Engineering Entangled States in Cavity QED

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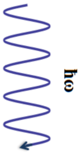
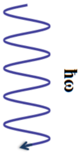
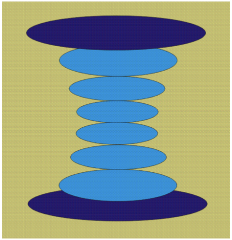
September 10, 2019

ELECTROMAGNETIC FIELD



ELECTROMAGNETIC FIELD





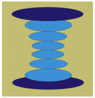
ELECTROMAGNETIC FIELD

$$
\begin{aligned}
\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \\
\nabla \cdot \mathbf{E} &= 0, \\
\nabla \cdot \mathbf{B} &= 0.
\end{aligned}
$$

taking curl of first equation, reveals a wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial^2 t} = 0.$$

ELECTROMAGNETIC FIELD

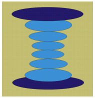


$$E_z(x,t) = \left(\frac{2\omega^2}{V\epsilon_0}\right)^{1/2} q(t)\sin(kx).$$

$$B_y(x,t) = \left(\frac{\mu_0\epsilon_0}{k}\right)\left(\frac{2\omega^2}{V\epsilon_0}\right)^{1/2} \dot{q}(t)\cos(kx).$$

$$\begin{aligned}\text{Energy} &= \frac{1}{2}\int dV\left(\epsilon_0 \mathbf{E}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t) + \frac{1}{\mu_0}\mathbf{B}(\mathbf{r},t)\cdot\mathbf{B}(\mathbf{r},t)\right) \\ &= \frac{1}{2}(\,\omega^2 q^2(t) + \dot{q}^2(t)\,).\end{aligned}$$

FACTORIZATION



$$
\begin{aligned}
\hat{a} &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} + i\dot{\hat{q}}\right) \\
\hat{a}^{\dagger} &= \frac{1}{\sqrt{2\hbar\omega}}\left(\omega\hat{q} - i\dot{\hat{q}}\right)
\end{aligned}
$$

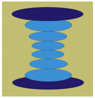
that leads to

$$
\hat{H}_F = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\hat{I}\right).
$$

and electric field appears as

$$
\hat{E}_z(x,t) = \sqrt{\frac{\hbar}{2\omega}}\left(\hat{a}(t) + \hat{a}^{\dagger}(t)\right)\sin(kx).
$$

LADDER OPERATORS



$$\hat{a} = \sqrt{n}|n-1\rangle\langle n|$$
$$\hat{a}^\dagger = \sqrt{n}|n\rangle\langle n-1|$$

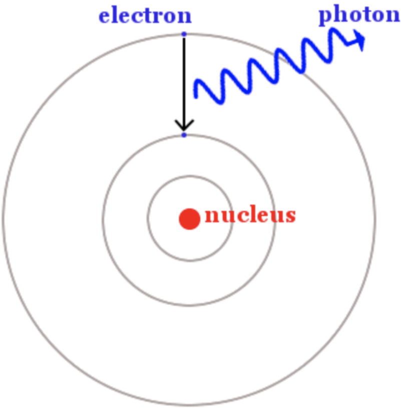
that yields

$$\hat{H}_F|n\rangle = E_n|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle$$

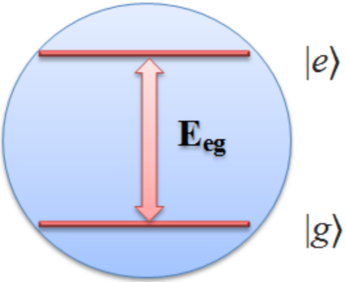
therefore

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right)$$

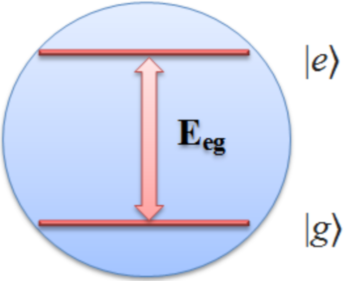
BOHR ATOM



BOHR ATOM

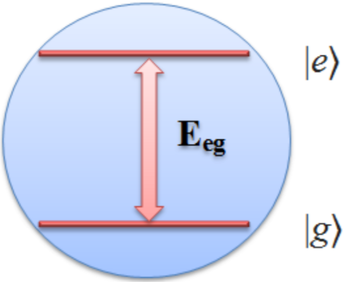


BOHR ATOM



excited state $|e\rangle$, with energy $E_e = \hbar\omega_e$, such that $\hat{H}_A|e\rangle = \hbar\omega_e|e\rangle$
ground state $|g\rangle$, with energy $E_g = \hbar\omega_g$, such that $\hat{H}_A|g\rangle = \hbar\omega_g|g\rangle$

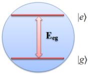
BOHR ATOM



excited state $|e\rangle$, with energy $E_e = \hbar\omega_e$, such that $\hat{H}_A|e\rangle = \hbar\omega_e|e\rangle$
ground state $|g\rangle$, with energy $E_g = \hbar\omega_g$, such that $\hat{H}_A|g\rangle = \hbar\omega_g|g\rangle$

$$\omega_{eg} = \omega_e - \omega_g$$

BOHR ATOM



hence

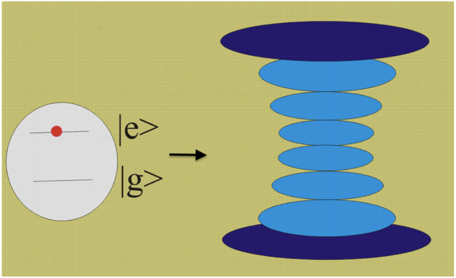
$$\hat{H}_A = \mathbf{1} \cdot \hat{H}_A \cdot \mathbf{1}$$

$$\hat{H}_A = (\,|g\rangle\langle g| + |e\rangle\langle e|\,)\,\hat{H}_A\,(\,|g\rangle\langle g| + |e\rangle\langle e|\,)$$

$$\hat{H}_A = \hbar\omega_g|g\rangle\langle g| + \hbar\omega_e|e\rangle\langle e|$$

