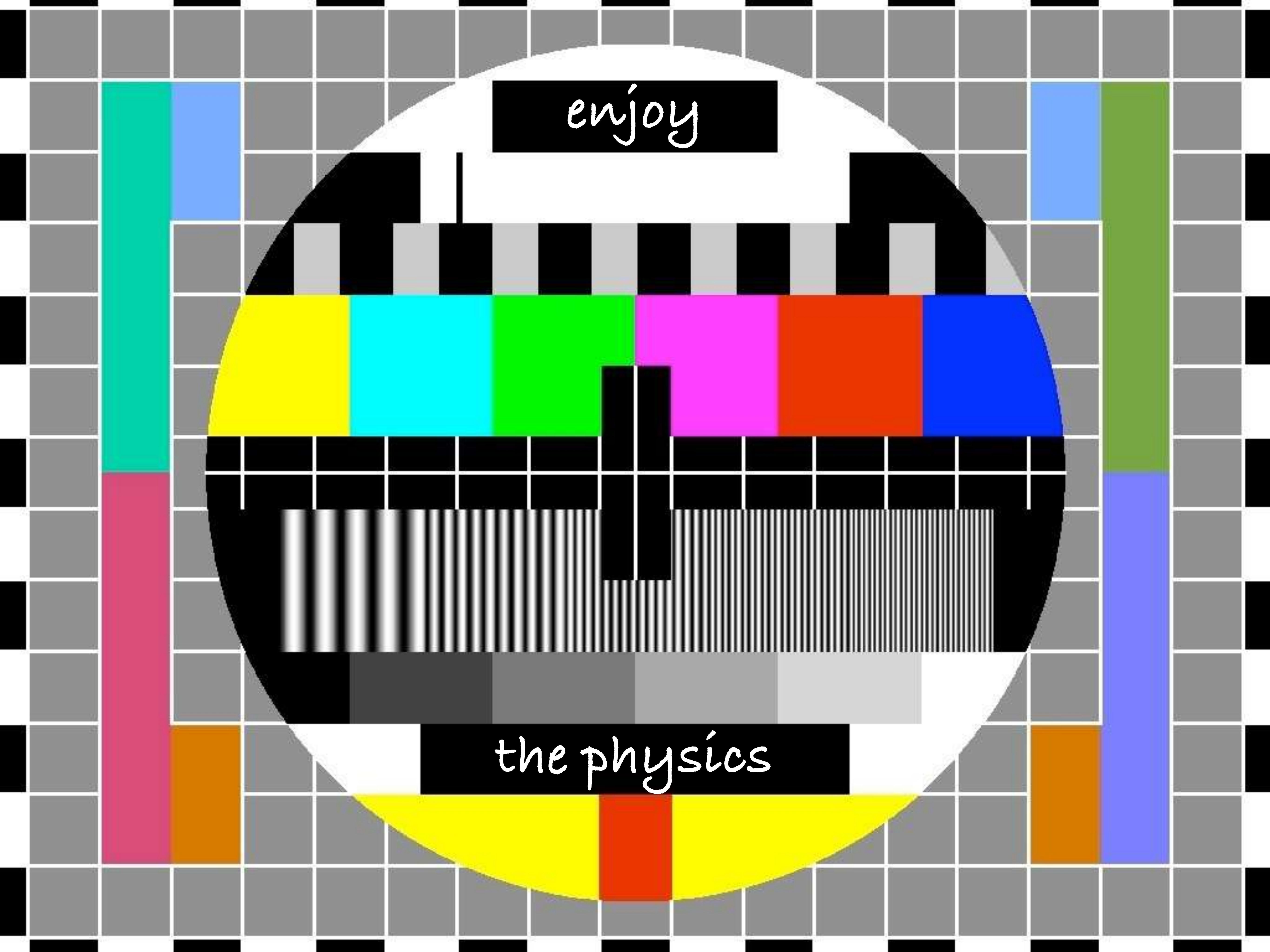
enjoy
the physics

Quantum sensing and metrology: the quantum Cramer-Rao bound and beyond

Matteo G. A. Paris
Quantum Technology Lab
Dipartimento di Fisica “Aldo Pontremoli”
Università degli Studi di Milano, Italy

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Quantum sensing and metrology: the quantum Cramer-Rao bound and beyond

- Basic ideas about measurement and estimation
- Classical estimation theory
- Quantum estimation theory and the Cramer-Rao bound to precision in quantum metrology
- Examples of applications
- Quantum sensing and metrology beyond the Cramer-Rao bound

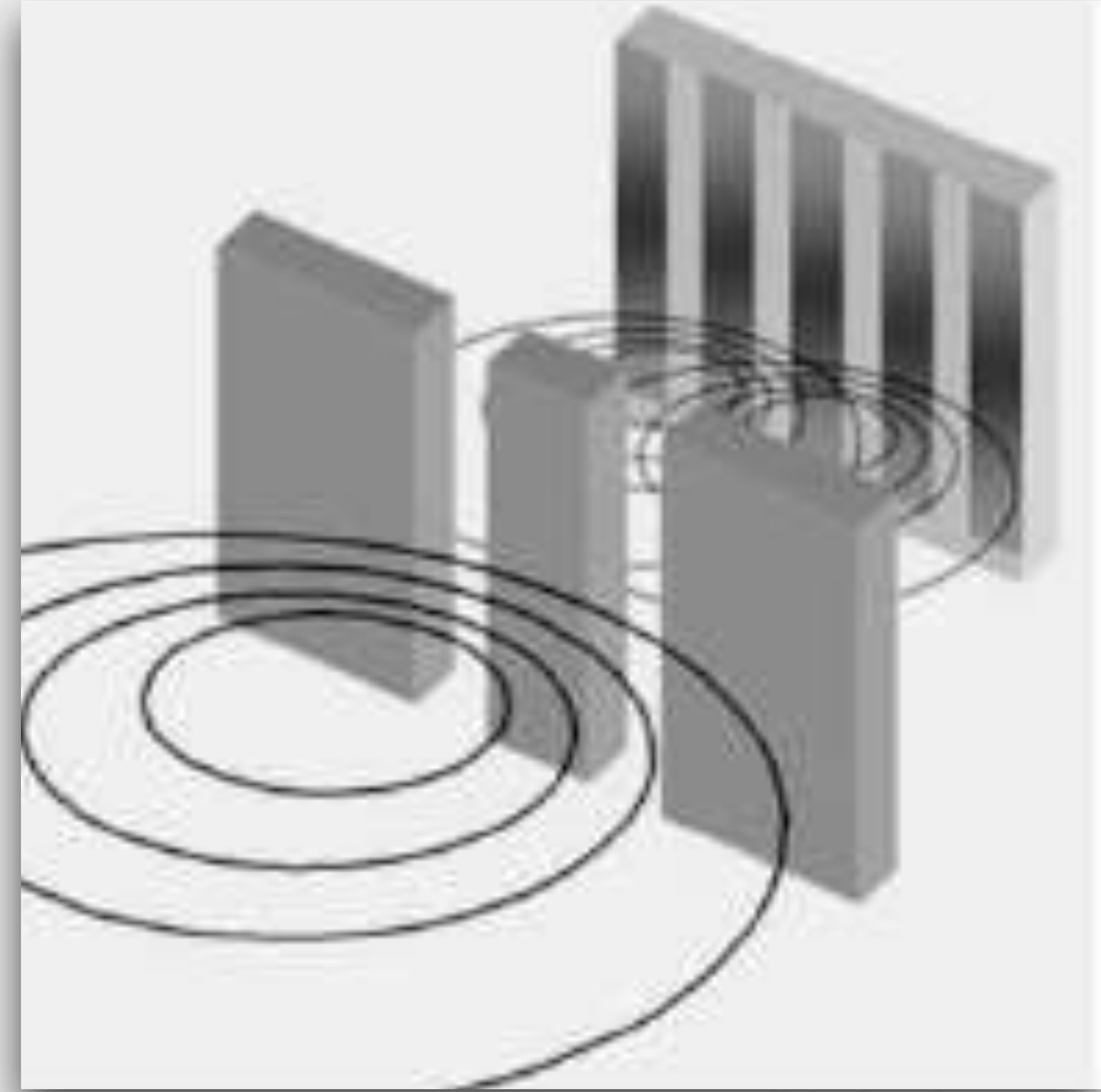
Measurement and estimation

(tum.de)

Do we measure physical quantities?

