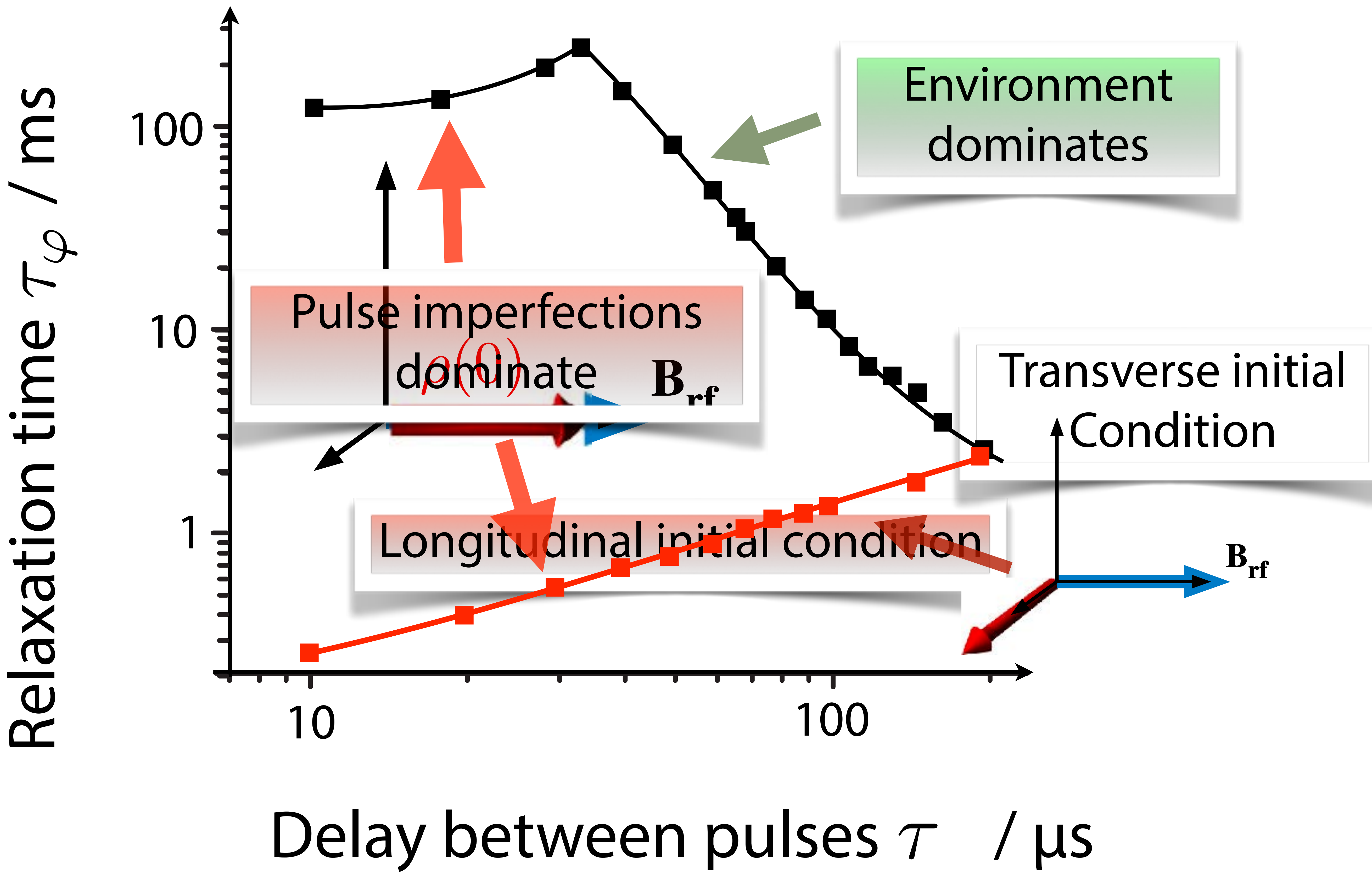
Performance of DD



Dependence on Delay



Dependence on Input



Robust Dynamical Decoupling

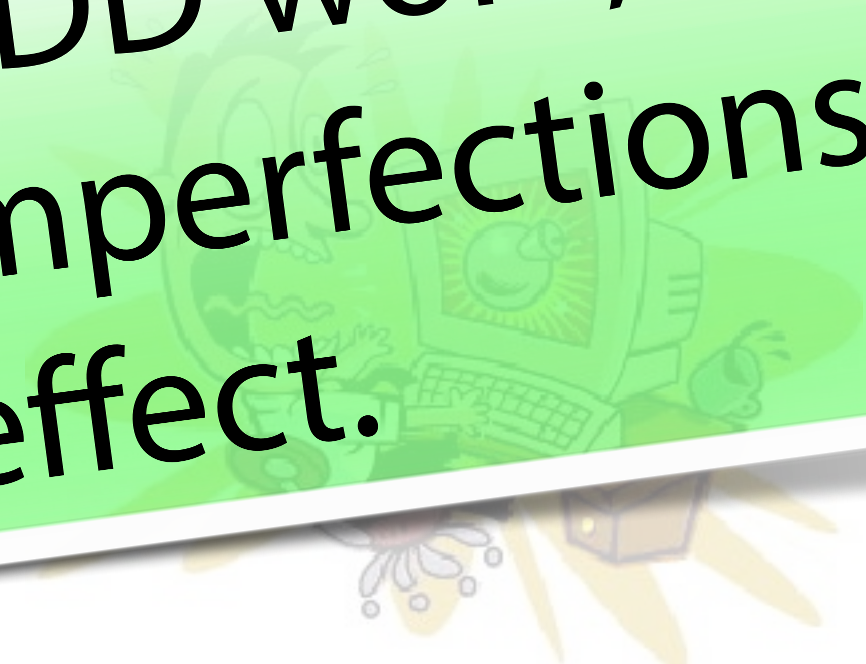
The problem

Dynamical decoupling requires ideal many inversion pulses

Real pulses have i

- Finite strength
- Finite duration
- Flip angle errors
- Offset
- ??

To make experimental DD work, we must consider pulse imperfections and reduce their effect.