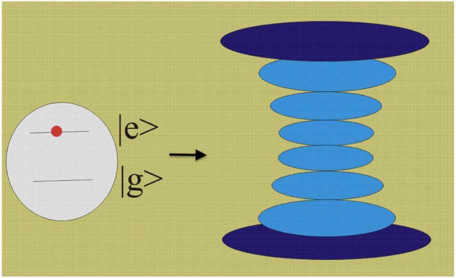
ATOM FIELD RESONANT INTERACTION

ATOM FIELD INTERACTION



$$\hat{H}_{int} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(x, t)$$

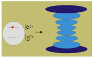
ATOM FIELD INTERACTION



$$\hat{H}_{int} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(x, t)$$

$$\Delta = \omega - \omega_{eg} = 0 \quad \omega = \omega_{eg}$$

ATOM FIELD INTERACTION



$$H_{int} = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(x, t)$$

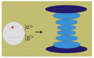
where

$$\begin{aligned}
\hat{\mathbf{d}} &= \mathbf{1} \cdot \hat{d} \cdot \mathbf{1} \\
&= e(|g\rangle\langle g| + |e\rangle\langle e|)\mathbf{r}(|g\rangle\langle g| + |e\rangle\langle e|) \\
&= d_{eg}|e\rangle\langle g| + d_{ge}^{*}|g\rangle\langle e| \\
&= d_{eg}\sigma^{\dagger} + d_{ge}^{*}\sigma \\
&= d_{eg}(\sigma^{\dagger} + \sigma)
\end{aligned}$$

such that

$$\sigma^{\dagger} = |e\rangle\langle g| \qquad \sigma = |g\rangle\langle e|$$

DIPOLE AND ROTATING WAVE APPROXIMATIONS

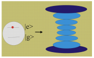


hence

$$
\begin{aligned}
\hat{H}_{int} &= -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(x, t) \\
&= -\hbar\mu(\sigma^{\dagger} + \sigma)(\hat{a}^{\dagger} + \hat{a}) \\
&= -\hbar\mu(\sigma^{\dagger}\hat{a}^{\dagger} + \sigma\hat{a}^{\dagger} + \sigma^{\dagger}\hat{a} + \sigma\hat{a}) \\
&\approx -\hbar\mu(\sigma\hat{a}^{\dagger} + \sigma^{\dagger}\hat{a})
\end{aligned}
$$

where

$$
\mu = \frac{d_{eg} \cdot \mathbf{e}_z E_0}{\hbar}
$$

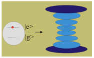


We write the Hamiltonian of the system $\hat{H}$ as,

$$
\begin{aligned}
\hat{H} &= \hat{H}_F + \hat{H}_A + \hat{H}_{int} \\
\hat{H} &= \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_g|g\rangle\langle g| + \hbar\omega_e|e\rangle\langle e| - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(x,t) \\
&= \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_g|g\rangle + \hbar\omega_e|e\rangle - \hbar\mu(\sigma\hat{a}^\dagger + \sigma^\dagger\hat{a})
\end{aligned}
$$

For a resonance between field frequency ω and transition frequency $\omega_{eg} = \omega_e - \omega_g$, we have the interaction Hamiltonian

$$\hat{H}_I = e^{iH_0t/\hbar}H_{int}e^{-iH_0t/\hbar} = \hbar\mu(\sigma\hat{a}^\dagger + \sigma^\dagger\hat{a})$$



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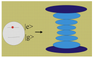
$$\begin{aligned}
\hat{H} &= \hat{H}_F + \hat{H}_A + \hat{H}_{int} \\
\hat{H} &= \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_g|g\rangle\langle g| + \hbar\omega_e|e\rangle\langle e| - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(x,t) \\
&= \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_g|g\rangle + \hbar\omega_e|e\rangle - \hbar\mu(\sigma\hat{a}^\dagger + \sigma^\dagger\hat{a})
\end{aligned}$$

For a resonance between field frequency ω and transition frequency $\omega_{eg} = \omega_e - \omega_g$, we have the interaction Hamiltonian

$$\hat{H}_I = e^{iH_0 t/\hbar} H_{int} e^{-iH_0 t/\hbar} = \hbar\mu(\sigma\hat{a}^\dagger + \sigma^\dagger\hat{a})$$

$$|\Psi(t)\rangle = e^{-iH_I t/\hbar}|\Psi(0)\rangle$$

GENERAL SOLUTION for $\Delta = 0$



For an initial condition

$$
\begin{aligned}
|\Psi(0)\rangle &= |\Psi_A\rangle \otimes |\Psi_F\rangle \\
&= \left(\, \psi_g(0)\,|g\rangle + \psi_e(0)\,|e\rangle \,\right) \otimes \sum_n w_n |n\rangle
\end{aligned}
$$