Or perhaps we are mostly estimating them?

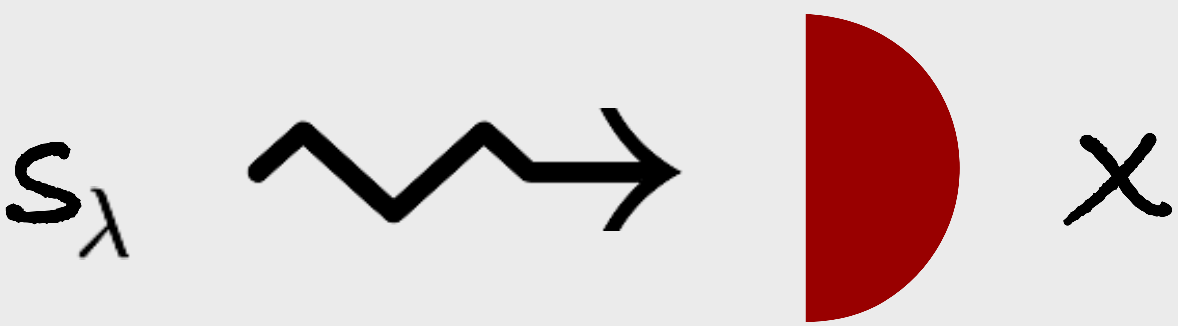
Measurement and estimation



Measurement and estimation

- ~~direct measurements~~
- indirect measurements

influence on a different quantity



$$\boldsymbol{\chi} = (x_1, x_2, \ldots)$$

- choice of the measurement $\quad p(x|\lambda)$

- choice of the estimator $\quad \boldsymbol{\chi} \mapsto \hat{\lambda} = f(\boldsymbol{\chi})$

Measurement and estimation

- global estimation theory
  (when you have no a priori information)
  look for a measurement which is optimal in average
  (over the possible values of the parameter)

- local estimation theory
  (when you have some a priori information)
  look for a measurement which is optimal for a
  specific value of the parameter (better, but...)

local estimation theory: Cramer - Rao bound

- variance of unbiased estimators

$$\mathrm{Var}_\lambda[\widehat{\lambda}] \geq \frac{1}{MF(\lambda)}$$

- M -> number of measurements
- F -> Fisher Information

$$F(\lambda) = \int dx\, p(x|\lambda)\left[\partial_\lambda \log p(x|\lambda)\right]^2$$

local estimation theory: Cramer - Rao bound

The proof of the Cramer-Rao bound is obtained by observing that given two functions $f_1(x)$ and $f_2(2)$ the average

$$\langle f_1, f_2 \rangle = \int dx\, p(x|\lambda)\, f_1(x)\, f_2(x)$$

defines a scalar product. Upon chosing $f_1(x) = \hat{\lambda}(x) - \lambda$ and $f_2(x) = \partial_\lambda \ln p(x|\lambda)$ we have

$$\boxed{\int dx\, \partial_\lambda p(x|\lambda) = 0}$$

$$||f_1||^2 = \mathrm{Var}(\lambda)$$
$$||f_2||^2 = F(\lambda)$$
$$\langle f_1, f_2 \rangle = 1$$

$x_1, x_2, ..., x_M$ independent we have $p(x_1, x_2, ..., x_M|\lambda) = \prod_{k=1}^M \log p(x_k|\lambda)$ and, in turn,

$$F_M(\lambda) = \int dx_1 ... dx_M\, p(x_1, x_2, ..., x_M|\lambda)\, [\partial_\lambda \ln p(x_1, x_2, ..., x_M|\lambda)]^2$$
$$= M \int dx\, p(x|\lambda)\, [\partial_\lambda \ln p(x|\lambda)]^2 = M F(\lambda)\,.$$

Optimal estimation scheme (classical)

$$\boldsymbol{\chi} = (x_1, x_2, \ldots)$$

$$\boldsymbol{\chi} \mapsto \widehat{\lambda} = f(\boldsymbol{\chi})$$

$$\mathrm{Var}_\lambda[\widehat{\lambda}] \geq \frac{1}{MF(\lambda)}$$

- Optimal measurement -> maximum Fisher (no recipes on how to find it)

- Optimal estimator -> saturation of CR inequality (e.g. Bayesian or MaxLik asymptotically)

Quantum estimation

- What about time and temperature in quantum mechanics?
- The "resources" involved in quantum-enahnced technology are entanglement, nonlocality, entropy, interferometric phase-shift, etc.. In general they are not observable quantities in strict sense (do not correspond to a selfadjoint operator)

  - No correspondence principle
  - No uncertainty relations

Quantum estimation

- What about time and temperature in quantum mechanics?
- The "resources" involved in quantum-enahnced technology are entanglement, nonlocality, entropy, interferometric phase-shift, etc.. In general they are not observable quantities in strict sense (do not correspond to a selfadjoint operator)

Quantum estimation theory

Quantum estimation

$$\{\Pi_x\}_{x\in\mathcal{X}}$$

$$\chi = (x_1, x_2, \dots)$$

- Optimal measurements
- Ultimate bounds to precision

Quantum estimation

$$\{\Pi_x\}_{x \in \mathcal{X}}$$

$$\chi = (x_1, x_2, \dots)$$

- Probability density $p(x|\lambda) = \mathrm{Tr}\left[\varrho_\lambda\, \Pi_x\right]$

Let's go quantum (local) (1)

$$\{\Pi_x\}_{x\in\mathcal{X}}$$

$$\mathcal{X} = (x_1, x_2, \dots)$$

- probability density $p(x|\lambda) = \mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]$

- symm. log. derivative (SLD) $$\frac{L_\lambda \varrho_\lambda + \varrho_\lambda L_\lambda}{2} = \frac{\partial \varrho_\lambda}{\partial \lambda}$$

  selfadjoint, zero mean $\mathrm{Tr}\left[\varrho_\lambda L_\lambda\right] = 0$

- Fisher information $$F(\lambda) = \int dx\, \frac{\mathrm{Re}\left(\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]\right)^2}{\mathrm{Tr}[\varrho_\lambda \Pi_x]}$$

Let's go quantum (local) (2)

$$F(\lambda) \le \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2$$

parameter independent POVM

$$= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2$$

$$\le \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right]$$

$$= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]$$

Helstrom 1976
Braunstein & Caves 1994

- Fisher vs Quantum Fisher

$$F(\lambda) \le H(\lambda) \equiv \mathrm{Tr}[\varrho_\lambda L_\lambda^2] = \mathrm{Tr}[\partial_\lambda \varrho_\lambda L_\lambda]$$

- ultimate bound on precision $\mathrm{Var}(\lambda) \ge \dfrac{1}{MH(\lambda)}$

Optimal estimation scheme (quantum, local)

$$\{\Pi_x\}_{x\in\mathcal{X}}$$

$$\boldsymbol{\chi} = (x_1, x_2, \ldots)$$

Optimal measurement -> Fisher = quantum Fisher

It is projective! The spectral measure of the SLD

Optimal estimator -> saturation of CR inequality (classical postprocessing, e.g. Bayesian or MaxLik)

$$\boldsymbol{\chi} \mapsto \widehat{\lambda} = f(\boldsymbol{\chi})$$

General formulas (basis indepedent)

$$\frac{L_\lambda \varrho_\lambda + \varrho_\lambda L_\lambda}{2} = \frac{\partial \varrho_\lambda}{\partial \lambda}$$

Lyapunov equation

- Symmetric logarithmic derivative

- Quantum Fisher Information

$$H(\lambda) = 2 \int_0^\infty dt \, \mathrm{Tr}\left[\partial_\lambda \varrho_\lambda \exp\{-\varrho_\lambda t\} \, \partial_\lambda \varrho_\lambda \exp\{-\varrho_\lambda t\}\right]$$

General formulas

- Family of quantum states

$$\varrho_\lambda = \sum_n \varrho_n |\psi_n\rangle\langle\psi_n|$$

- Symmetric logarithmic derivative

$$L_\lambda = \sum_p \frac{\partial_\lambda \varrho_p}{\varrho_p} |\psi_p\rangle\langle\psi_p| + 2\sum_{n\neq m} \frac{\varrho_n - \varrho_m}{\varrho_n + \varrho_m} \langle\psi_m|\partial_\lambda\psi_n\rangle\, |\psi_m\rangle\langle\psi_n|$$