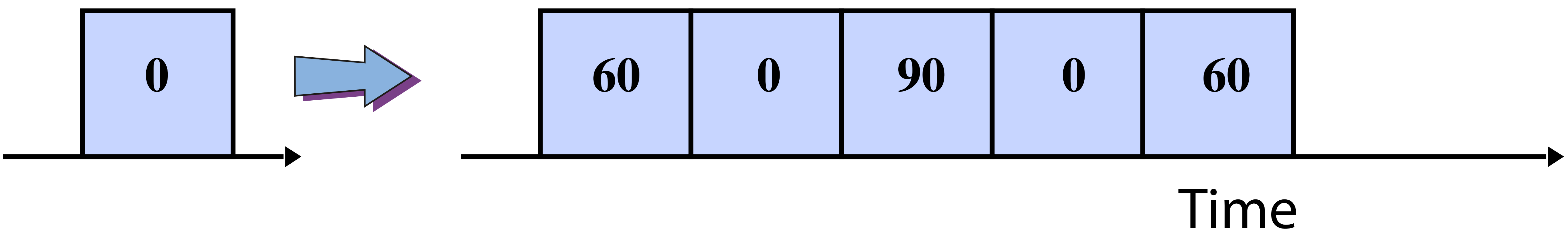
Towards a solution

Robust pulses and robust sequences are insensitive to pulse imperfections

Robust Pulse

Composite pulses = robust pulses

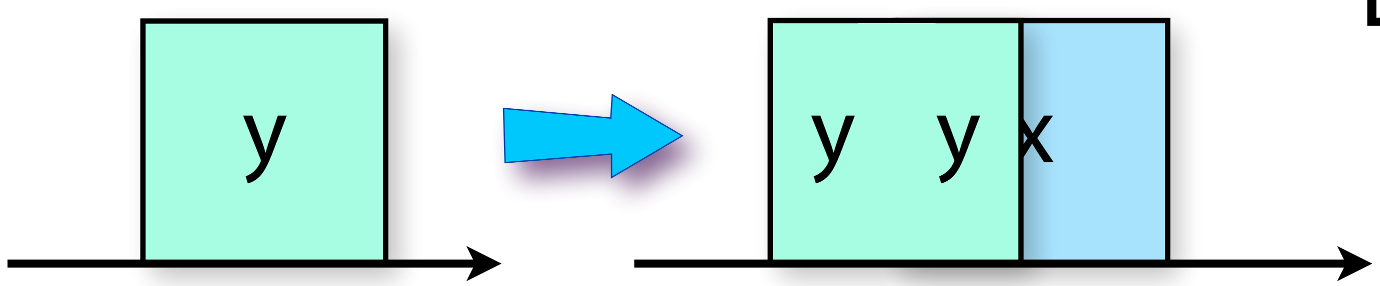
“Knill pulse”



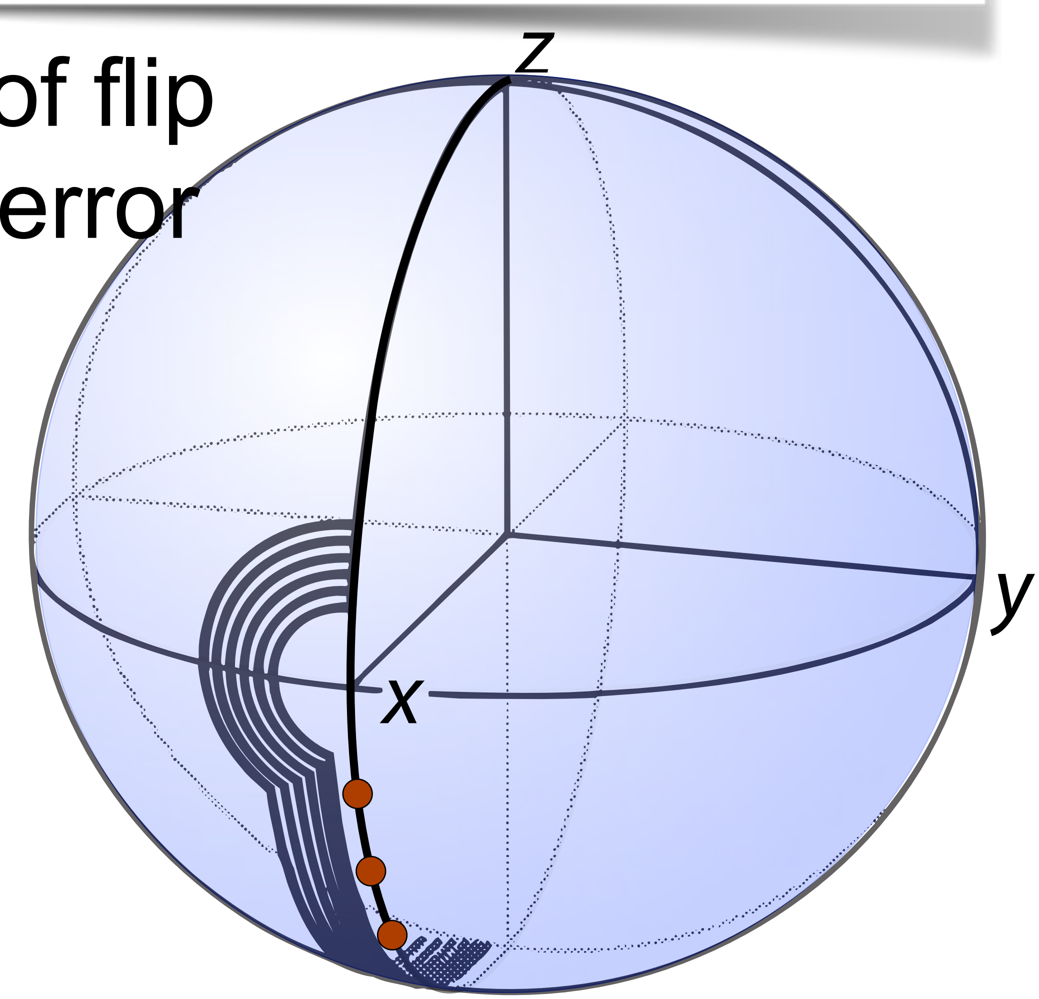
R. Tycko, A. Pines, and J. Guckenheimer, J. Chem. Phys. 83, 2775 (1985).