Hence we obtain

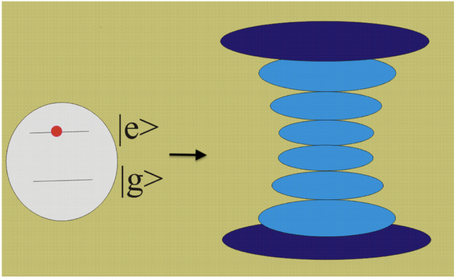
$$
|\Psi(t)\rangle = \sum_n \left(\, \psi_{g,n}(t)\,|g,n\rangle + \psi_{e,n}(t)\,|e,n\rangle \,\right)
$$

where

$$
\begin{aligned}
\psi_{e,n}(t) &= \cos(\Omega_n t)\psi_{e,n}(0) - i\sin(\Omega_n t)\psi_{g,n+1}(0) \\
\psi_{g,n}(t) &= -i\sin(\Omega_n t)\psi_{e,n}(0) + \cos(\Omega_n t)\psi_{g,n+1}(0)
\end{aligned}
$$

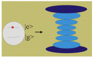
where $\Omega_n = \mu\sqrt{n+1}$ and $\mu = d\,E_0/\hbar$

EXEMPLE



$$|\Psi(0)\rangle = |e\rangle \otimes |0\rangle$$

EXAMPLE



$$|\Psi(0)\rangle = |e\rangle \otimes |0\rangle$$

evolves in time as

$$|\Psi(t)\rangle = \psi_{g,1}(t)\,|g,1\rangle + \psi_{e,0}(t)\,|e,0\rangle$$

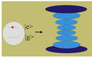
where

$$\begin{aligned}\psi_{e,0}(t) &= \cos(\Omega_0 t)\\ \psi_{g,1}(t) &= -i\sin(\Omega_0 t)\end{aligned}$$

where $\Omega_0 = \mu$ is vacuum Rabi frequency, and

$$|\Psi(t)\rangle = \cos(\Omega_0 t)\,|g,1\rangle - i\sin(\Omega_0 t)\,|e,0\rangle$$

EXAMPLE

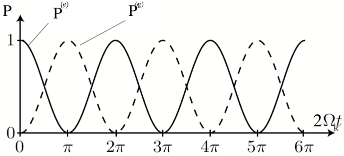


$$|\Psi(t)\rangle = \cos(\Omega_0 t)\,|g,1\rangle - i\sin(\Omega_0 t)\,|e,0\rangle$$

where

$$\begin{aligned} |(\psi_{e,0}(t)|^2 &= \cos^2(\Omega_0 t) = \frac{1}{2}(1+\cos 2\Omega_0 t) \\ |\psi_{g,1}(t)|^2 &= \sin^2(\Omega_0 t) = \frac{1}{2}(1-\cos 2\Omega_0 t) \end{aligned}$$

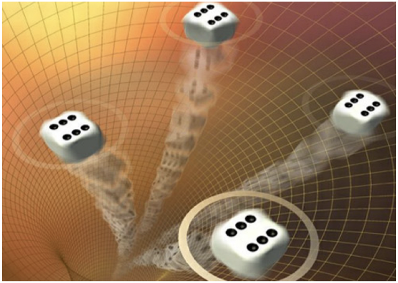
where $\Omega_0 = \mu$ is vacuum Rabi frequency.



Engineering Entanglement

Engineering Entanglement

- Schroedinger (1935)
- Einstein Podolsky and Rosen (1935)
  EPR states

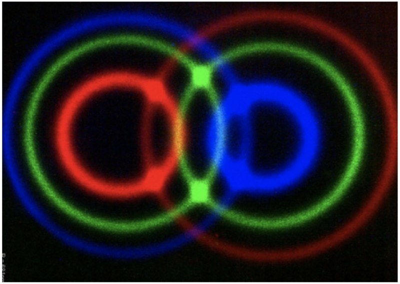


1

[1] E. Schroedinger, Proc. Camb. Phil. Soc. 31, 555 (1935); A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935).

Engineering Entanglement

- Mathematically: Non-factorizable states
- Physically: Correlations between parties
- Technologically: Quantum channels

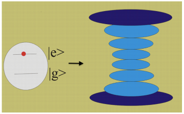


2

[2] E. Schroedinger, Proc. Camb. Phil. Soc. 31, 555 (1935); A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935).

EXAMPLES

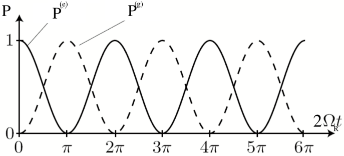
ATOM-FIELD ENTANGLEMENT



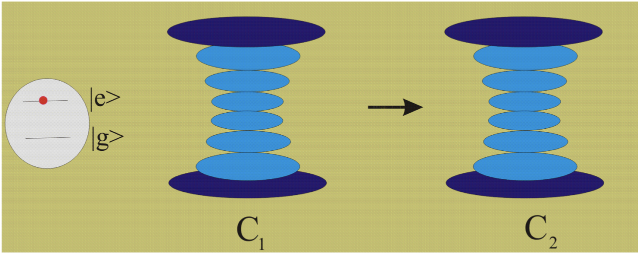
$$|\Psi(t)\rangle = \cos(\Omega_0 t)\,|g,1\rangle - i\sin(\Omega_0 t)\,|e,0\rangle$$

where

$$\begin{aligned}|(\psi_{e,0}(t)|^2 &= \cos^2(\Omega_0 t) = \frac{1}{2}(1+\cos 2\Omega_0 t)\\ |\psi_{g,1}(t)|^2 &= \sin^2(\Omega_0 t) = \frac{1}{2}(1-\cos 2\Omega_0 t)\end{aligned}$$