- Quantum Fisher Information

$$H(\lambda) = \sum_p \frac{\left(\partial_\lambda \varrho_p\right)^2}{\varrho_p} + 2\sum_{n\neq m} \frac{\left(\varrho_n - \varrho_m\right)^2}{\varrho_n + \varrho_m} \left|\langle\psi_m|\partial_\lambda\psi_n\rangle\right|^2$$

General formulas

- Family of quantum states

$$\varrho_\lambda = \sum_n \varrho_n |\psi_n\rangle\langle\psi_n|$$

- Symmetric logarithmic derivative

$$L_\lambda = \sum_p \frac{\partial_\lambda \varrho_p}{\varrho_p} |\psi_p\rangle\langle\psi_p| + 2 \sum_{n \neq m} \frac{\varrho_n - \varrho_m}{\varrho_n + \varrho_m} \langle\psi_m|\partial_\lambda \psi_n\rangle \, |\psi_m\rangle\langle\psi_n|$$

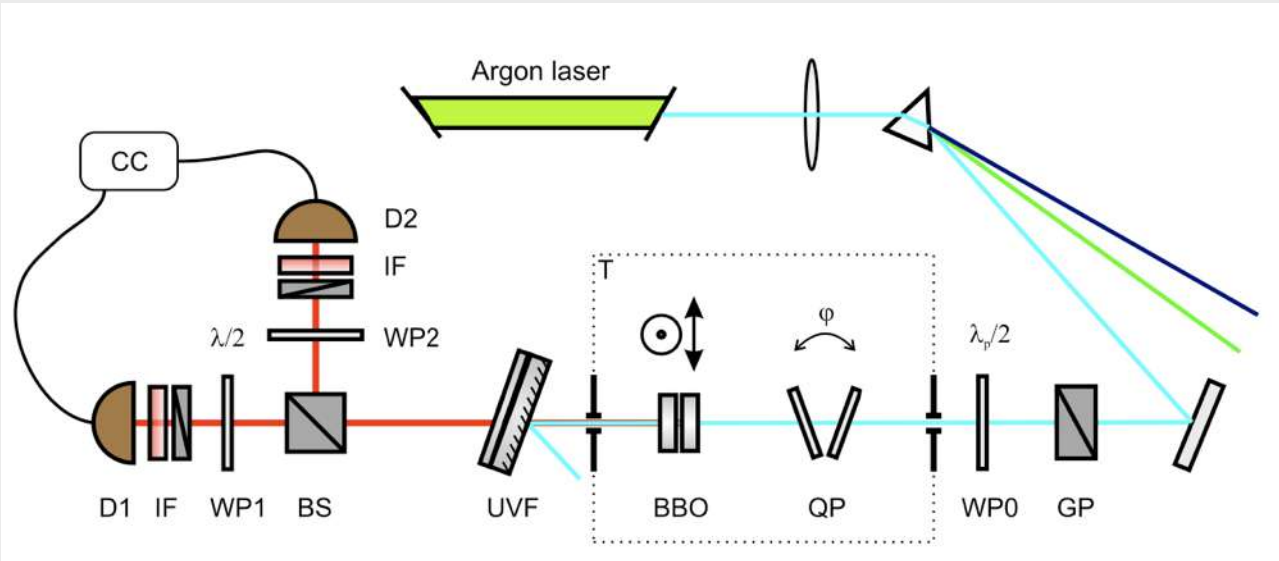
- Quantum Fisher Information

$$H(\lambda) = 8 \lim_{\epsilon \to 0} \frac{1 - F(\varrho_\lambda, \varrho_{\lambda+\epsilon})}{\epsilon^2}$$

Applications

- Quantum Interferometry
- Estimation of Gaussian states and operations
- Coupling constants (e.g. nonlinear interactions)
- Wave function of finite-dimensional systems
- Estimation of entanglement (and discord)
- Estimation in quantum critical systems
- Assessing quantum probes for complex systems
- Assessing quantum resources in metrology
- Assessing local vs global measurements
- Assessing criticality as a resource in metrology
- Probing quantum phase transitions
- Probing Hamiltonian terms
- New physics at gravity/QM interface
- ...

Estimation of entanglement (@INRIM)



$$|\psi_\phi\rangle = \cos\phi|HH\rangle + \sin\phi|VV\rangle$$

$$D_\phi = \cos^2\phi|HH\rangle\langle HH| + \sin^2\phi|VV\rangle\langle VV|$$

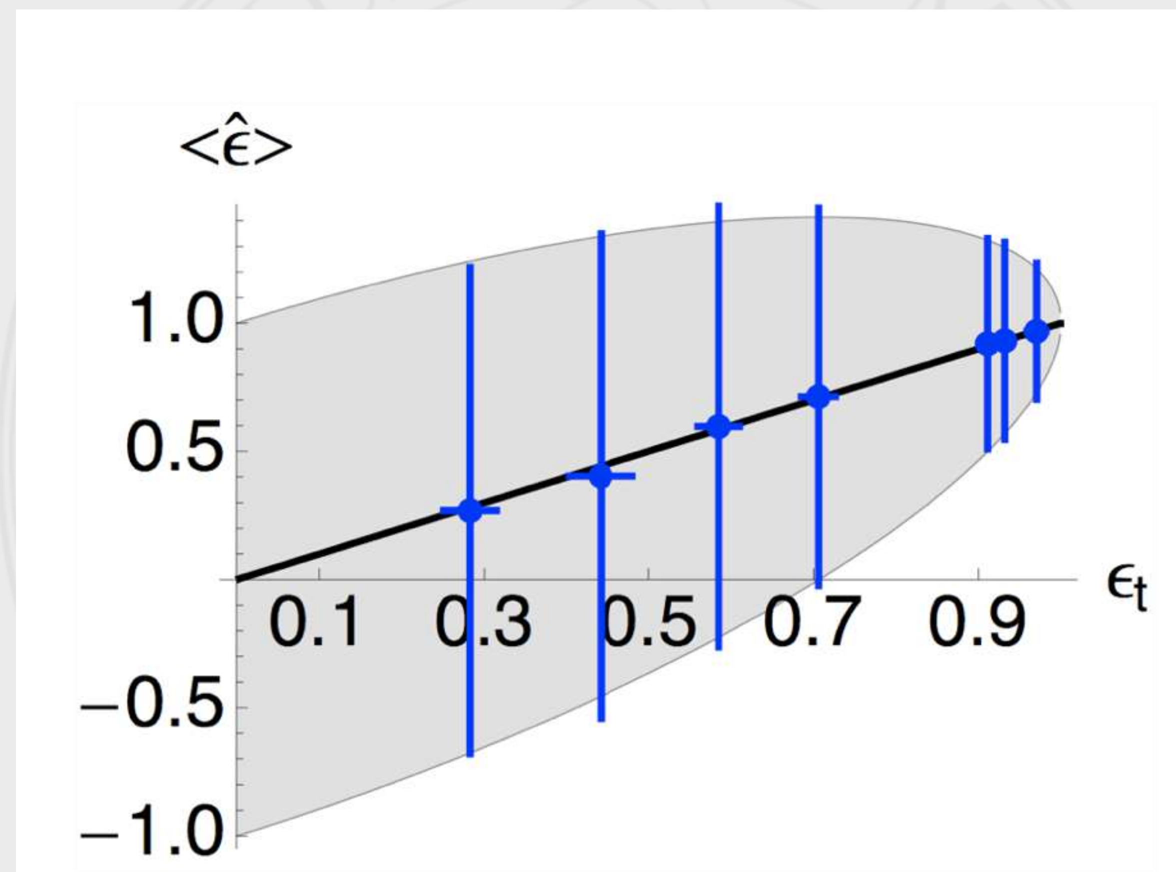
$$\varrho_\epsilon = p|\psi_\phi\rangle\langle\psi_\phi| + (1-p)D_\phi$$