Error Compensation

Composite pulses = robust pulses = compensated pulses

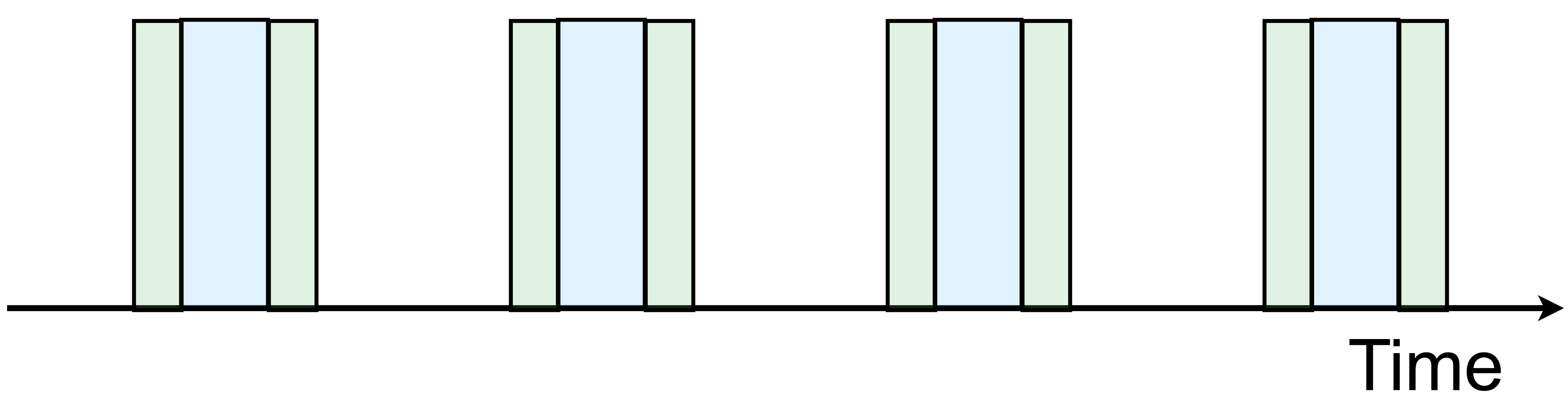


Effect of flip angle error

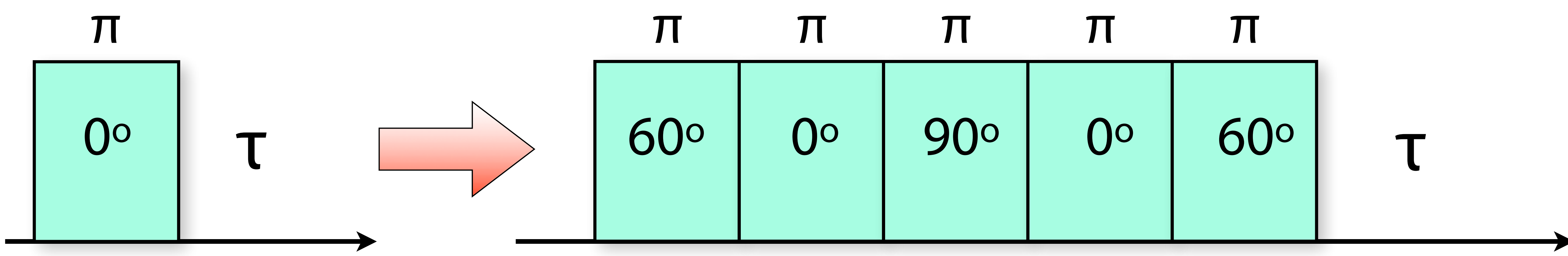


Levitt and Freeman, J. Magn. Reson. 33, 473 (1979).
Khodjasteh and Viola, Phys. Rev. A, 80, 032314 (2009).

Making DD sequences robust:



A Robust Pulse



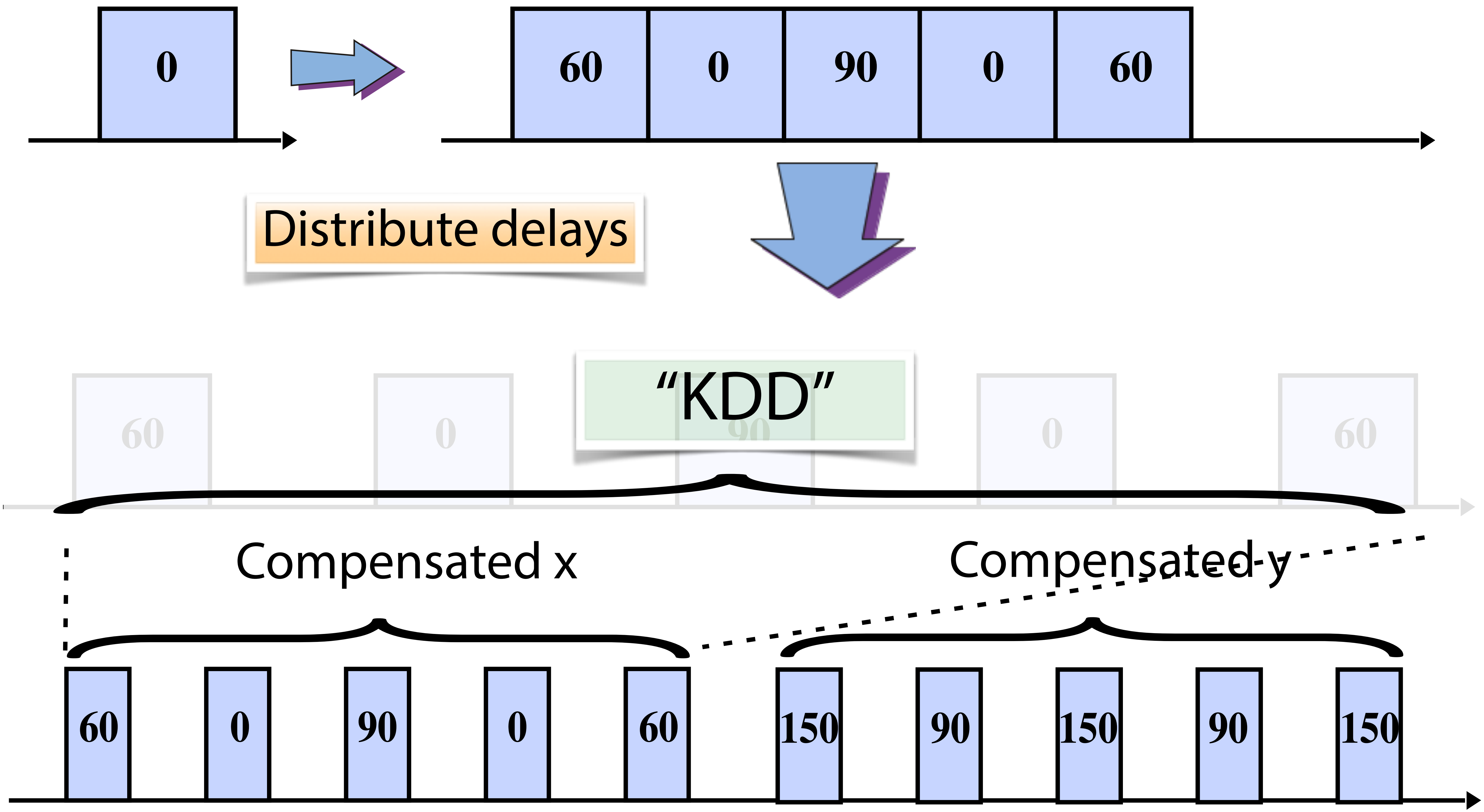
- Eliminates pulse imperfections

- Same decoupling performance

- Power deposition 5× higher

KDD

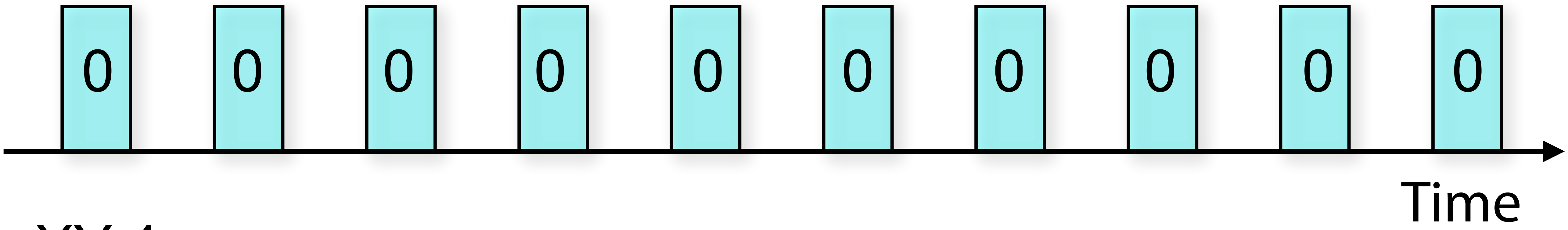
Composite pulses = robust pulses



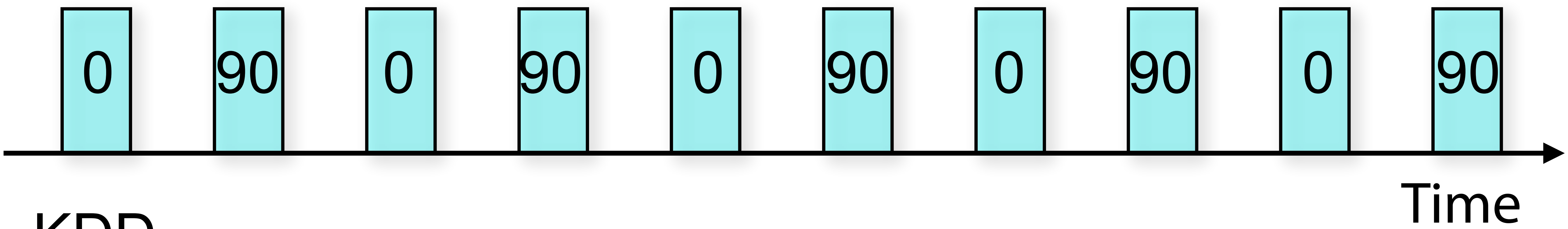
Robust Sequences

Concept can be extended to sequences of pulses