ENTANGLEMENT OF *TWO CAVITIES IN SAME MODE*

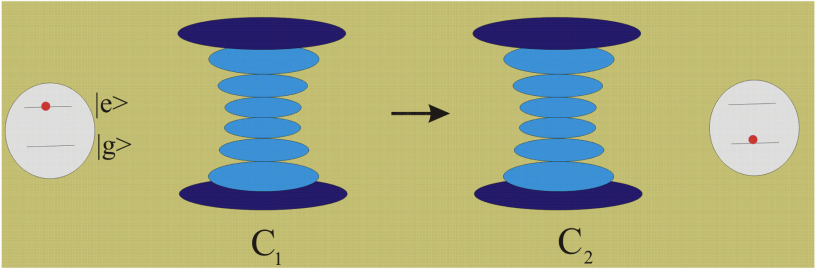


$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}(|e,0\rangle + e^{i\varphi}|g,1\rangle) \otimes |0\rangle$$

3

[3] L. Davidovich, N. Zagury, M. Brune, J.M. Raimond, and S. Haroche, PRA50, R895 (1994)

ENTANGLEMENT OF *TWO CAVITIES IN SAME MODE*

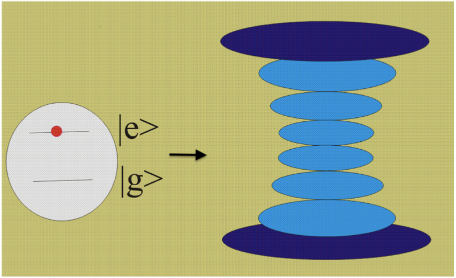


Interaction of the atom with the second cavity C_2 for an interaction time $2\Omega_0 t_2 = \pi$ leads to caviti-cavity entanglement as

$$|\Psi(t_2)\rangle = |g\rangle \otimes \frac{1}{\sqrt{2}}(1,0\rangle + e^{i\varphi}|0,1\rangle)$$

4

[4] L. Davidovich, et al, Phys. Rev. A 50, R895 (1994)

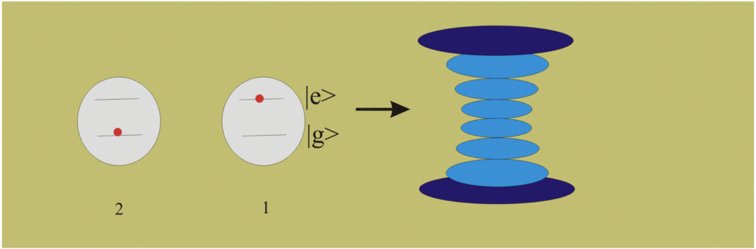


$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}(|e_1, 0\rangle + e^{i\varphi}|g_1, 1\rangle) \otimes |g_2\rangle$$

[5]

[5] I. Cirac and P. Zoller, Phys. Rev. Lett 72 (1994)

ENTANGLEMENT BETWEEN TWO ATOMS

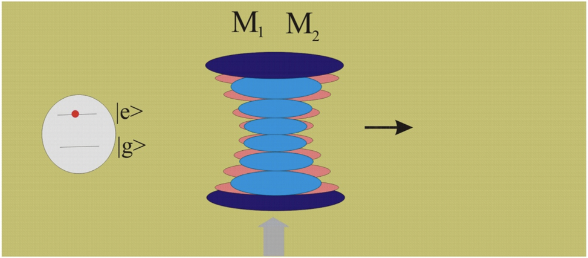


Interaction of the second atom, initially in ground state $|g_2\rangle$ with the cavity for an interaction time $2\Omega_0 t_2 = \pi$ leads to atom-atom entanglement

$$|\Psi(t_2)\rangle = |0\rangle \otimes \frac{1}{\sqrt{2}}(e_1, g_2\rangle + e^{i\varphi}|g_1, e_2\rangle)$$

6

[6] I. Cirac and P. Zoller, Phys. Rev. Lett 72 (1994)

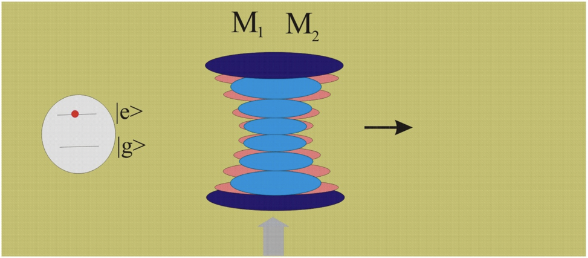


$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}(|e,0\rangle + e^{i\varphi}|g,1\rangle) \otimes |0_2\rangle$$

[7]

[7] A. Rauchenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A 64, 050301 (2001)

ENTANGLEMENT B/W *TWO CAVITY FIELD MODES*-I

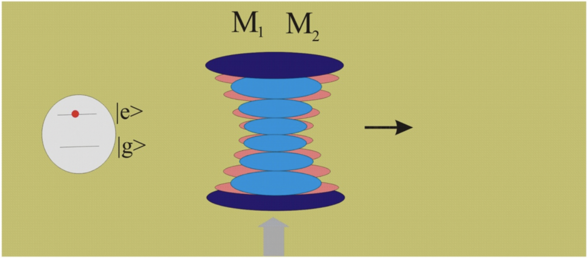


$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}(|e,0\rangle + e^{i\varphi}|g,1\rangle) \otimes |0_2\rangle$$

8

[8] A. Rauchenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A 64, 050301 (2001)

