Part 2

Fast forward of adiabatic quantum dynamics of spin clusters: Geometry-dependent driving interactions

Review the FF scheme

Consider the Hamiltonian for spin systems to be characterized by a slowly-time-changing parameter $R(t)$ such as the exchange interaction, magnetic field, etc. Then we can study the eigenvalue problem for the time-independent Schrödinger equation

$$H_0(R)\mathbf{C}^{(n)}(R) = E_n(R)\mathbf{C}^{(n)}(R), \tag{2.1}$$

with

$$\mathbf{C}^{(n)}(R) = \begin{pmatrix} C_1^{(n)}(R) \\ \vdots \\ C_N^{(n)}(R) \end{pmatrix}, \tag{2.2}$$

$R(t) = R_0 + \epsilon t$ adiabatically-changing

$\epsilon \ll 1$ parameter

$$\Psi_0^{(n)}(R(t)) = \mathbf{C}^{(n)}(R(t)) \exp\left(-\frac{i}{\hbar}\int_0^t E_n(R(t'))dt'\right) e^{i\xi_n(R(t))} \tag{2.4}$$

is a quasiadiabatic state, i.e., adiabatically evolving state, where ξ_n is the adiabatic phase

$$\xi_n(R(t)) = i\int_0^t dt' \mathbf{C}^{(n)\dagger}\partial_t \mathbf{C}^{(n)} = i\epsilon\int_0^t dt' \mathbf{C}^{(n)\dagger}\partial_R \mathbf{C}^{(n)}. \tag{2.5}$$

The $\Psi_0^{(n)}(R(t))$ in Eq. (2.4) is not a solution of the TDSE. To make it satisfy the TDSE, we must regularize the Hamiltonian as

$$H_0^{\rm reg}(R(t)) = H_0(R(t)) + \epsilon\tilde{\mathcal{H}}_n(R(t)). \tag{2.6}$$

Then the TDSE becomes

$$i\hbar\frac{\partial}{\partial t}\Psi_0^{(n)}(R(t)) = (H_0 + \epsilon\tilde{\mathcal{H}}_n)\Psi_0^{(n)}(R(t)). \tag{2.7}$$

We see

the eigenvalue problem in Eq. (2.1) of $O(\epsilon^0)$ and the algebraic equation for $\tilde{\mathcal{H}}_n$,

$$\tilde{\mathcal{H}}_n\mathbf{C}^{(n)}(R) = i\hbar\partial_R\mathbf{C}^{(n)}(R) - i\hbar(\mathbf{C}^{(n)\dagger}\partial_R\mathbf{C}^{(n)})\mathbf{C}^{(n)}(R), \tag{2.8}$$

of $O(\epsilon^1)$. Equation (2.8) is the core of the present study.

The state (2.4) and TDSE (2.7) are working on a very slow timescale. We will modify them so that they can work on a laboratory timescale.

With time t rescaled by the advanced time $\Lambda(t)$, the fast-forward state is introduced as

$$\begin{aligned}\Psi_{\mathrm{FF}}^{(n)}(t) &\equiv \Psi_0^{(n)}(R(\Lambda(t))) \\ &= \mathbf{C}^{(n)}(R(\Lambda(t)))\exp\left(-\frac{i}{\hbar}\int_0^t E_n(R(\Lambda(t')))dt'\right) \\ &\quad \times e^{i\xi_n(R(\Lambda(t)))},\end{aligned} \tag{2.9}$$

where $\Lambda(t)$ is defined by

$$\Lambda(t) = \int_0^t \alpha(t')dt', \tag{2.10}$$

$$\alpha(t) = \bar{\alpha} - (\bar{\alpha} - 1)\cos\left(\frac{2\pi}{T_{\mathrm{FF}}}t\right), \tag{2.11}$$

where $\bar{\alpha}$ is the mean value of $\alpha(t)$ and is given by $\bar{\alpha} = T/T_{\mathrm{FF}}$.

Then, by taking the time derivative of $\Psi_{\mathrm{FF}}^{(n)}$ in Eq. (2.9) and using the equalities $\partial_t \mathbf{C}^{(n)}(R(\Lambda(t))) = \alpha\epsilon\,\partial_R \mathbf{C}^{(n)}$ and $\partial_t \xi_n(R(\Lambda(t))) = i\mathbf{C}^{(n)\dagger}\partial_t \mathbf{C}^{(n)} = i\alpha\epsilon\mathbf{C}^{(n)\dagger}\partial_R \mathbf{C}^{(n)}$, we have

$$
\begin{aligned}
i\hbar\dot{\Psi}_{\mathrm{FF}}^{(n)} = \{&i\hbar\alpha\epsilon[\partial_R \mathbf{C}^{(n)} - (\mathbf{C}^{(n)\dagger}\partial_R \mathbf{C}^{(n)})\mathbf{C}^{(n)}] + E\mathbf{C}^{(n)}\} \\
&\times \exp\left(-\frac{i}{\hbar}\int_0^t E_n(R(\Lambda(t')))dt'\right) e^{i\xi_n(R(\Lambda(t)))}.
\end{aligned}
\tag{2.13}
$$