$$\epsilon = p\sin 2\phi$$

optimal estimation by visibility measurements

Fisher information is monotone with entanglement

Estimation of entanglement is inherently inefficient

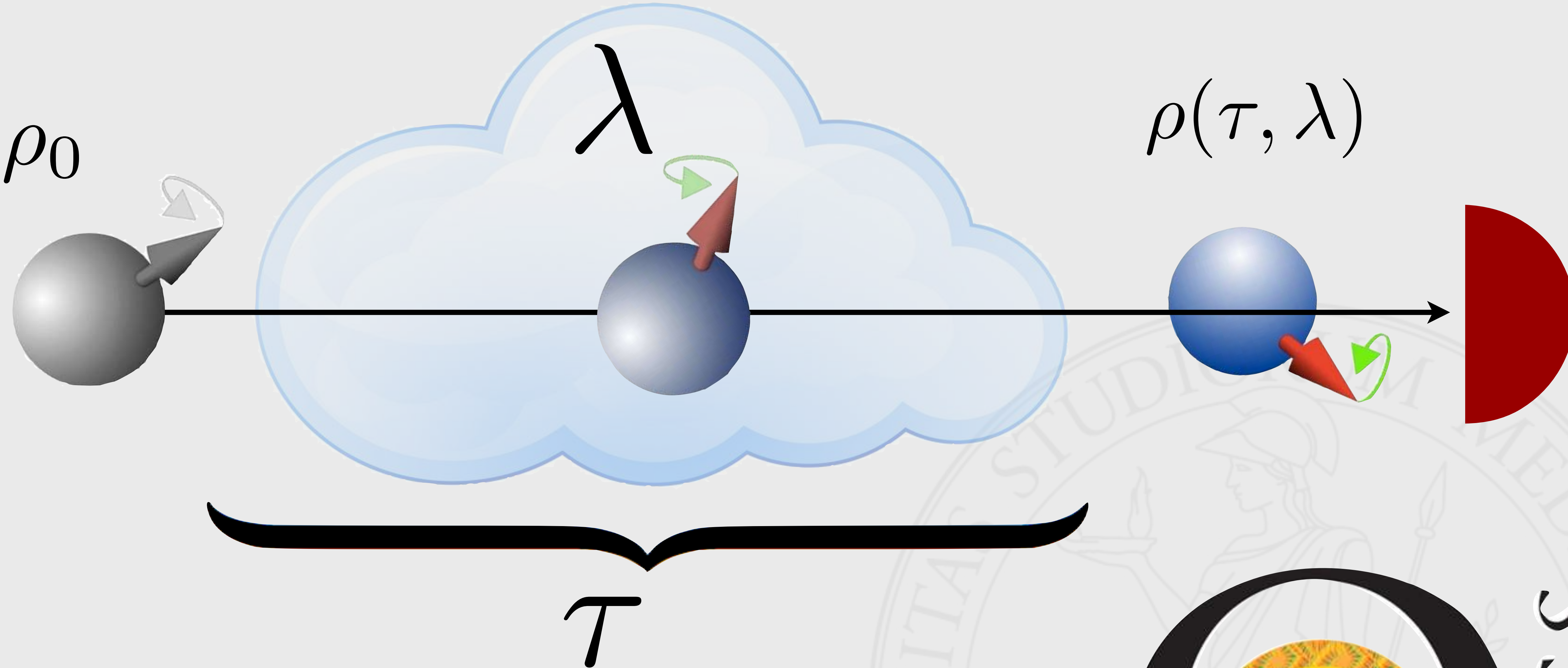


PRL 104, 100501 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

Experimental Estimation of Entanglement at the Quantum Limit

Giorgio Brida,[1] Ivo Pietro Degiovanni,[1] Angela Florio,[1,2] Marco Genovese,[1] Paolo Giorda,[3] Alice Meda,[1] Matteo G. A. Paris,[4,5] and Alexander Shurupov[6,1,7]

Quantum probes for complex systems



Maximizing the extraction of information by optimizing the preparation of the probe, the interaction time and the measurement at the output.

Quantum probes for complex systems

Quantum probes for the cutoff frequency of Ohmic environments

Claudia Benedetti,[1] Fahimeh Salari Sehdaran,[2] Mohammad H. Zandi,[2] and Matteo G. A. Paris[1]
[1]*Quantum Technology Lab, Physics Department, Università degli Studi di Milano, Milano, Italy*
[2]*Faculty of Physics, Shahid Bahonar University of Kerman, Kerman, Iran*

PRA **97**, 012126 (2018)

Quantum thermometry by single-qubit dephasing

Sholeh Razavian,[1] Claudia Benedetti,[2] Matteo Bina,[2] Yahya Akbari-Kourbolagh,[3] and Matteo G. A. Paris[2,4]
[1]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[2]*Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[3]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[4]*INFN, Sezione di Milano, I-20133 Milano, Italy*

Eur. Phys. J. Plus **134,** 284 (2019)

Quantum metrology out of equilibrium

Sholeh Razavian[1,2], Matteo G. A. Paris[1,3]
[1]*QCSE School, Villa del Grumello, I-22100, Como, Italy*
[2]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[3]*Quantum Technology Lab, Dipartimento di Fisica 'Aldo Pontremoli', Università degli Studi di Milano, I-20133 Milano, Italy*

Physica A **525**, 825 (2019)

Universal Quantum Magnetometry with Spin States at Equilibrium

Filippo Troiani[1,*] and Matteo G. A. Paris[2,3,†]
[1]*Centro S3, CNR-Istituto di Nanoscienze, I-41125 Modena, Italy*
[2]*Quantum Technology Lab, Dipartimento di Fisica dell'Università degli Studi di Milano, I-20133 Milano, Italy*
[3]*INFN, Sezione di Milano, I-20133 Milano, Italy*

(Received 2 November 2017; published 29 June 2018)

PRL **120**, 260503 (2018)

Quantum probes for complex systems

Continuous-variable quantum probes for structured environments

Matteo Bina,[1,*] Federico Grasselli,[2,1] and Matteo G. A. Paris[1,3]
[1] *Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[2] *Institut für Theoretische Physik III, Heinrich-Heine-Universität Düsseldorf, D-40225 Düsseldorf, Germany*
[3] *INFN, Sezione di Milano, I-20133 Milano, Italy*

PRA **97**, 012125 (2018)

The walker speaks its graph: global and nearly-local probing of the tunnelling amplitude in continuous-time quantum walks

J. Phys. A: Math. Theor. **52** (2019) 10530

Luigi Seveso, Claudia Benedetti and Matteo G A Paris

Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli,
Università degli Studi di Milano, I-20133 Milano, Italy

The quantum walker probes her coin parameter

Shivani Singh[*] and C. M. Chandrashekar[†]
The Institute of Mathematical Sciences, C. I. T, campus, Taramani, Chennai, 600113, India and
Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India

Matteo G. A. Paris[‡]
Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli,
Università degli Studi di Milano, I-20133 Milano, Italia

PRA **99**, 052117 (2019)

Parameter-dependent measurements: no Cramer-Rao

$$
\begin{aligned}
F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\
&= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\
&\leq \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right] \\
&= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]
\end{aligned}
$$