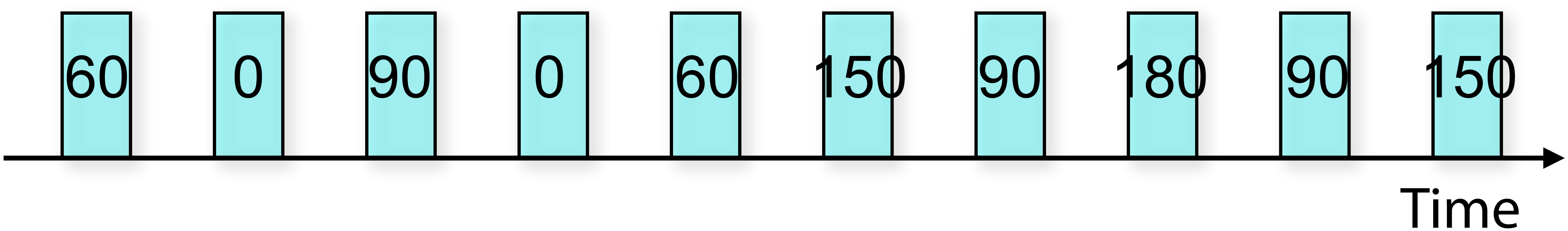
CPMG



XY-4

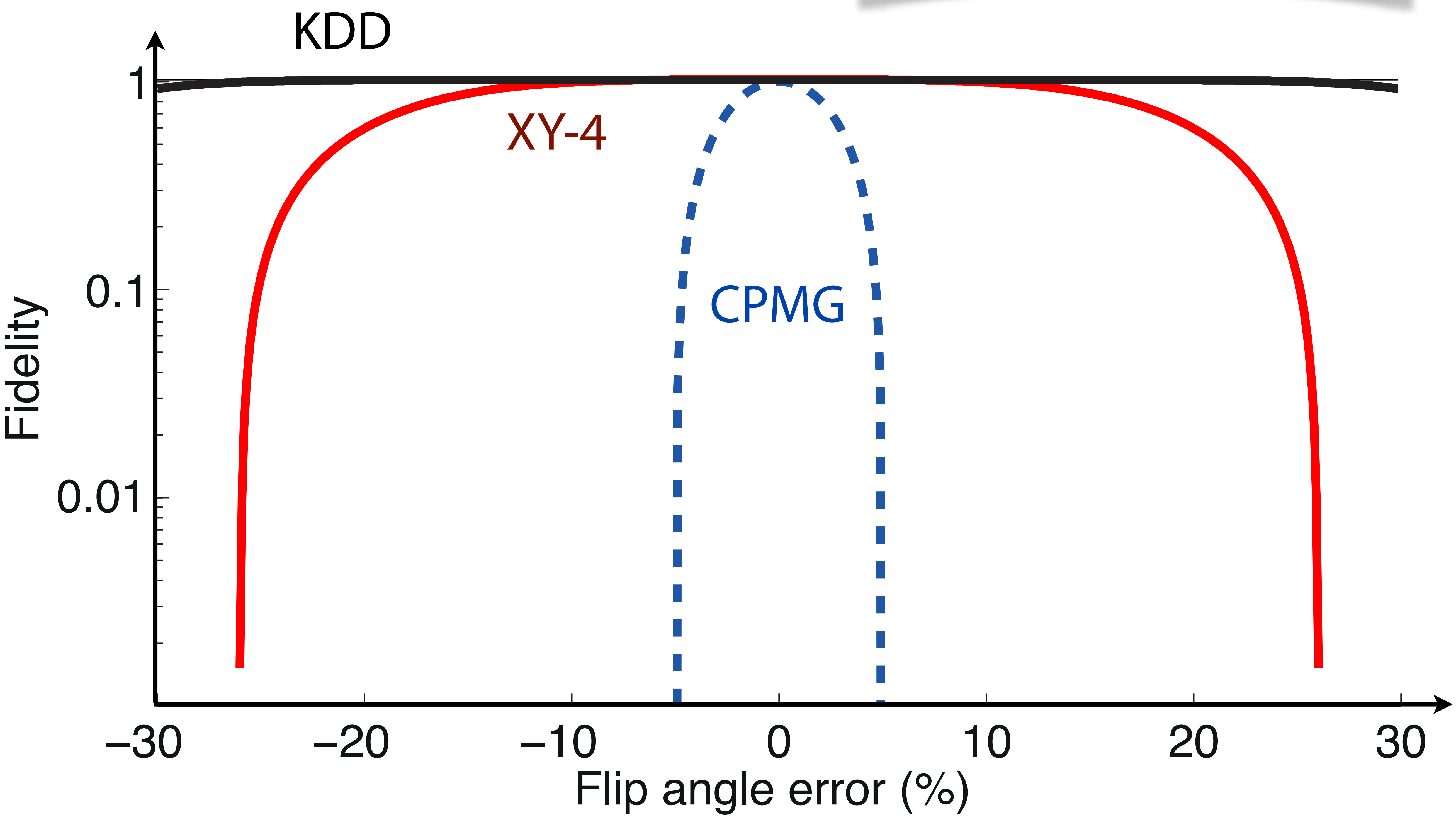


KDD



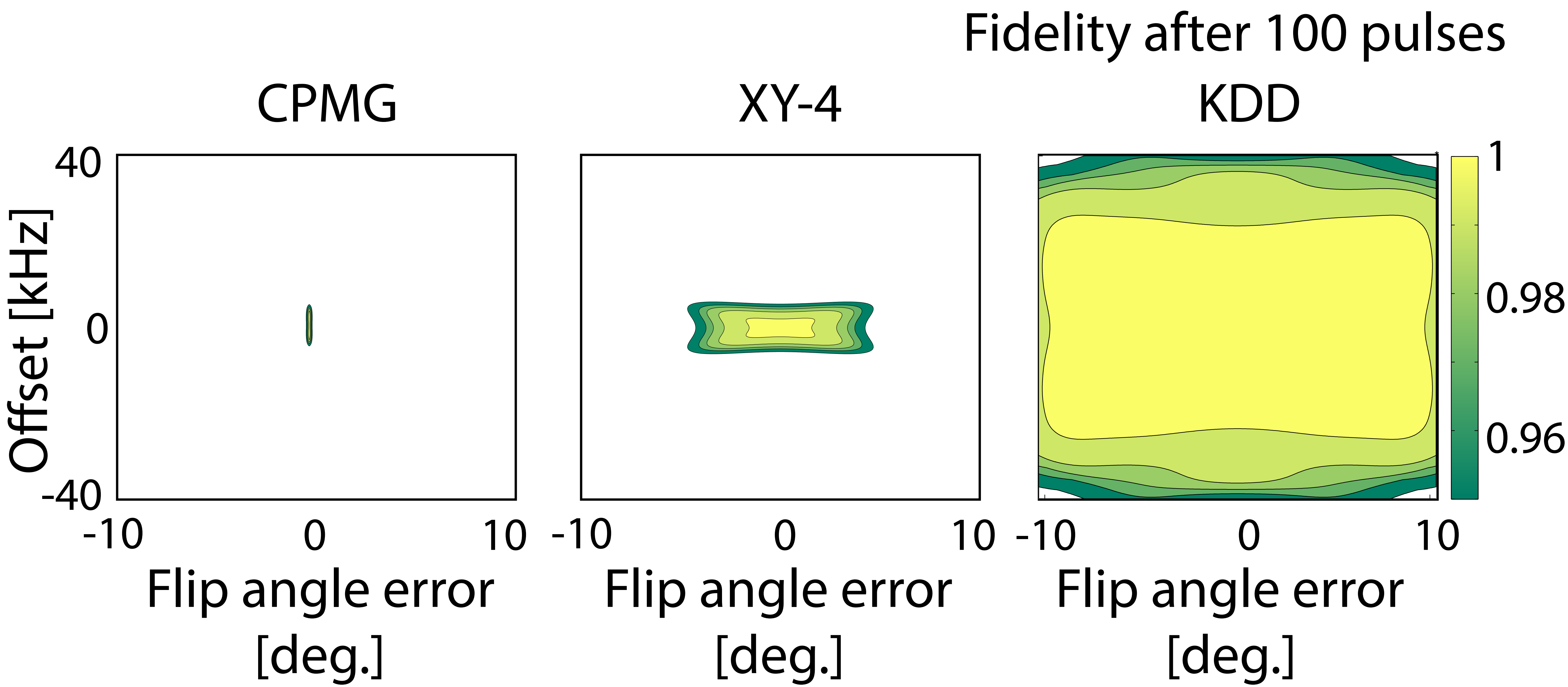
Effect of Flip Angle Errors

Fidelity after 20 pulses



2 Types of Errors

Compensation of both errors simultaneously

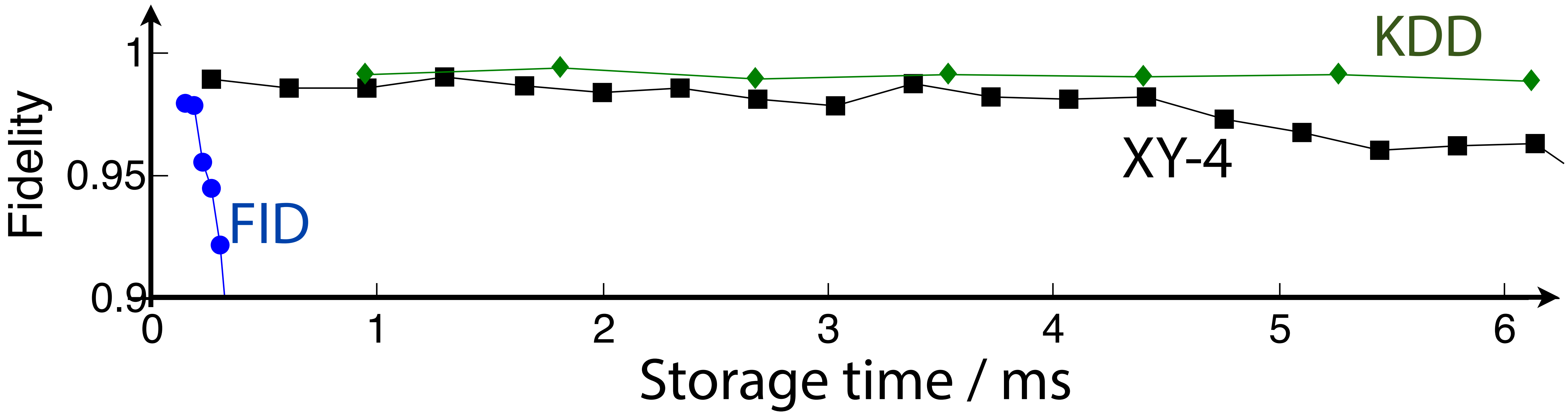


PRL 106, 240501 (2011).