Using the definition of $\Psi_{\mathrm{FF}}^{(n)}(t)$ and taking the asymptotic limit $\bar{\alpha} \to \infty$ and $\epsilon \to 0$ under the constraint that $\bar{\alpha} \cdot \epsilon \equiv \bar{v}$ is finite, we obtain

$$\begin{aligned} i\hbar \frac{\partial \Psi_{\mathrm{FF}}^{(n)}}{\partial t} &= [H_0(R(\Lambda(t))) + v(t)\tilde{\mathcal{H}}_n(R(\Lambda(t)))]\Psi_{\mathrm{FF}}^{(n)} \\ &\equiv H_{\mathrm{FF}}^{(n)}\Psi_{\mathrm{FF}}^{(n)}. \end{aligned} \tag{2.14}$$

Time scaling factor is now replaced by velocity function as

$$\begin{aligned} v(t) &= \lim_{\epsilon \to 0, \alpha \to \infty} \epsilon \alpha(t) \\ &= \bar{v}\left(1 - \cos\frac{2\pi}{T_{FF}}t\right), \end{aligned} \tag{2.14}$$

where $\bar{v} = \lim_{\epsilon \to 0, \alpha \to \infty} \epsilon \bar{\alpha} (= finite)$ is the mean of $v(t)$.

$$\begin{aligned} R(\Lambda(t)) &= R_0 + \lim_{\epsilon \to 0, \bar{\alpha} \to \infty} \varepsilon \Lambda(t) \\ &= R_0 + \int_0^t v(t')dt' \\ &= R_0 + \bar{v}\left[t - \frac{T_{FF}}{2\pi}\sin\left(\frac{2\pi}{T_{FF}}t\right)\right], \\ &\quad \text{for } 0 \le t \le T_{FF}. \end{aligned}$$

Fast forward of adiabatic dynamics of regular spin clusters

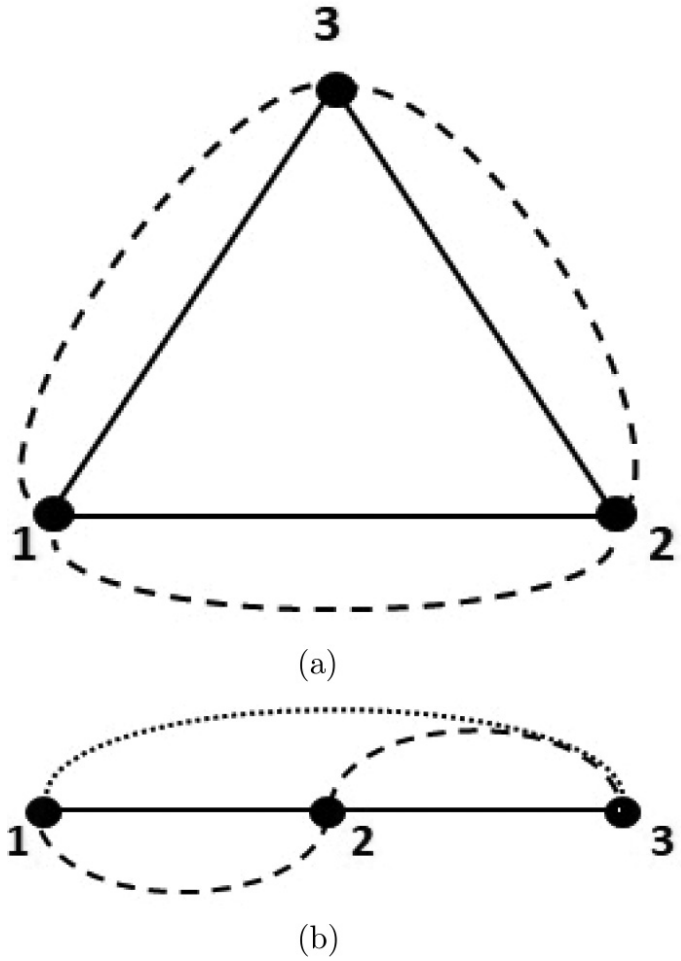


FIG. 1. (a) Regular triangle and (b) open linear three-spin chain. Solid lines stand for the original exchange interactions. Dashed and dotted lines show the pairwise regularization interactions. Each line species denotes the geometrically identical regularization interaction.