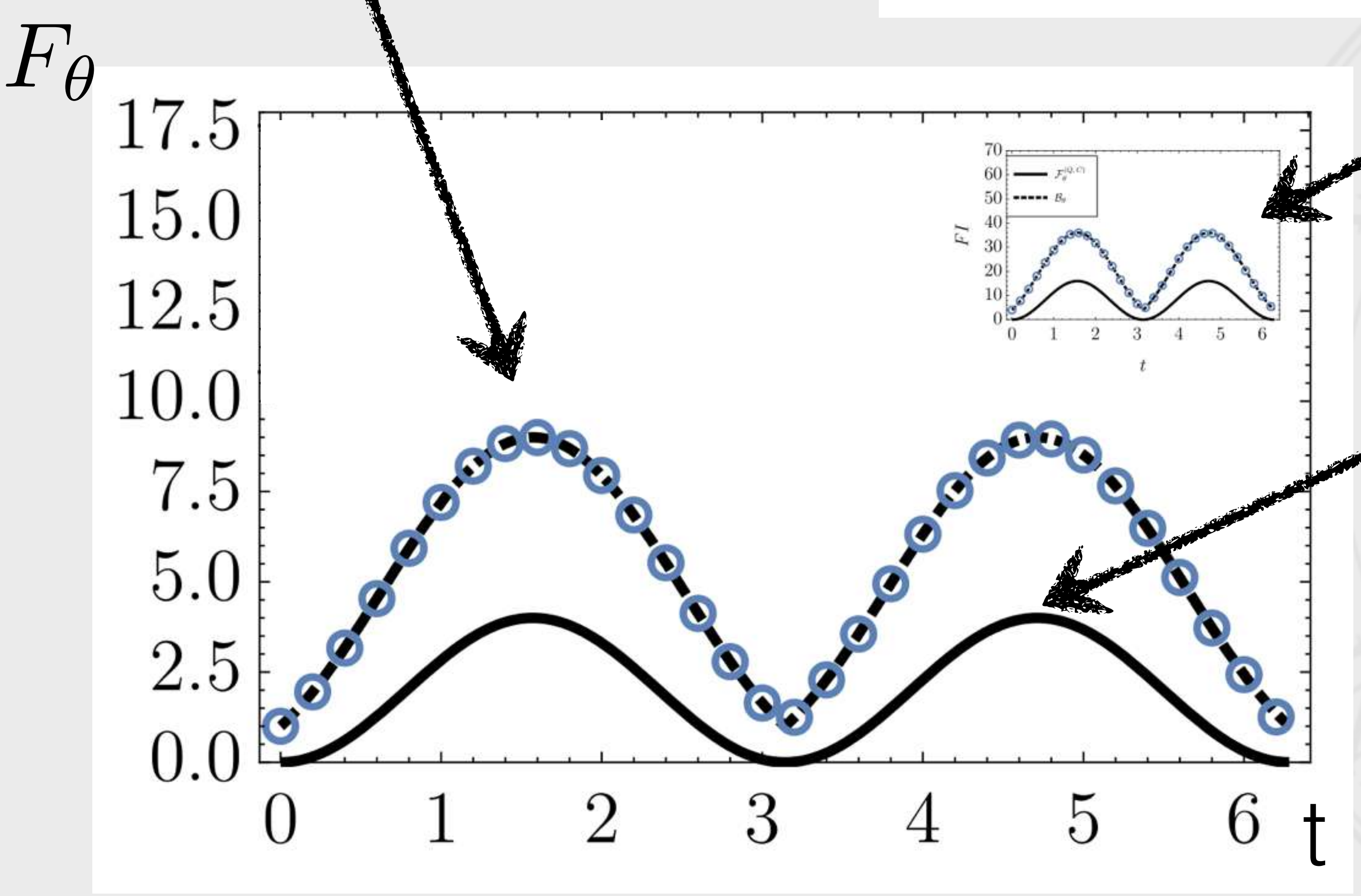
Estimation of the direction of an external field

$$H_\theta = \omega \left(\sin\theta\, \sigma_x + \cos\theta\, \sigma_z \right)$$

The same for qutrit

FI of energy measurement (optimized over the initial preparation)

$$(2\,|\sin(\omega t)| + 1)^2$$



QFI (optimized over the initial preparation)

$$4 \sin^2(\omega t)$$

Parameter-dependent measurements: no Cramer-Rao

$$
\begin{aligned}
F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\
&= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\
&\leq \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right] \\
&= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]
\end{aligned}
$$

Parameter-dependent measurements

$$\int dx\, \Pi(x) = \mathbb{I}$$

$$\int dx\, m_\lambda(x)\, \Pi_\lambda(x) = \mathbb{I}$$

Parameter-dependent <u>sample space</u>
(possible also in classical
estimation problem)

Parameter-dependent <u>POVM</u>
(an entirely novel quantum
degree of freedom)

New bound for parameter dependent sample space

$$\int dx\, m_\lambda(x)\, |x\rangle\langle x| = \mathbb{I}$$

$$\begin{aligned}
F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\
&= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\
&\leq \int dx\, \mathrm{Tr}[\Pi_x L_\lambda \varrho_\lambda L_\lambda] \\
&= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]
\end{aligned}$$

$$\mathrm{Var}(\hat{\lambda}) \geq \frac{1}{M\, F_X^{m_\lambda}}$$

$$F_X^{m_\lambda} = \int dx\, m_\lambda(x)\, \frac{[\mathrm{tr}(\Pi(x)\partial_\lambda \rho_\lambda)]^2}{\mathrm{tr}(\Pi(x)\rho_\lambda)} + \mathscr{I}_m$$

$$\geq 0$$

$$\mathscr{I}_m = \int dx\, m_\lambda(x)\; \mathrm{tr}(\Pi(x)\rho_\lambda)\, (\partial_\lambda \log m_\lambda(x))^2$$

L Seveso *et al* Phys. Rev. A **95**, 012111 (2017)

New bound for parameter dependent sample space

$$\int dx\, \boxed{m_\lambda(x)\ |x\rangle\langle x|} = \mathbb{I}$$

$$\cancel{\begin{aligned} F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\ &= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\ &\leq \int dx\, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right] \\ &= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2] \end{aligned}}$$

$$\mathrm{Var}(\hat{\lambda}) \geq \frac{1}{M\, F_X^{m_\lambda}}$$

$$F_X^{m_\lambda} = \int dx\, m_\lambda(x) \frac{[\mathrm{tr}(\Pi(x)\partial_\lambda \rho_\lambda)]^2}{\mathrm{tr}(\Pi(x)\rho_\lambda)} + \mathscr{I}_m$$

$$\geq 0$$

Achievable ?

What about optimal measurement?

$$\mathscr{I}_m = \int dx\, m_\lambda(x)\ \mathrm{tr}(\Pi(x)\rho_\lambda)\,(\partial_\lambda \log m_\lambda(x))^2$$

L Seveso *et al* Phys. Rev. A **95**, 012111 (2017)

Parameter-dependent measurements

$$\int dx\, m_\lambda(x)\, \Pi_\lambda(x) = \mathbb{I}$$

Parameter-dependent <u>sample space</u>
(possible also in classical estimation problem)

Parameter-dependent <u>POVM</u>
(an entirely novel quantum degree of freedom)

New bound for parameter dependent POVMs

$$F_X(\lambda) = \int d\nu \left\{ \frac{[\mathrm{tr}(\Pi_\lambda(x)\partial_\lambda\rho_\lambda)]^2}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} + \frac{[\mathrm{tr}(\partial_\lambda\Pi_\lambda(x)\rho_\lambda)]^2}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} + \frac{2\,\mathrm{tr}(\Pi_\lambda(x)\partial_\lambda\rho_\lambda)\,\mathrm{tr}(\partial_\lambda\Pi_\lambda(x)\rho_\lambda)}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} \right\}$$

$$F_X(\lambda) \leq \left[\sqrt{H(\lambda)} + \sqrt{\mathscr{K}_X(\lambda)}\right]^2$$

(projective POVMs)

$$\mathscr{K}_X(\lambda) = 4\int dx\, \langle \partial_\lambda x|\rho_\lambda|\partial_\lambda x\rangle$$

Achievable ?

What about optimal measurement?

L Seveso *et al* Phys. Rev. A **95**, 012111 (2017)