Fidelity = 1 : perfect gate

Protected Quantum Memory

Preserving a state = Quantum memory

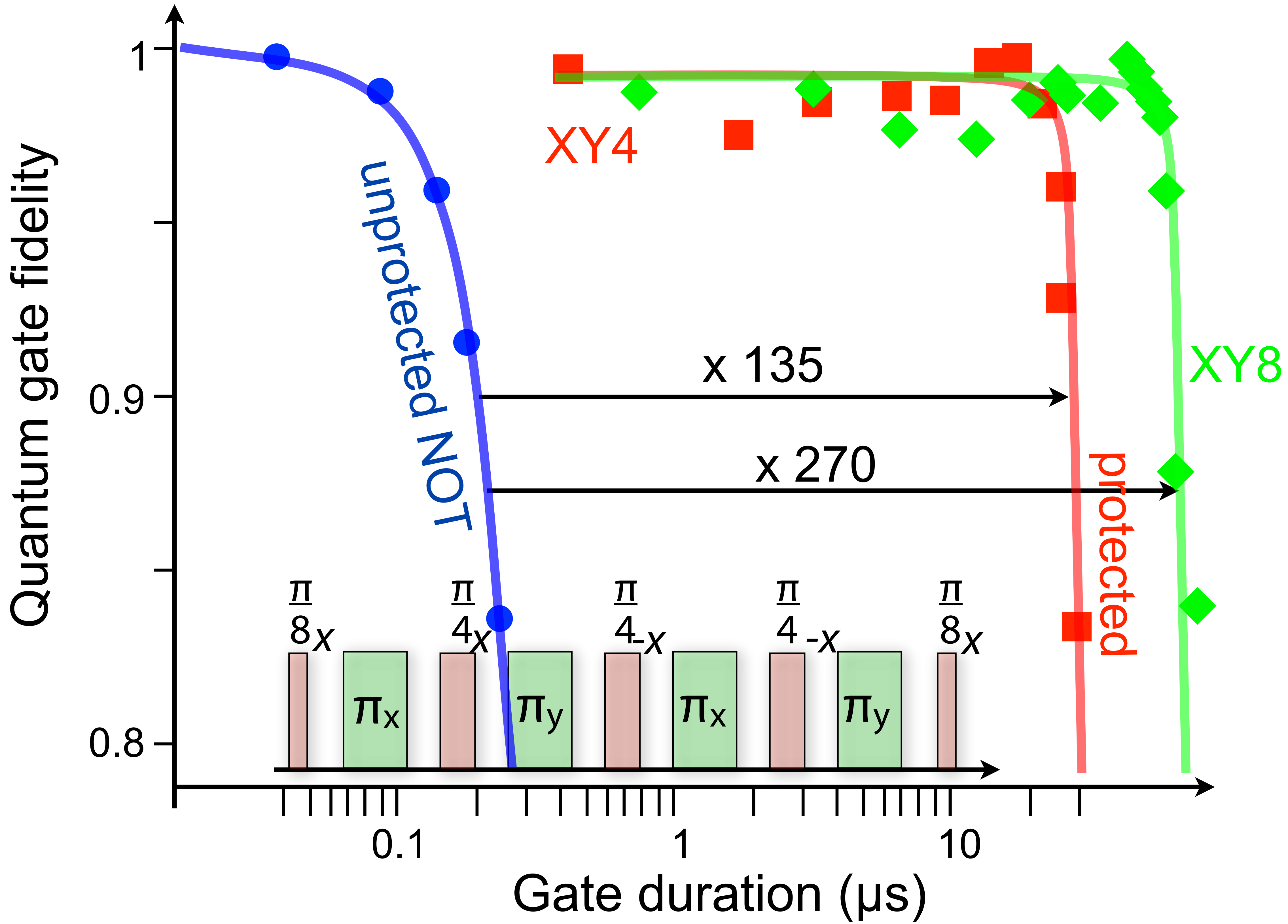


Q: Can we combine DD with gate operations ?

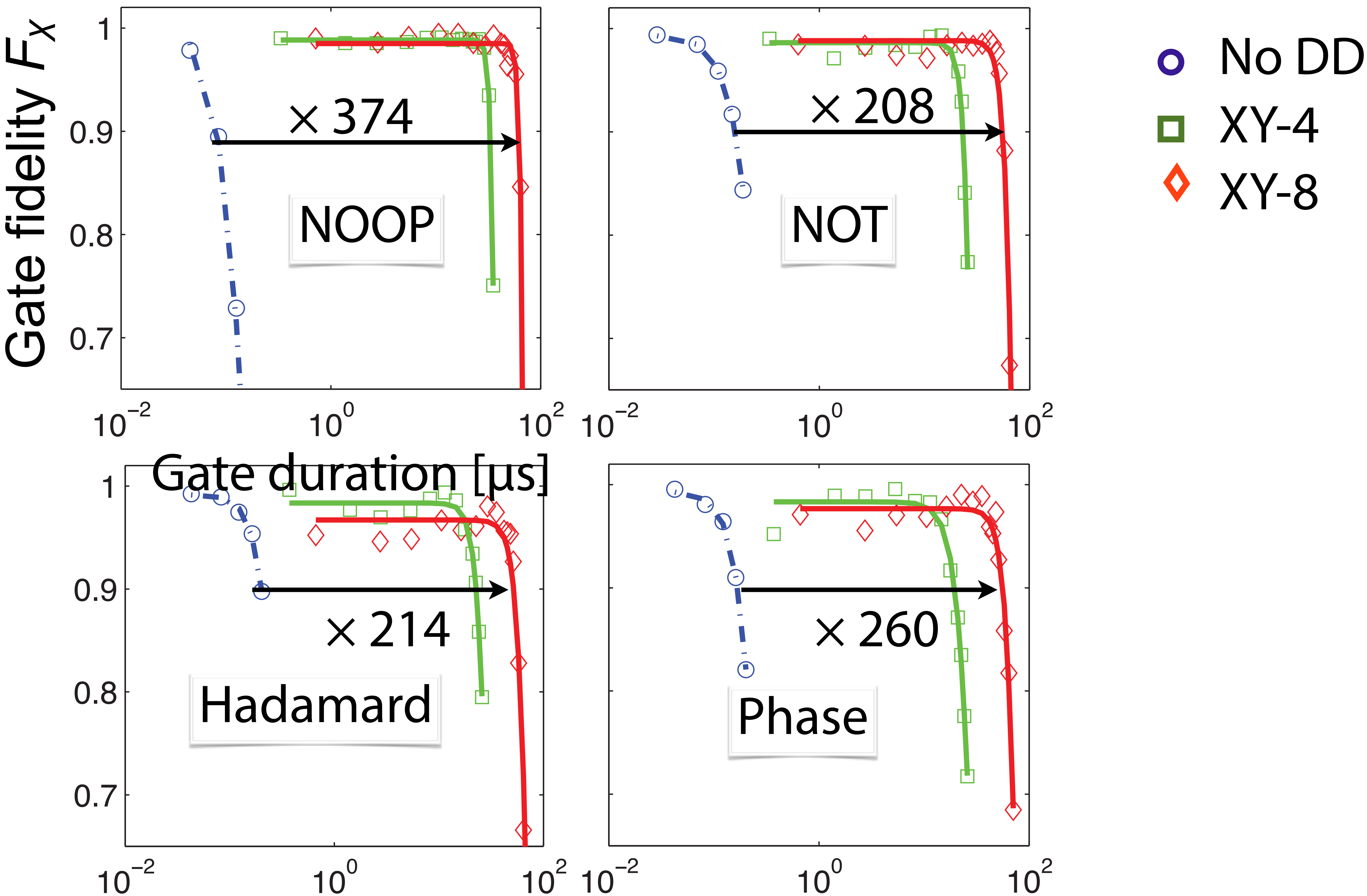
A1: Not directly: refocusing eliminates effect of control fields!

A2: Use modified, adapted control fields!