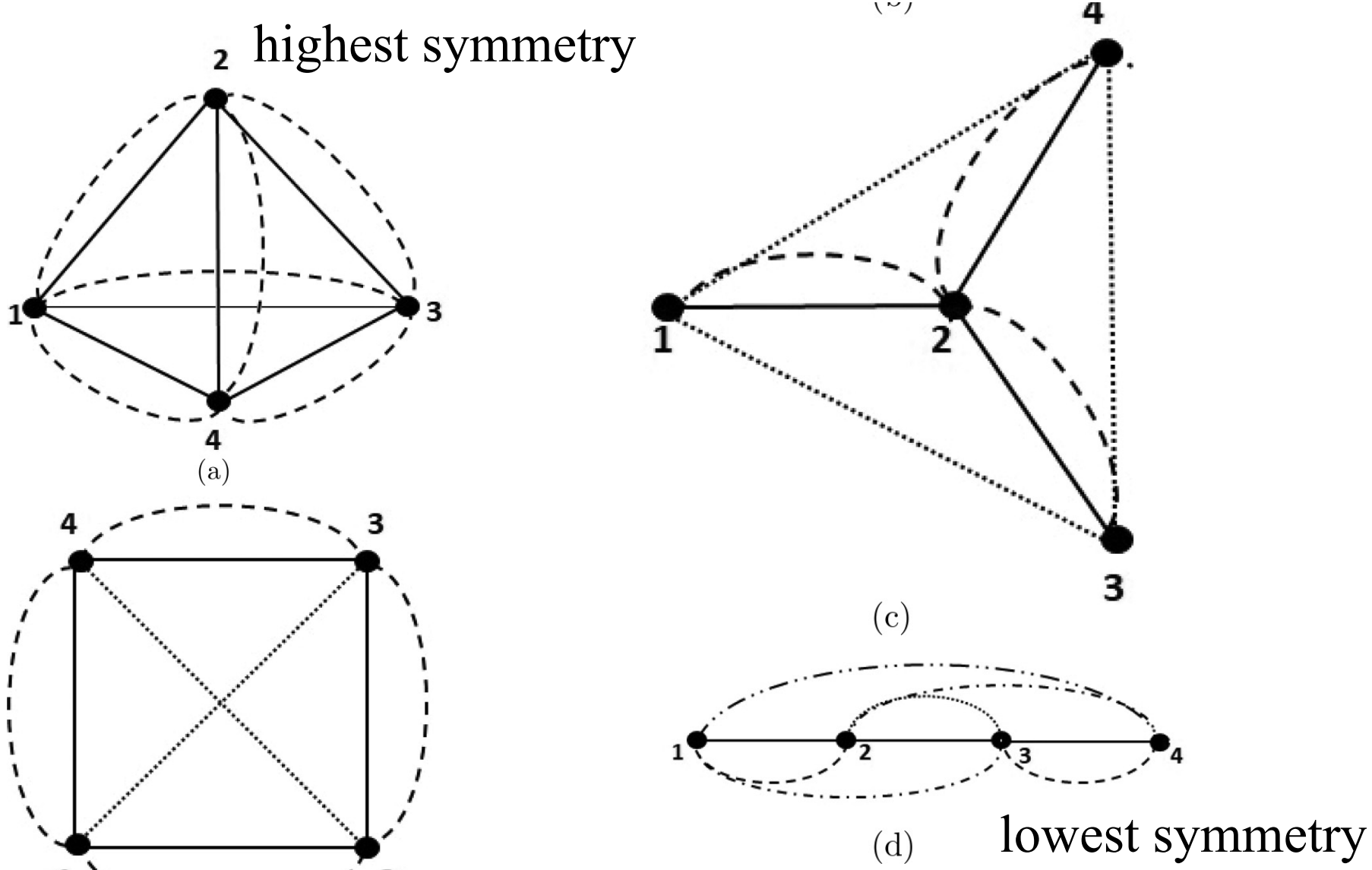


FIG. 2. (a) Triangular pyramid, (b) square, (c) primary star graph, and (d) open linear four-spin chain. Solid lines stand for the original exchange interactions. Dashed, dotted, dot-dashed, and double-dot–dashed lines show the pairwise regularization interactions. Each line species denotes the geometrically identical regularization interaction.

As an original (reference) model, we choose the transverse Ising mode, whose Hamiltonian for N spin systems is written as

$$H_0 = J(R(t)) \sum_{(i,j)\in \mathrm{NN}} \sigma_i^z \sigma_j^z - \tfrac{1}{2} B_x(R(t)) \sum_{i=1}^{N} \sigma_i^x, \qquad (3.1)$$

where $J(R(t)) = R(t) = R_0 + \epsilon t$ and $B_x(R(t)) = B_0 - R(t)$, with $\epsilon \ll 1$, are the adiabatically changing exchange interaction and transverse magnetic field, respectively, and $(i, j) \in$ NN denotes nearest-neighboring pairs. Using the spin configuration bases, the dimension of Hilbert space is 2^N.

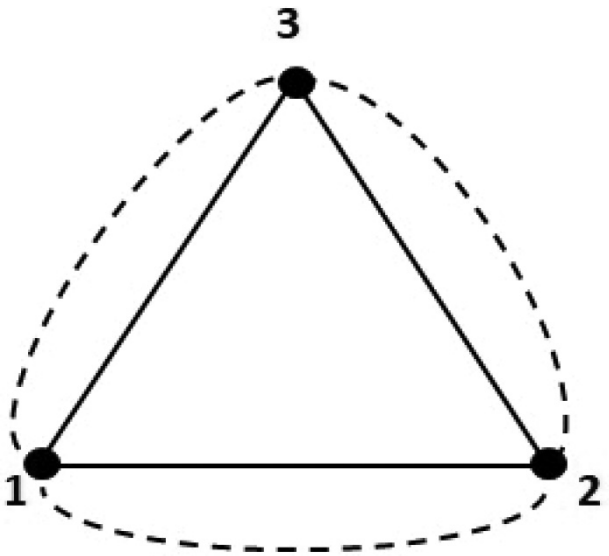
the regularization term consisting of pairwise interactions described by $\tilde{W}_{ij}^{yz} = \tilde{W}_{ij}^{yz}(\epsilon t)$ and three-body interactions $\tilde{Q}_{ijk}^{xyz} = \tilde{Q}_{ijk}^{xyz}(\epsilon t)$. Other possible contributions such as a single-particle energy due to the y component of the magnetic field ($\tilde{B}_y$), pairwise interaction $\tilde{W}_{ij}^{xy}$, and three-body interaction $\tilde{Q}_{ijk}^{xxy}$ lead to an incompatible algebraic equation (2.8) and should be excluded. The candidate for the regularization Hamiltonian $\tilde{\mathcal{H}}$ then takes the form