New bound for parameter dependent POVMs

- gravimetry with a quantum mechanical oscillator

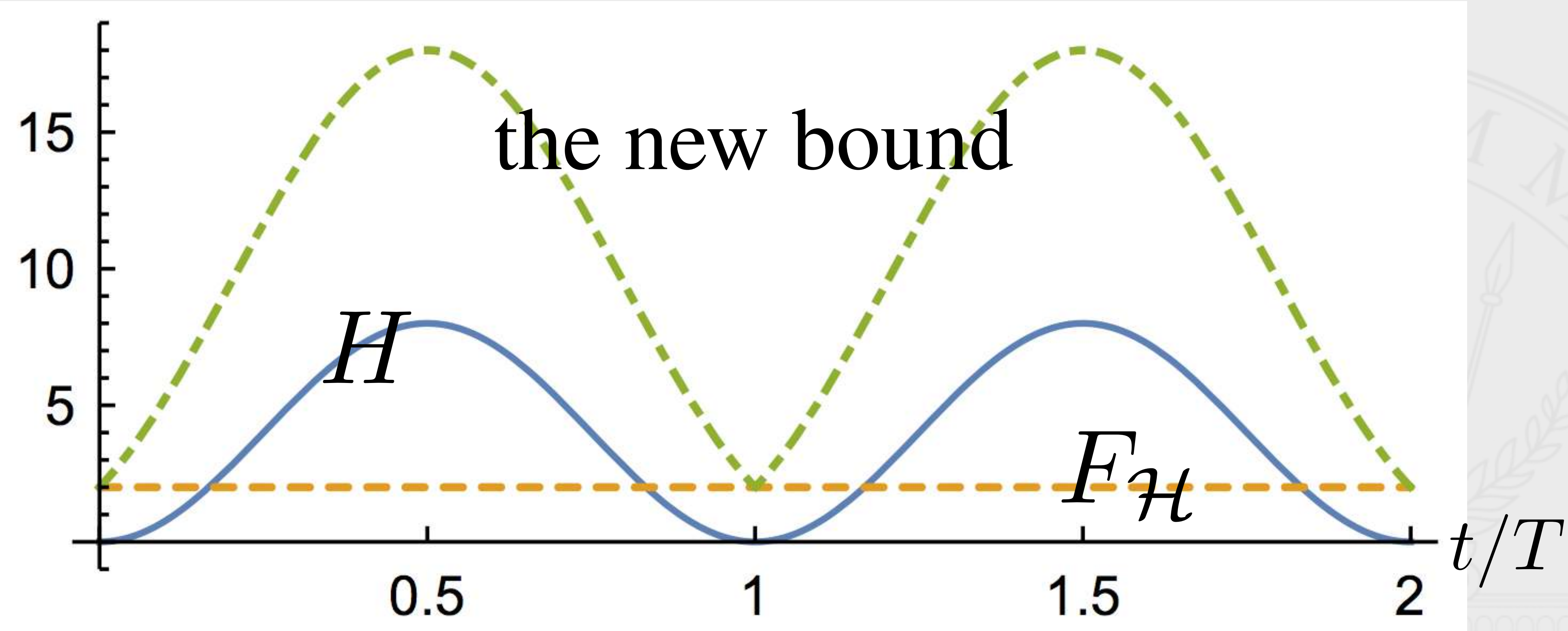
$$\mathcal{H} = p^2/2m + kx^2/2 + mgx$$

- prepare the oscillator in a coherent state

$$H(g) = 8m/\omega^3 \sin^2 \omega t/2$$

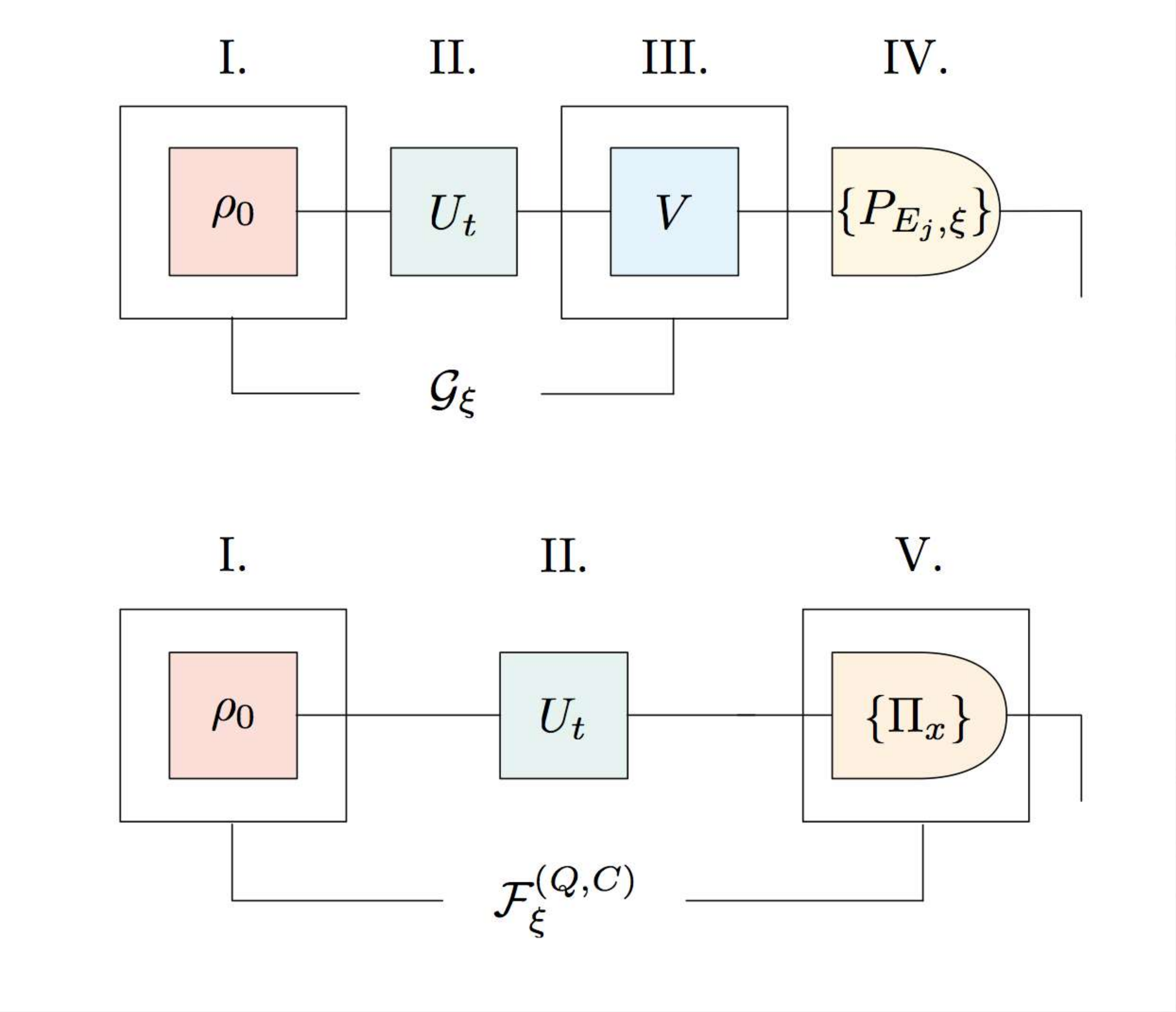
$$F_{\mathcal{H}}(g) = 2m/\omega^3$$ measurement of energy (Hamiltonian)

New bound for parameter dependent POVMs



New bound: estimating a Hamiltonian parameter beyond CR

$$H \equiv H_\xi \qquad H \neq \xi G$$



CTRL-energy measurements

regular strategies

New bound: estimating a Hamiltonian parameter beyond CR

PROPOSITION 1. *Given a finite-dimensional quantum system with Hamiltonian $H_\xi \in \mathsf{Her}_d(\mathbb{C})$ and general parameter $\xi \in \Xi$, the performance of any non-regular estimation strategy based on a controlled energy measurement $\mathscr{M}_{V,\xi}$ is bounded as follows. The maximum extractable information $\mathcal{G}_\xi$ obeys the inequality*

$$\mathcal{G}_\xi \leq \left(\sigma[\mathfrak{g}_{U,\xi}] + \sigma[\mathfrak{g}_{S,\xi}]\right)^2 \ ,$$

where $U_t = exp(-itH)$ is the unitary encoding, S_ξ is the similarity transformation diagonalizing H_ξ, $\mathfrak{g}_{U,\xi}$ (resp., $\mathfrak{g}_{S,\xi}$*) is the generator of U_t* (resp., S_ξ*), i.e.*

$$\mathfrak{g}_{U,\xi} = i\partial_\xi U_t U_t^\dagger \ , \qquad \mathfrak{g}_{S,\xi} = i\partial_\xi S_\xi S_\xi^\dagger \ ,$$

and $\sigma(\cdot)$ denotes the spectral gap.