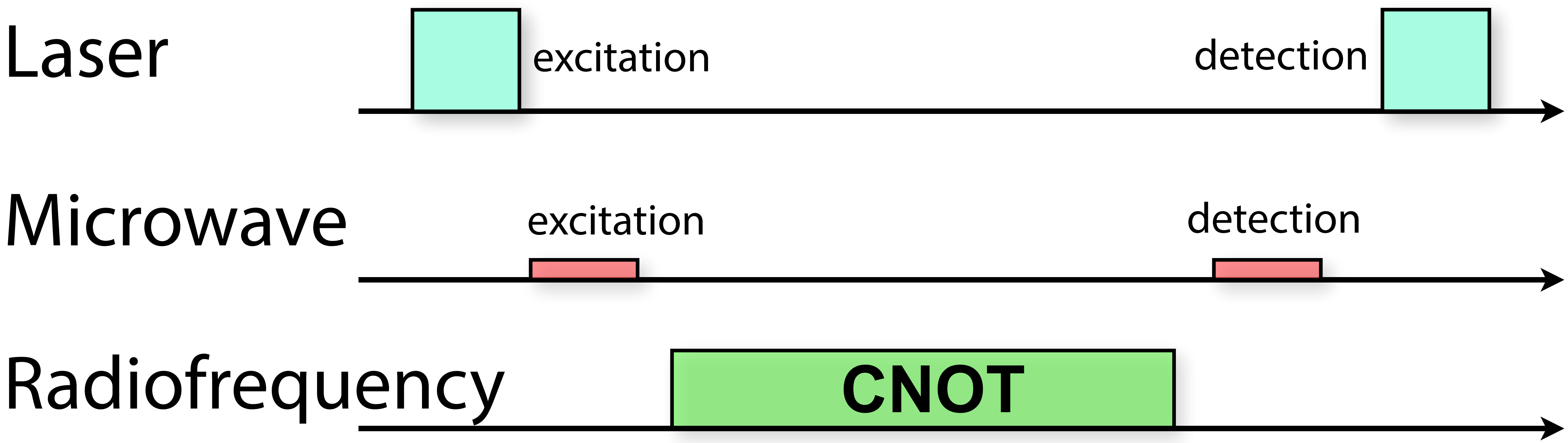
Protected NOT: Experiment

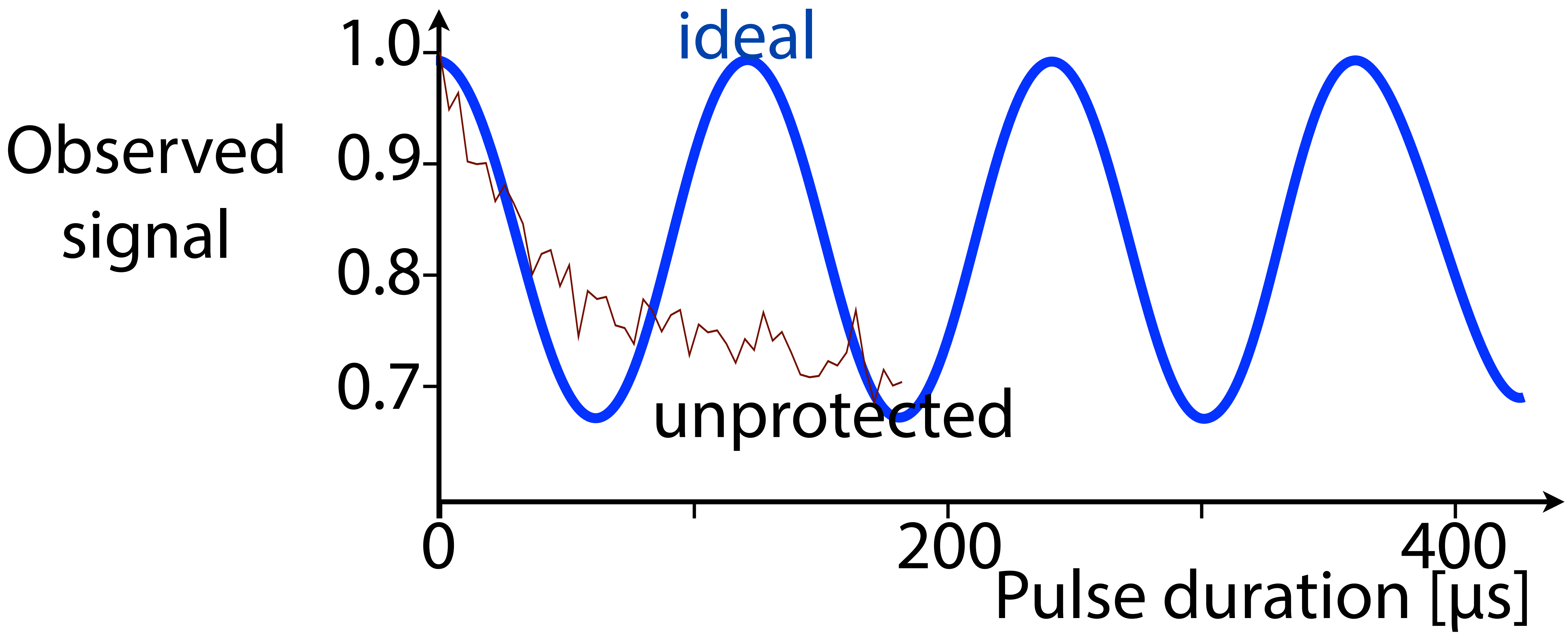


Protected Gates

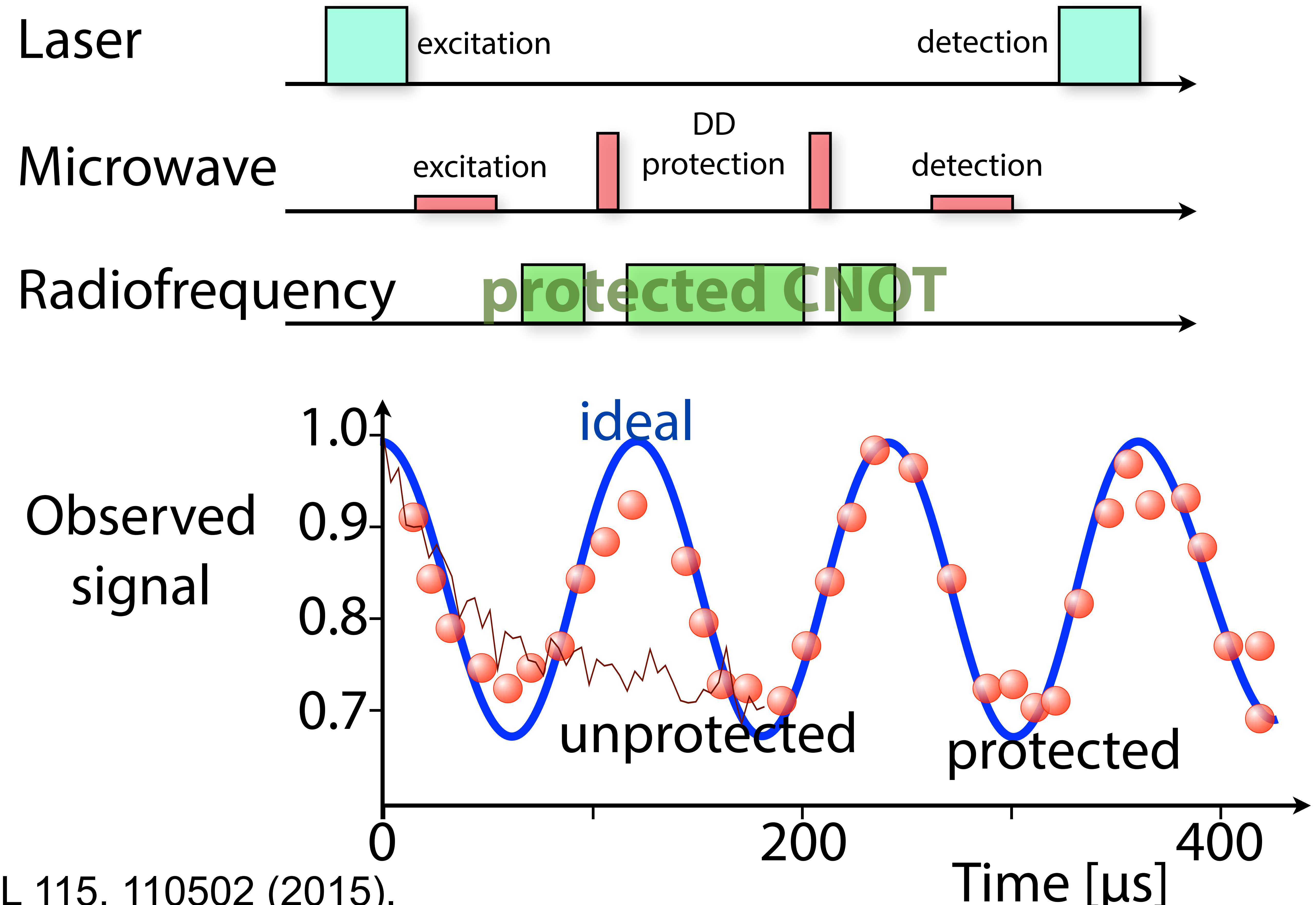


2-Qubit Gate


Laser
excitation
detection
Microwave
excitation
detection
Radiofrequency
CNOT


Observed signal
1.0
0.9
0.8
0.7
ideal
unprotected
0
200
400
Pulse duration [μs]

Protected 2-Qubit Gates



PRL 115, 110502 (2015).

Threshold and Gate Fidelity

A quantum computation can be as long as required with any desired accuracy **as long as the noise level is below a threshold value.**

Experimental error per gate:

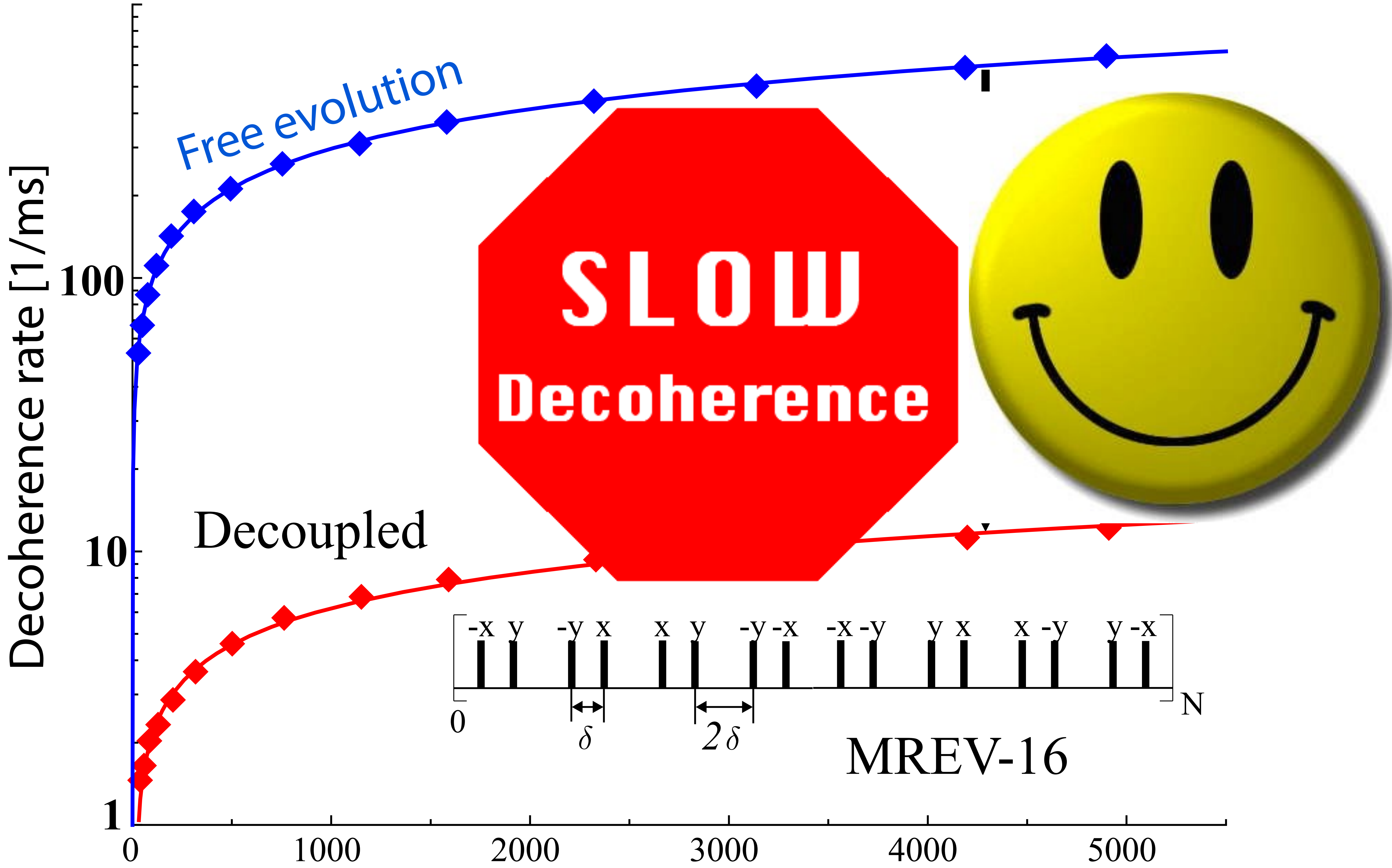
Gate	BB1	(a)	(b)	(c)	(d)	(e)	
τ [μs]	76	88	116	152	336	384	
EPG_m [10^{-4}]	5	6	8	10	22	25	estimated from max. DD
EPG_M [10^{-4}]	317	364	472	604	1191	1322	from Hahn echo
EPG_{exp} [10^{-4}]	32± 3	34± 3	28± 3	22± 3	172± 6	47± 3	measured

Required value for reliable QIP ~ 10^{-2} .. 10^{-4} (depends on QEC scheme)

Experimental : $2 \cdot 10^{-3}$

Phys. Rev. A 92, 062332 (2015).

Decoupling Quantum Registers



PRL 97, 150503 (2006)

Literature

REVIEWS OF MODERN PHYSICS, VOLUME 88, OCTOBER–DECEMBER 2016

Colloquium: Protecting quantum information against environmental noise

Dieter Suter

Fakultät Physik, TU Dortmund, 44221 Dortmund, Germany

Gonzalo A. Álvarez

Weizmann Institute of Science, 76100 Rehovot, Israel,
and Centro Atómico Bariloche, CNEA, CONICET, 8400 S. C. de Bariloche, Argentina

(published 10 October 2016)

D. Suter and G. A. Álvarez,
"Protecting quantum information against environmental noise",
Rev. Mod. Phys. 88, 041001 (2016).

Additional notes to this lecture:

https://qnap.e3.physik.tu-dortmund.de/suter/Vorlesung/ProtectingQI.pdf

Conclusions

- Reliable Quantum Computers require protection against experimental uncertainties and environmental noise.
- Active and passive protection techniques must be combined.
- Reaching the threshold is difficult but appears possible.

Thank you for your attention !