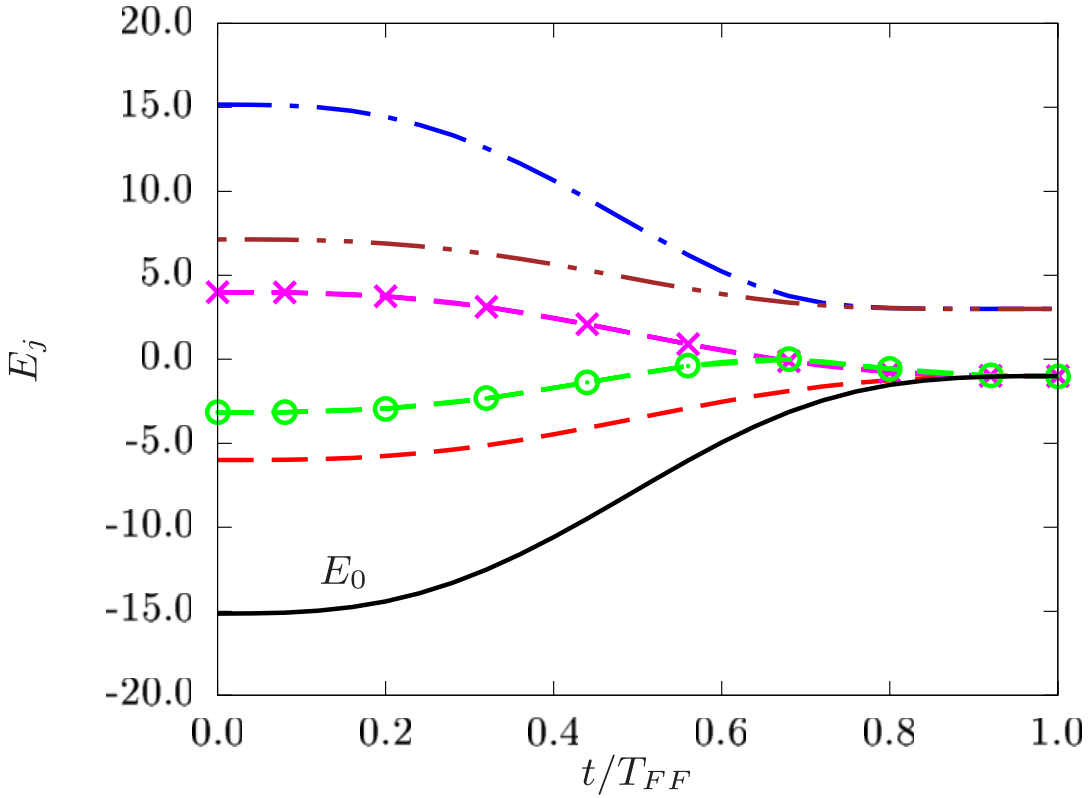
$$\tilde{\mathcal{H}} = \sum_{(i,j)\in\text{all}} \tilde{W}_{ij}^{yz}\left(\sigma_i^y\sigma_j^z + \sigma_i^z\sigma_j^y\right) + \sum_{(i,j,k)\in\text{all}} \tilde{Q}_{ijk}^{xyz}\left(\sigma_i^x\sigma_j^y + \sigma_i^y\sigma_j^x\right)\cdot\sigma_k^z, \tag{3.2}$$

Since regular spin clusters have geometric symmetry, some of the pair-wise interaction ($\tilde{W}_{ij}^{yz}$) are degenerate and the reduced number of independent interactions should be equal to the number of independent equations in the core equation.

A. Regular triangle

In the case of the regular triangle, the eigenvalue for the ground state is $E_0 = -\sqrt{B_x^2 + 2B_xJ + 4J^2} - \frac{B_x}{2} + J$. We have confirmed in Fig. 3(a) that all eight eigenvalues show no mutual energy crossing in the fast-forward time range where we choose $J(R(\Lambda(t))) \equiv R(\Lambda(t))$ and $B_x(R(\Lambda(t))) \equiv B_0 - R(\Lambda(t))$, with $R(\Lambda(t))$ defined in Eq. (2.16).