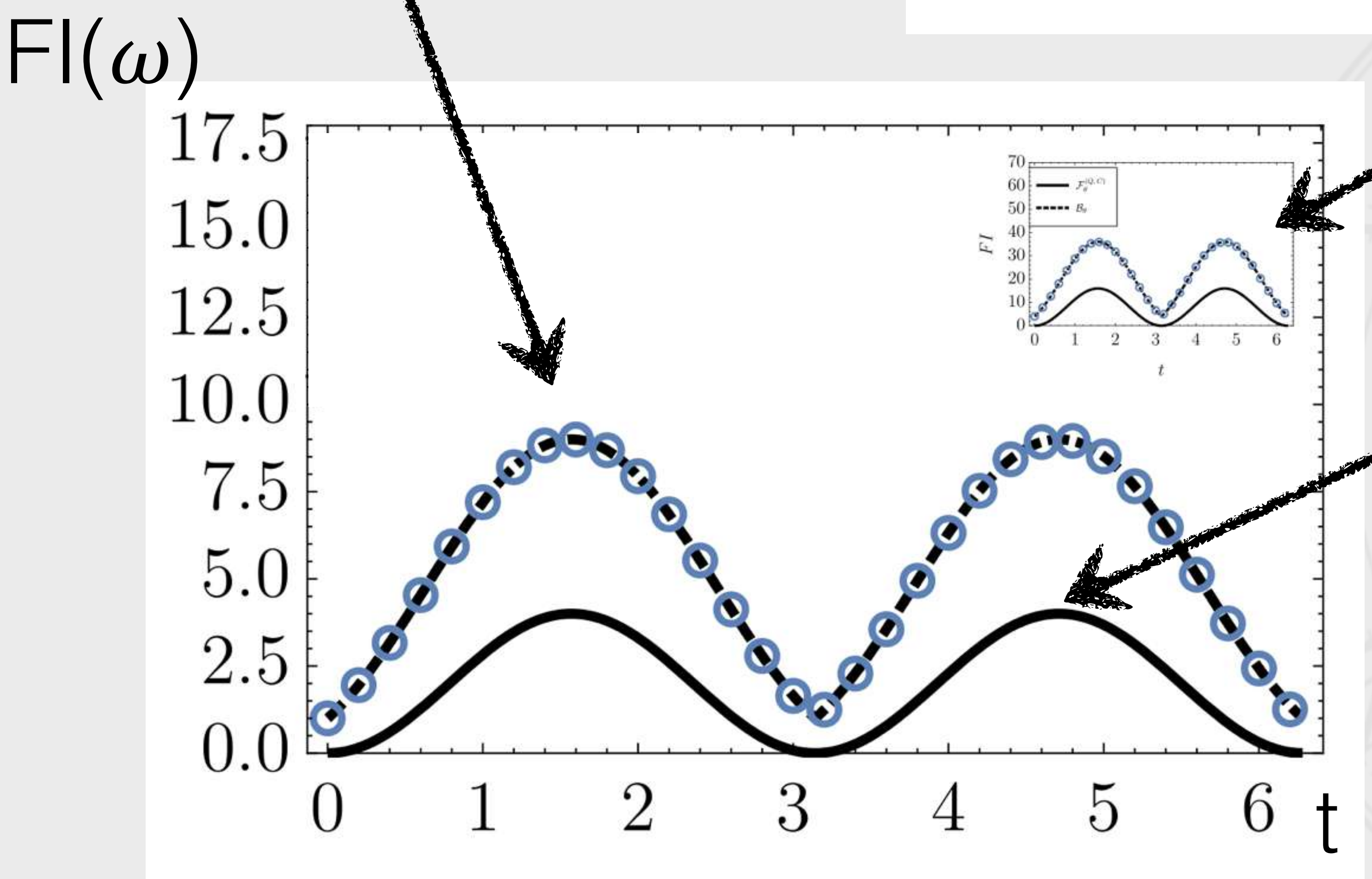
Estimation of the direction of an external field

$$H_\theta = \omega\,(\sin\theta\,\sigma_x + \cos\theta\,\sigma_z)$$

The same for qutrit

FI of energy measurement (optimized over the initial preparation)

$$(2\,|\sin(\omega t)| + 1)^2$$



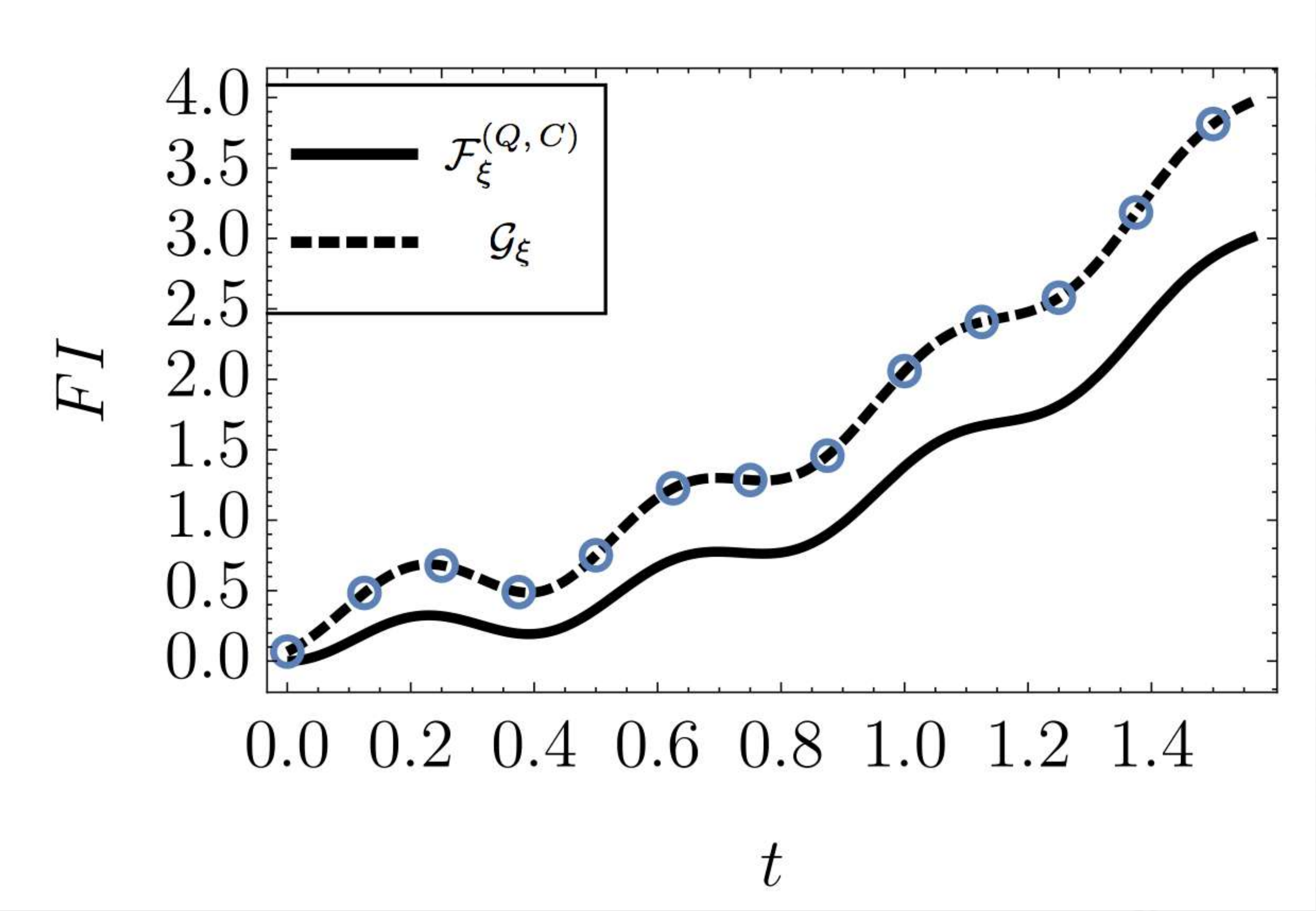
QFI (optimized over the initial preparation)

$$4\sin^2(\omega t)$$

NV-center (in diamond) magnetometry

$$H_{NV} = \mu\, B_z S_z + D\, S_z^2 + E\,(S_x^2 - S_y^2)$$

$$D \simeq \pi \times 1.44 GHz \quad E \simeq \pi \times 50 kHz$$



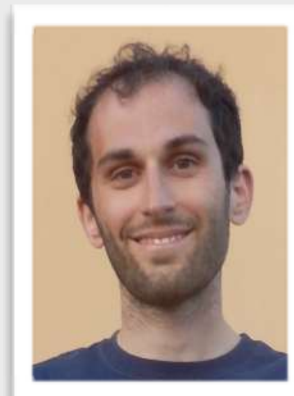
- Quantum estimation theory provides a set of fundamental tools and ideas of general interest for quantum information and quantum technology

- Parameter dependent quantum measurements are not bounded by the QCR bound and may be realised in practice: metrology may be more quantum-enhanced than previously thought.