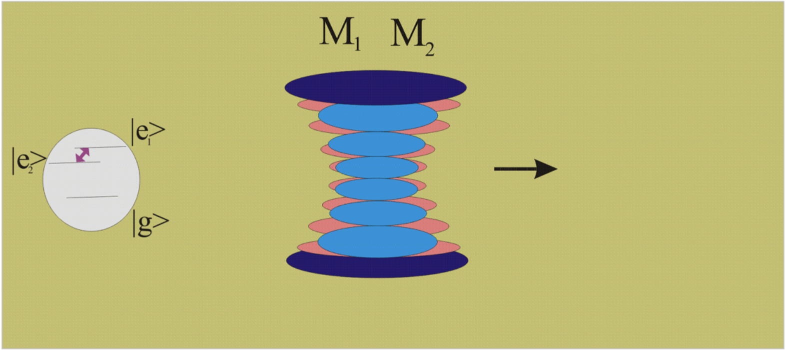
ENTANGLEMENT B/W *TWO CAVITY FIELD MODES*-I



$$|\Psi(t_2)\rangle = |g\rangle \otimes \frac{1}{\sqrt{2}}(|1,0\rangle + e^{i\varphi}|0,1\rangle)$$

9

9 A. Rauchenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A 64, 050301 (2001)



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle + |e_2\rangle) \otimes |0_A, 0_B\rangle$$

10

[10] M. Ikram and F. Saif, Phys. Rev. A 66, 014304 (2002)

ENTANGLEMENT B/W *TWO CAVITY FIELD MODES*-II

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle + |e_2\rangle) \otimes |0_A, 0_B\rangle$$

$$|\Psi(t)\rangle = (C_{1,0}\,|1_A, 0_B\rangle + C_{0,1}\,|0_A, 1_B\rangle) \otimes |g\rangle$$

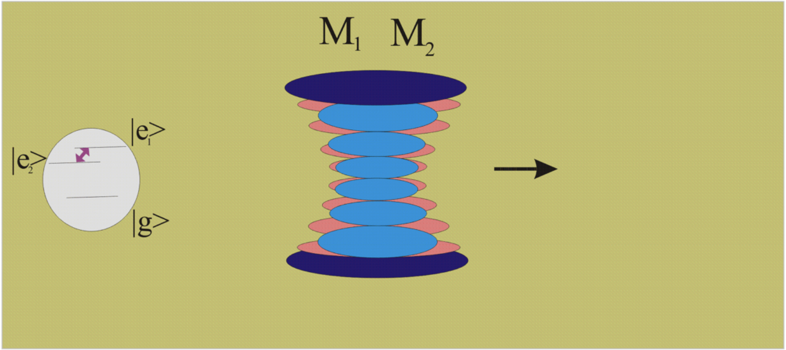
$$C_{1,0} = \sin(g_A t)$$
$$C_{0,1} = \sin(g_B t)$$

$$\sin(g_A t) = \sin(g_B t)$$

$$t_A = n\pi/g_A, \quad t_B = m\pi/g_B$$

n and m are odd integers

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(\,|1_A, 0_B\rangle + |0_A, 1_B\rangle\,) \otimes |g\rangle$$



$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|1_A, 0_B\rangle + |0_A, 1_B\rangle \right) \otimes |g\rangle$$

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[12] M. Ikram and F. Saif, Phys. Rev. A 66, 014304 (2002)

OFF-RESONANCE INTERACTION

In far-ff resonance or dispersive regime we consider

$$\Delta > \mu\sqrt{n+1}$$

hence in secular approximation we consider that

$$\frac{\partial \psi_e}{\partial t} = 0,$$

$$\frac{\partial^2 \psi_e}{\partial x^2} = 0,$$

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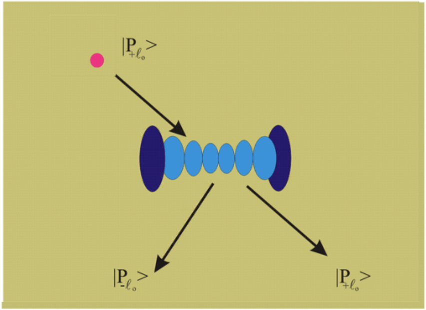
Hence, the atom evolves in its ground state with effective Hamiltonian

$$H = \frac{P^2}{2M} + \hbar\Omega_R\, \hat{a}^\dagger \hat{a}\, \sigma^\dagger \sigma \left(\cos 2kx + 1\right) \tag{1}$$

where $\Omega_R = \mu^2/\Delta$, and $\Delta = \omega - \omega_{eg}$.

OFF-RESONANCE INTERACTION



BRAGG SCATTERING

If $\mathbf{p}_{in} = l_0 \hbar k/2$ denotes initial atomic momentum,
Momentum conservation requires

$$\mathbf{p}_{out} = \mathbf{p}_{in} + l\hbar k$$

l_0 is an even integer and represent the order of Bragg diffraction
$\mathbf{p}_{out}$ is the atomic momentum after l interactions with the cavity field.
Energy conservation requires that

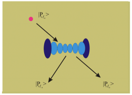
$$\frac{|\mathbf{p}_{in}|^2}{2M} = \frac{|\mathbf{p}_{out}|^2}{2M}$$

we obtain resonance condition for Bragg regime, that is

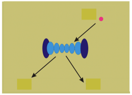
$$\frac{l\,(l + l_0)}{2M}\hbar^2 k^2 = 0$$

This equation has only two solutions: $l = 0$, corresponds to undeflected atomic beam; and $l = -l_0$, which corresponds to deflected beam.