In this section we investigate a regular triangle and open linear three-spin chains in Fig. 1. We use the spin configuration bases as $|1\rangle = |\uparrow\uparrow\uparrow\rangle$, $|2\rangle = |\uparrow\uparrow\downarrow\rangle$, $|3\rangle = |\uparrow\downarrow\uparrow\rangle$, $|4\rangle = |\downarrow\uparrow\uparrow\rangle$, $|5\rangle = |\uparrow\downarrow\downarrow\rangle$, $|6\rangle = |\downarrow\uparrow\downarrow\rangle$, $|7\rangle = |\downarrow\downarrow\uparrow\rangle$, and $|8\rangle = |\downarrow\downarrow\downarrow\rangle$.

The components of the eigenvector for the ground state are $C_1 = V_1\zeta$, $C_2 = V_2\zeta$, $C_3 = V_3\zeta$, $C_4 = V_4\zeta$, $C_5 = V_5\zeta$, $C_6 = V_6\zeta$, $C_7 = V_7\zeta$, and $C_8 = V_8\zeta$, where $V_1 = V_8 = 1$, $V_2 = V_3 = V_4 = V_5 = V_6 = V_7 = \frac{2\sqrt{B_x^2+2B_xJ+4J^2}+B_x+4J}{3B_x}$, and $\zeta = \frac{1}{\sqrt{2+6V_2^2}}$. Here we see the symmetry $C_1 = C_8$ and $C_2 = C_3 = C_4 = C_5 = C_6 = C_7$. From the R derivative of the normalization ($\sum_{j=1}^{8} C_j^2 = 2C_1^2 + 6C_2^2 = 1$), we see that

$$C_1 \frac{\partial C_1}{\partial R} + 3C_2 \frac{\partial C_2}{\partial R} = 0, \tag{4.1}$$

and then the adiabatic phase $\xi = 0$.

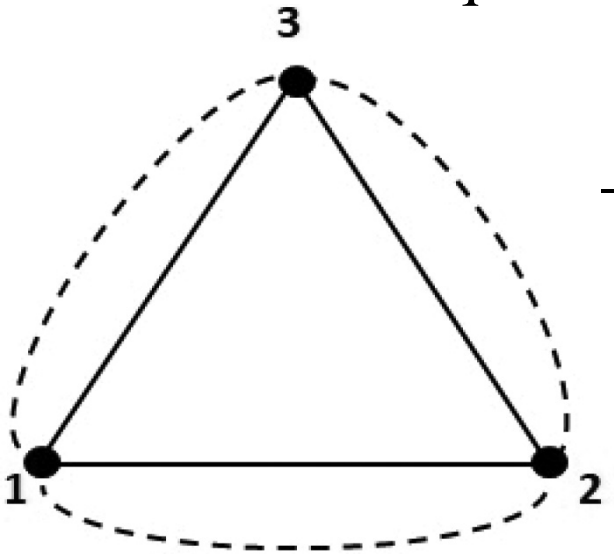
Initial ground state = product state

$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = \frac{1}{2\sqrt{2}}$.

three-body interaction. Three $\tilde{W}_{ij}^{yz}$'s should be identical due to the triangular symmetry in Fig. 1(a). Therefore, the unknown pairwise interaction is the only one, $\tilde{W} \equiv \tilde{W}_{ij}^{yz}$, independent of the pairs (i, j).

By using the spin configuration bases as above, the regularization Hamiltonian (3.2) is characterized by the matrix elements: $\tilde{\mathcal{H}}_{1j} = -\tilde{\mathcal{H}}_{j1} = -2i\tilde{W}$ with $j = 2, 3, 4$, $\tilde{\mathcal{H}}_{8j} = -\tilde{\mathcal{H}}_{j8} = -2i\tilde{W}$ with $j = 5, 6, 7$, and all other elements equal to zero. The explicit expression for $\tilde{\mathcal{H}}$ will help us solve Eq. (2.8).

Due to the symmetry of $\{C_j\}$, there are only two independent equations in Eq. (2.8):



$$-6\tilde{W}C_2 = \hbar\frac{\partial C_1}{\partial R}, \quad 2\tilde{W}C_1 = \hbar\frac{\partial C_2}{\partial R}. \tag{4.2}$$

$$\tilde{W} = \hbar \frac{\partial_R C_2}{2C_1} = \hbar(C_1 \partial_R C_2 - C_2 \partial_R C_1)$$
$$= \frac{B_x \frac{\partial J}{\partial R} - J \frac{\partial B_x}{\partial R}}{4\left(B_x^2 + 2B_x J + 4J^2\right)}. \tag{4.3}$$

The second equality in (4.3) is due to the normalization condition and Eq. (4.1). Including the regularization term followed by rescaling of time, the fast-forward Hamiltonian is written as

$$H_{\mathrm{FF}} = H_0(R(\Lambda(t))) + v(t)\tilde{\mathcal{H}}(R(\Lambda(t))), \tag{4.4}$$

with $H_0 = J(R(\Lambda(t)))(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z + \sigma_3^z\sigma_1^z) - \frac{1}{2}(\sigma_1^x + \sigma_2^x + \sigma_3^x)B_x(R(\Lambda(t)))$ and $v\tilde{\mathcal{H}} = v(t)\tilde{W}(R(\Lambda(t)))[(\sigma_1^y\sigma_2^z + \sigma_1^z\sigma_2^y) + (\sigma_2^y\sigma_3^z + \sigma_2^z\sigma_3^y) + (\sigma_3^y\sigma_1^z + \sigma_3^z\sigma_1^y)]$.