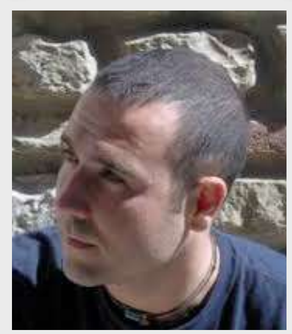


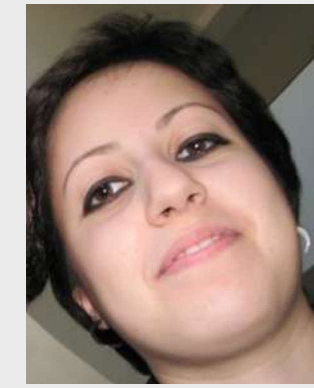




could not find a picture of Hamza



$$
\begin{aligned}
F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\
&= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\
&\leq \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right] \\
&= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]
\end{aligned}
$$





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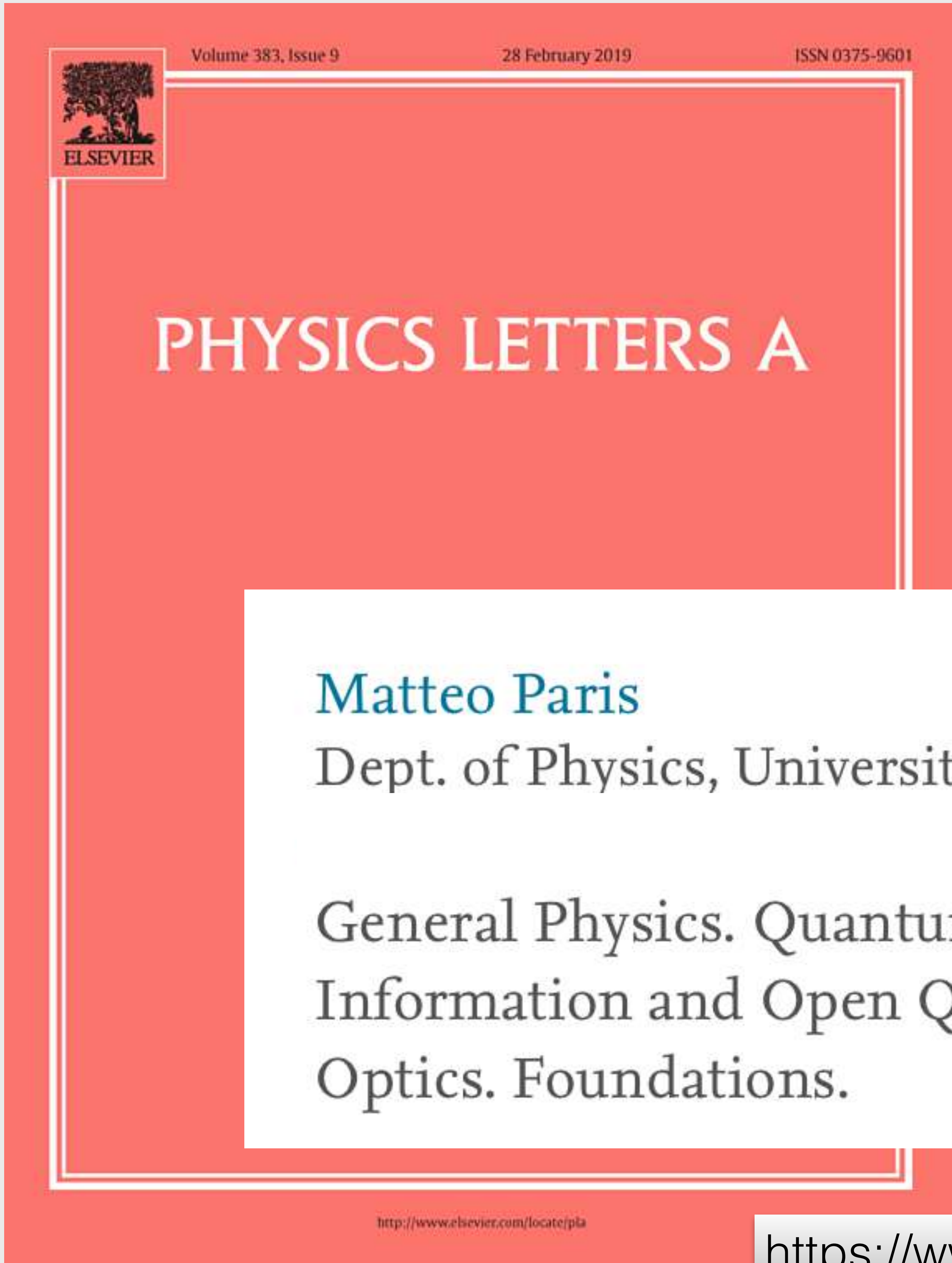

Matteo Paris
Dept. of Physics, University of Milan, Milan, Italy

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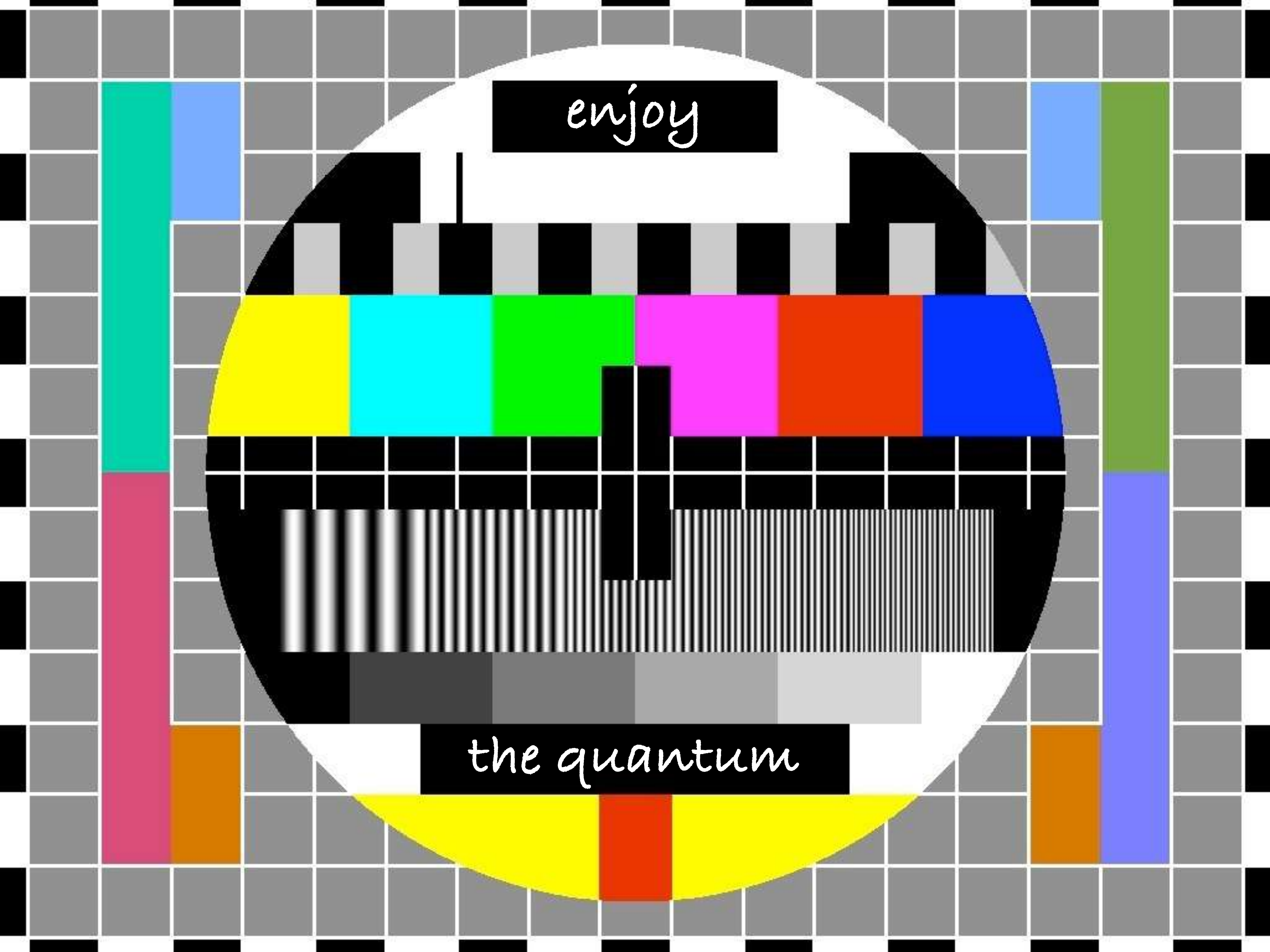
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