BRAGG SCATTERING

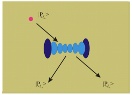


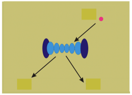
$$|\Psi(t=0)\rangle = |\Psi_A\rangle \otimes |\Psi_F\rangle = |P_{+l_0}\rangle \otimes |n\rangle$$



$$|\Psi(t=0)\rangle = |\Psi_A\rangle \otimes |\Psi_F\rangle = |P_{-l_0}\rangle \otimes |n\rangle$$

BRAGG SCATTERING





$$|\Psi(t=0)\rangle = |\Psi_A\rangle \otimes |\Psi_F\rangle = |P_{\pm l_0}\rangle \otimes |n\rangle$$

after an evolution over time t, yields

$$|\Psi(t)\rangle = C_{n,+l_0}(t)|P_{+l_0}, n\rangle + C_{n,-l_0}(t)|P_{-l_0}, n\rangle$$

where

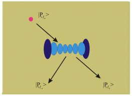
$$C^{(j)}_{n,\pm l_0}(t) = e^{-iA_n t}\left[C^{(j)}_{n,\pm l_0}(0)\cos\left(\frac{1}{2}B_n t\right) + C^{(j)}_{n,\mp l_0}(0)\sin\left(\frac{1}{2}B_n t\right)\right]$$

$$A_n = \begin{cases} 0 & l_0 = 2\,, \\ -\frac{(\chi n/2)^2}{\omega_{rec}(l_0-2)(2)} & l_0 \neq 2. \end{cases}$$

and

$$|B_n| = \begin{cases} \chi n & l_0 = 2\,, \\ \frac{(\chi n)^{l_0/2}}{(2\omega_{rec})^{l_0/2-1}[(l_0-2)(l_0-4)...4.2]^2} & l_0 \neq 2\,. \end{cases}$$

BRAGG SCATTERING



$$|\Psi(t=0)\rangle = |\Psi_A\rangle \otimes |\Psi_F\rangle = |P_{+l_0}\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

after an evolution over time t, yields

$$|\Psi(t)\rangle = C_{n,+l_0}(t)|P_{+l_0},0\rangle + C_{n,-l_0}(t)|P_{-l_0},1\rangle$$

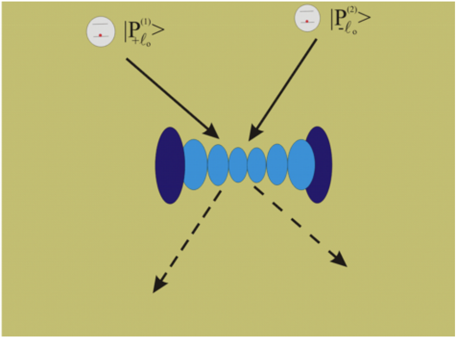
for first order Bragg diffraction, that is $l_0 = 2$.

$$|C_{n,+l_0}(t)|^2 = \cos^2(\Omega_n t)$$
$$|C_{n,-l_0}(t)|^2 = \sin^2(\Omega_n t)$$

If $2\Omega_1 t = 2\chi t = \pi/2$, where $\chi = \Omega^2/2\Delta$ is effective Rabi frequency

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(|P_{+l_0},0\rangle + |P_{-l_0},1\rangle$$

ENTANGLEMENT OF TWO MOMENTUM STATES



$$|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |P_{+l_0}\rangle \otimes |P_{-l_0}\rangle$$

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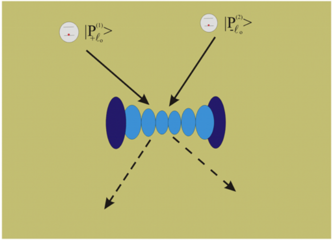
[13] A. Khalique, F. Saif, Physics Letters A 314, 37-43 (2003)

ENTANGLEMENT OF TWO MOMENTUM STATES

$$\hat{H}_{eff} = \frac{\hat{P}_{x_1}^2}{2M} + \frac{\hat{P}_{x_2}^2}{2M} + \frac{\hbar|g|^2}{2\Delta}\sum_{j=1,2}\hat{n}\hat{\sigma}_-^{(j)}\hat{\sigma}_+^{(j)}(\cos 2k\hat{x}_j + 1)$$



$$|\Psi(\tau)\rangle = \frac{1}{\sqrt{2}}\left\{\left|P_{+\ell_0}^{(1)}, P_{-\ell_0}^{(2)}\right\rangle|0\rangle + e^{-i\varphi}\left|P_{-\ell_0}^{(1)}, P_{+\ell_0}^{(2)}\right\rangle|1\rangle\right\}$$

Probe Atom in ground state and Ramsey Field

$$|\Psi(\tau)\rangle = \frac{1}{\sqrt{2}}\left\{\left|P_{+\ell_0}^{(1)}, P_{-\ell_0}^{(2)}\right\rangle + e^{-i\varphi}\left|P_{-\ell_0}^{(1)}, P_{+\ell_0}^{(2)}\right\rangle\right\}$$

$$\frac{1}{2}B_1\tau = \frac{s\pi}{2} \qquad s = \text{odd integer}$$

$$\varphi = 2s\pi A_1/B_1$$

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[14] A. Khalique, F. Saif, Physics Letters A 314, 37-43 (2003)

ENTANGLEMENT OF TWO MOMENTUM STATES

$$|\Psi(t)\rangle = \frac{1}{2}\left\{|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}, 0\rangle + e^{-i\varphi}|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}, 1\rangle\right\} \otimes |g\rangle$$

resonant interaction for an interaction time $2\Omega_R t = \pi$, yields

$$|\Psi(t)\rangle = |0\rangle \otimes \frac{1}{2}\left\{|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}, g\rangle + e^{-i\varphi}|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}, e\rangle\right\}$$

Using a $\pi/2$-pulse , we generate superposition state of the internal state as

$$|g\rangle \longrightarrow \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \qquad |e\rangle \longrightarrow \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$$

$$\begin{aligned} |\Psi(t)\rangle &= \frac{1}{\sqrt{2}}\left\{|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}\rangle + e^{-i\varphi}|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}\rangle\right\} \otimes |g\rangle \\ &+ \frac{1}{\sqrt{2}}\left\{|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}\rangle - e^{-i\varphi}|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}\rangle\right\} \otimes |e\rangle \end{aligned}$$