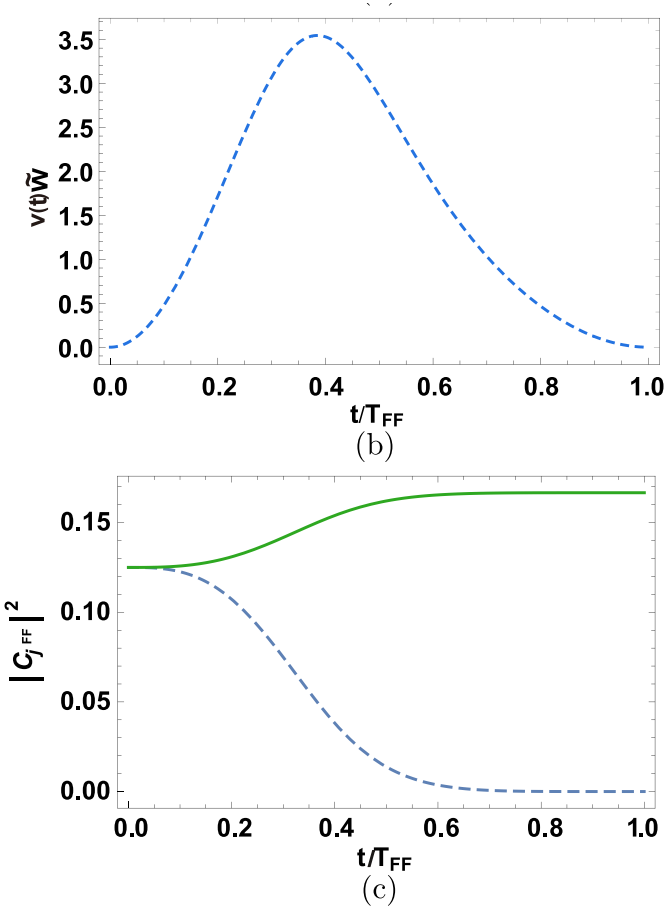
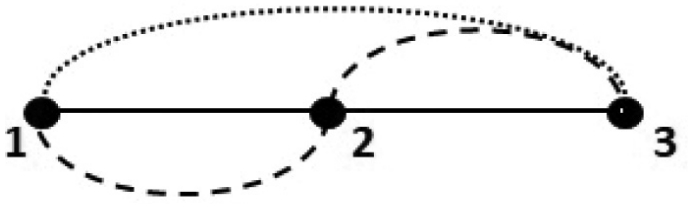
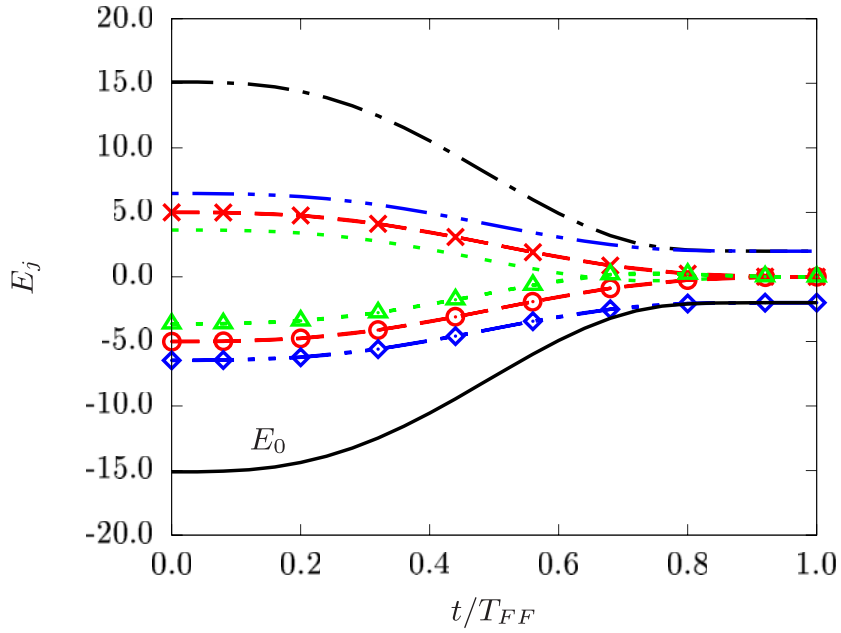


FIG. 3. Time dependence in the case of the regular triangle in the fast-forward time range where we choose $J = R(\Lambda(t))$ and $B_x = B_0 - R(\Lambda(t))$, with $R(\Lambda(t))$ defined in Eq. (2.16). The other parameters are $B_0 = 10$, $\bar{v} = 100$, $T_{\rm FF} = 0.1$, and $R_0 = 0$. (a) All eight eigenvalues. From the bottom, the second and fourth lines are doubly degenerate. (b) Regularization term $v(t)\tilde{W}$. (c) Probability amplitudes for the solution $\Psi_{\rm FF}(t)$ of the TDSE: $|C_2^{\rm FF}|^2 = |C_3^{\rm FF}|^2 = |C_4^{\rm FF}|^2 = |C_5^{\rm FF}|^2 = |C_6^{\rm FF}|^2 = |C_7^{\rm FF}|^2$ (solid line) and $|C_1^{\rm FF}|^2 = |C_8^{\rm FF}|^2$ (dashed line).

B. Open linear three-spin chains

In a similar way we can obtain the regularization term and fast-forward Hamiltonian in the case of open linear three-spin chains. In this case the eigenvalue for the ground state is $E_0 = -\frac{1}{6}[B_x + (\beta + \bar{\beta}) - \sqrt{3}i(\beta - \bar{\beta})]$, where $\beta = (18J^2B_x - 8B_x^3 + 6Ji\sqrt{48J^4 + 39B_x^2J^2 + 24B_x^4})^{1/3}$. We have confirmed in Fig. 4(a) that all eight eigenvalues show no mutual energy crossing in the fast-forward time range where we choose $J(R(\Lambda(t))) \equiv R(\Lambda(t))$ and $B_x(R(\Lambda(t))) \equiv B_0 - R(\Lambda(t))$, with $R(\Lambda(t))$ defined in Eq. (2.16).





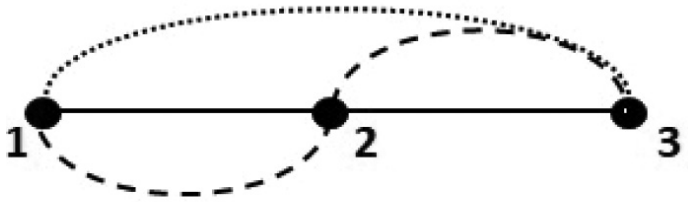
The components of the eigenvector for the ground state are $C_1 = C_8 = V_1\zeta$, $C_2 = C_4 = C_5 = C_7 = V_2\zeta$, and $C_3 = C_6 = V_3\zeta$, where $V_1 = \frac{3B_x^2 - 8JB_x - 4B_xE_0 - 4E_0^2 - 8E_0J}{4JB_x}$, $V_2 = -\frac{1}{2}V_1 - \frac{2J+E_0}{B_x}$, $V_3 = 1$, and $\zeta = \frac{1}{\sqrt{2V_1^2 + 4V_2^2 + 2}}$. Here we see the symmetry $C_1 = C_8$, $C_2 = C_4 = C_5 = C_7$, and $C_3 = C_6$. From the R derivative of the normalization ($\sum_{j=1}^{8} C_j^2 = 2C_1^2 + 4C_2^2 + 2C_3^2 = 1$), we see that

$$C_1 \frac{\partial C_1}{\partial R} + 2C_2 \frac{\partial C_2}{\partial R} + C_3 \frac{\partial C_3}{\partial R} = 0, \tag{4.5}$$

Initial ground state = product state

$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = \frac{1}{2\sqrt{2}}$.

The regularization Hamiltonian for the linear three-spin system can also be available without using the three-body interaction. Because of the geometric symmetry seen in Fig. 1(b), $\tilde{\mathcal{H}}$ is then characterized by two independent pairwise interactions $\tilde{W}_1 \equiv \tilde{W}_{12}^{yz} = \tilde{W}_{23}^{yz}$ and $\tilde{W}_2 \equiv \tilde{W}_{31}^{yz}$, where $\tilde{W}_1$ and $\tilde{W}_2$ correspond to the nearest-neighbor (NN) and next-nearest-neighbor (NNN) interactions, respectively. With use of the spin configuration bases, the matrix form for $\tilde{\mathcal{H}}$ in Eq. (3.2) is given by



$$\tilde{\mathcal{H}} = i\begin{pmatrix} 0 & -\tilde{W}_1-\tilde{W}_2 & -2\tilde{W}_1 & -\tilde{W}_1-\tilde{W}_2 & 0 & 0 & 0 & 0 \\ \tilde{W}_1+\tilde{W}_2 & 0 & 0 & 0 & 0 & -\tilde{W}_1+\tilde{W}_2 & 0 & 0 \\ 2\tilde{W}_1 & 0 & 0 & 0 & \tilde{W}_1-\tilde{W}_2 & 0 & \tilde{W}_1-\tilde{W}_2 & 0 \\ \tilde{W}_1+\tilde{W}_2 & 0 & 0 & 0 & 0 & -\tilde{W}_1+\tilde{W}_2 & 0 & 0 \\ 0 & 0 & -\tilde{W}_1+\tilde{W}_2 & 0 & 0 & 0 & 0 & \tilde{W}_1+\tilde{W}_2 \\ 0 & \tilde{W}_1-\tilde{W}_2 & 0 & \tilde{W}_1-\tilde{W}_2 & 0 & 0 & 0 & 2\tilde{W}_1 \\ 0 & 0 & -\tilde{W}_1+\tilde{W}_2 & 0 & 0 & 0 & 0 & \tilde{W}_1+\tilde{W}_2 \\ 0 & 0 & 0 & 0 & -\tilde{W}_1-\tilde{W}_2 & -2\tilde{W}_1 & -\tilde{W}_1-\tilde{W}_2 & 0 \end{pmatrix}.$$