ENTANGLED CLUSTER STATES

$$\left|\Psi(\tau)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^{(1)}_{+\ell_0}, P^{(2)}_{-\ell_0}\right\rangle + e^{-i\varphi}\left|P^{(1)}_{-\ell_0}, P^{(2)}_{+\ell_0}\right\rangle\right\}$$

$$\left|\Psi(\tau)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}\right\rangle + \left|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}\right\rangle\right\}$$

$$\left|\Psi(\tau)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^{(1)}_{+l_0}, P^{(2)}_{-l_0}\right\rangle - \left|P^{(1)}_{-l_0}, P^{(2)}_{+l_0}\right\rangle\right\}$$

$$\left|\Psi(\tau)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^{(1)}_{+l_0}, P^{(2)}_{+l_0}\right\rangle + \left|P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\right\rangle\right\}$$

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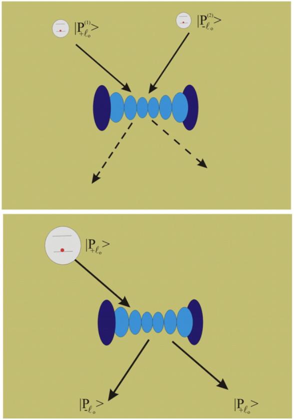
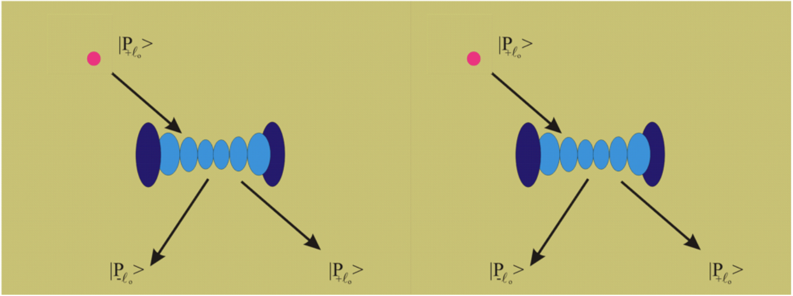


[15]

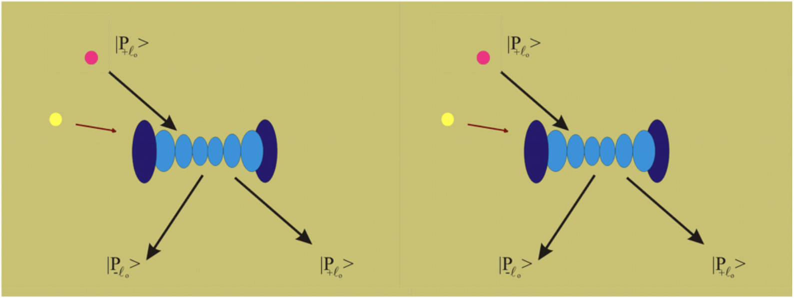
[15] A. Khalique, F. Saif, Physics Letters A 314, 37-43 (2003)

ENTANGLED CLUSTER STATES



16

[16] I. Rameez, A. H. Khosa, F. Saif, J.A. Bergou, Quantum Inf Process 12:129–148 (2013)



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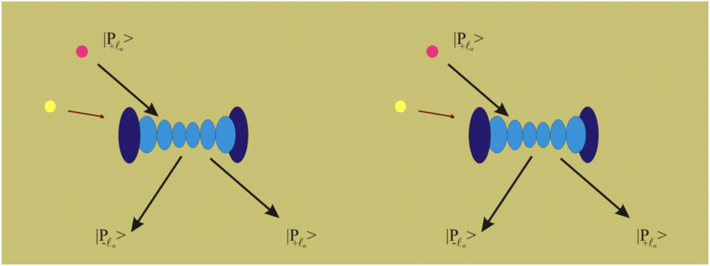
[17] I. Rameez, A. H. Khosa, F. Saif, J.A. Bergou, Quantum Inf Process 12:129–148 (2013)

ENTANGLED CLUSTER STATES



$$|\Psi(0)\rangle = \frac{1}{2^{n/2}} \prod_{j=1}^{n} \left[|0_j, P_0^{(j)}\rangle + |1_j, P_{-2}^{(j)}\rangle \right] \otimes |\bar{g}^{(j)}\rangle$$

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[18] I. Rameez, A. H. Khosa, F. Saif, J.A. Bergou, Quantum Inf Process 12:129–148 (2013)

Step I. Resonant Interaction of atom-1 *with cavity 1*

$$g_r t_1 = \pi/2$$

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0_1\rangle \otimes \left[|\bar{g}^{(1)}, P_0^{(1)}\rangle - i|\bar{e}^{(1)}, P_{-2}^{(1)}\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0_2, P_0^{(2)}\rangle + |1_2, P_{-2}^{(2)}\rangle\right] \otimes |\bar{g}^{(2)}\rangle$$

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[19] I. Rameez, A. H. Khosa, F. Saif, J.A. Bergou, Quantum Inf Process 12:129–148 (2013)

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Step II. Off-Resonant Interaction of atom-1 *with cavity 2*

$$e^{-i\mu\{(n+1)|e\rangle\langle e|-n|g\rangle\langle g|\}t}|n,s\rangle \qquad \text{where } s=e,g \qquad \mu=g_d^2/\Delta$$

$$e^{-i\mu\{(n+1)|e\rangle\langle e|-n|g\rangle\langle g|\}t}|0,g\rangle=|0,g\rangle$$

$$e^{-i\mu\{(n+1)|e\rangle\langle e|-n|g\rangle\langle g|\}t}|1,g\rangle=e^{+i\mu t}|1,g\rangle$$

$$e^{-i\mu\{(n+1)|e\rangle\langle e|-n|g\rangle\langle g|\}t}|0,e\rangle=e^{-i\mu t}|0,e\rangle$$

$$e^{-i\mu\{(n+1)|e\rangle\langle e|-n|g\rangle\langle g|\}t}|1,e\rangle=e^{-i2\mu t}|1,e\rangle$$

$$|\Psi(t_2)\rangle=\frac{1}{2}|0_1\rangle\otimes\begin{bmatrix}|\bar{g}^{(1)},0_2\rangle|P_0^{(1)},P_0^{(2)}\rangle-ie^{-i\mu t_2}|\bar{e}^{(1)},0_2\rangle|P_{-2}^{(1)},P_0^{(2)}\rangle\\+e^{+i\mu t_2}|\bar{g}^{(1)},1_2\rangle|P_0^{(1)},P_{-2}^{(2)}\rangle-ie^{-2i\mu t_2}|\bar{e}^{(1)},1_2\rangle|P_{-2}^{(1)},P_{-2}^{(2)}\rangle\end{bmatrix}\otimes|\bar{g}^{(2)}\rangle$$

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ENTANGLED CLUSTER STATES

Step III. Resonant Interaction of atom-2 *with cavity 2*

$$g_r t_3 = \pi/2$$

$$\left|\Psi(t_3)\right\rangle = \frac{1}{2}\left|0_1, 0_2\right\rangle \otimes \begin{bmatrix} \left|\bar{g}^{(1)}, \bar{g}^{(2)}\right\rangle \left|P_0^{(1)}, P_0^{(2)}\right\rangle - ie^{-i\mu t_2}\left|\bar{e}^{(1)}, \bar{g}^{(2)}\right\rangle \left|P_{-2}^{(1)}, P_0^{(2)}\right\rangle \\ -ie^{+i\mu t_2}\left|\bar{g}^{(1)}, \bar{e}^{(2)}\right\rangle \left|P_0^{(1)}, P_{-2}^{(2)}\right\rangle - e^{-2i\mu t_2}\left|\bar{e}^{(1)}, \bar{e}^{(2)}\right\rangle \left|P_{-2}^{(1)}, P_{-2}^{(2)}\right\rangle \end{bmatrix}$$

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[21] I. Rameez, A. H. Khosa, F. Saif, J.A. Bergou, Quantum Inf Process 12:129–148 (2013)

ENTANGLED CLUSTER STATES

The first atom interacts with the second cavity in dispersive regime, that is off-resonantly for time of interaction $\mu t_2 = \pi/2$
we find

$$-ie^{-i\mu t_2} = -ie^{-i\pi/2} = -1$$

$$-ie^{+i\mu t_2} = -ie^{+i\pi/2} = +1$$

$$-e^{-2i\mu t_2} = -e^{-i\pi} = +1$$

ENTANGLED CLUSTER STATES

$$\left|\bar{g}\right\rangle \xrightarrow{Hadamard} \frac{1}{\sqrt{2}}\left(\left|\bar{g}\right\rangle + \left|\bar{e}\right\rangle\right)$$

$$\left|\bar{e}\right\rangle \xrightarrow{Hadamard} \frac{1}{\sqrt{2}}\left(\left|\bar{g}\right\rangle - \left|\bar{e}\right\rangle\right)$$

$$\left|\Psi(t_3)\right\rangle = \frac{1}{2}\begin{bmatrix} \frac{1}{2}\left|\bar{g}^{(1)},\bar{g}^{(2)}\right\rangle \otimes \left\{\left|P_0^{(1)},P_0^{(2)}\right\rangle - \left|P_{-2}^{(1)},P_0^{(2)}\right\rangle + \left|P_0^{(1)},P_{-2}^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_{-2}^{(2)}\right\rangle\right\} \\ +\frac{1}{2}\left|\bar{e}^{(1)},\bar{e}^{(2)}\right\rangle \otimes \left\{\left|P_0^{(1)},P_0^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_0^{(2)}\right\rangle - \left|P_0^{(1)},P_{-2}^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_{-2}^{(2)}\right\rangle\right\} \\ +\frac{1}{2}\left|\bar{e}^{(1)},\bar{g}^{(2)}\right\rangle \otimes \left\{\left|P_0^{(1)},P_0^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_0^{(2)}\right\rangle + \left|P_0^{(1)},P_{-2}^{(2)}\right\rangle - \left|P_{-2}^{(1)},P_{-2}^{(2)}\right\rangle\right\} \\ -\frac{1}{2}\left|\bar{g}^{(1)},\bar{e}^{(2)}\right\rangle \otimes \left\{-\left|P_0^{(1)},P_0^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_0^{(2)}\right\rangle + \left|P_0^{(1)},P_{-2}^{(2)}\right\rangle + \left|P_{-2}^{(1)},P_{-2}^{(2)}\right\rangle\right\} \end{bmatrix}$$

Conclusions

- We can use atom field interaction in resonant regime and off-resonant regime to develop entanglement in cavity QED

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Conclusions

- We can use atom field interaction in resonant regime and off-resonant regime to develop entanglement in cavity QED
- We developed CAVITY-CAVITY Entanglement, ATOM-ATOM Entanglement, INTER-MODE Entanglement, INTER-ATOMIC MOMENTUM STATES Entanglements which can be enhanced to develop quantum networks
- We can extend these techniques to engineer other entangled states such GHZ states, Cluster States, NOON states and W-states.

THANKS