Due to the symmetry of $\{C_j\}$, there are three independent equations in Eq. (2.8):

$$-2(\tilde{W}_1+\tilde{W}_2)C_2 - 2\tilde{W}_1 C_3 = \hbar\frac{\partial C_1}{\partial R}, \tag{4.7a}$$

$$(\tilde{W}_1+\tilde{W}_2)C_1 + (-\tilde{W}_1+\tilde{W}_2)C_3 = \hbar\frac{\partial C_2}{\partial R}, \tag{4.7b}$$

$$2\tilde{W}_1 C_1 + 2(\tilde{W}_1-\tilde{W}_2)C_2 = \hbar\frac{\partial C_3}{\partial R}. \tag{4.7c}$$

By using Eq. (4.5), Eq. (4.7c), for example, proves trivial. Then Eqs. (4.7), whose coefficient matrix has the rank 2, gives the solution

$$\tilde{W}_1 = -\frac{\hbar}{2}\frac{\partial(C_1 - C_3)}{\partial R}(C_1 + 2C_2 + C_3)^{-1},$$
$$\tilde{W}_2 = -\frac{\hbar}{2}\frac{\partial(C_1 - 2C_2 + C_3)}{\partial R}(C_1 + 2C_2 + C_3)^{-1}. \quad (4.8)$$

Including the regularization terms followed by rescaling of time, the fast-forward Hamiltonian is written as

$$H_{\mathrm{FF}} = H_0(R(\Lambda(t))) + v(t)\tilde{\mathcal{H}}(R(\Lambda(t))), \quad (4.9)$$

with $H_0 = J(R(\Lambda(t)))(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z) - \frac{1}{2}(\sigma_1^x + \sigma_2^x + \sigma_3^x)$ $B_x(R(\Lambda(t)))$ and $v\tilde{\mathcal{H}} = v(t)\tilde{W}_1(R(\Lambda(t)))[(\sigma_1^y\sigma_2^z + \sigma_1^z\sigma_2^y)$ $+ (\sigma_2^y\sigma_3^z + \sigma_2^z\sigma_3^y)] + v(t)\tilde{W}_2(R(\Lambda(t)))(\sigma_1^y\sigma_3^z + \sigma_1^z\sigma_3^y)$. The

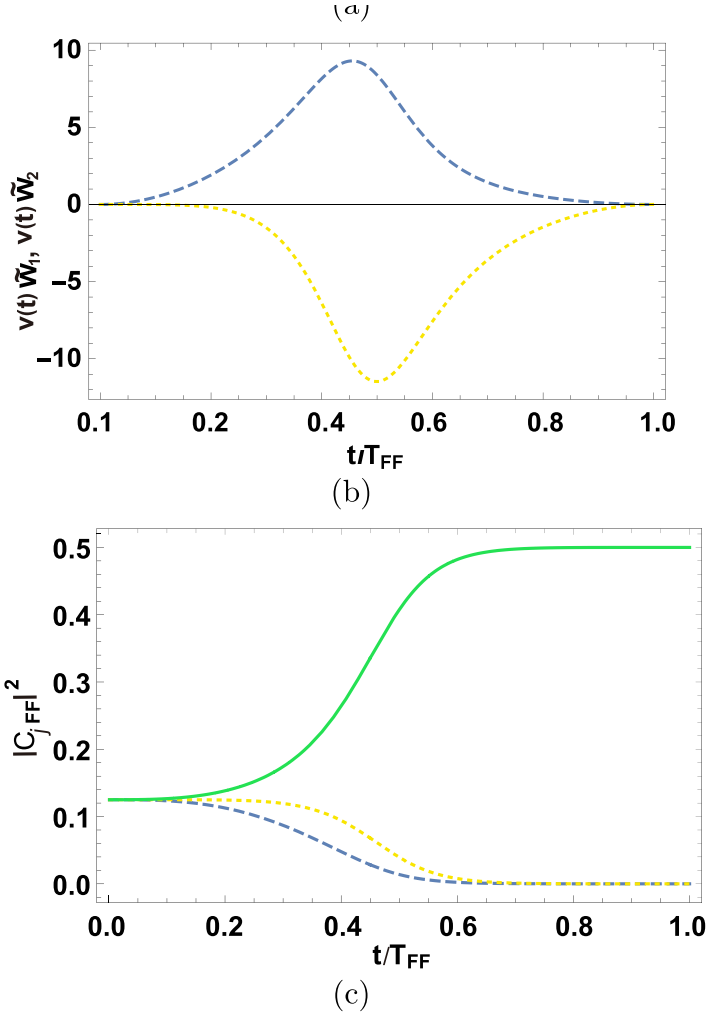


FIG. 4. Same time dependence as in Fig. 3, but in the case of the open linear three-spin chain. (a) All eight eigenvalues. (b) Regularization terms $v(t)\tilde{W}_1$ (dashed line) and $v(t)\tilde{W}_2$ (dotted line). (c) Probability amplitudes for the solution $\Psi_{\rm FF}(t)$ of the TDSE: $|C_3^{\rm FF}|^2 = |C_6^{\rm FF}|^2$ (solid line), $|C_1^{\rm FF}|^2 = |C_8^{\rm FF}|^2$ (dashed line), and $|C_2^{\rm FF}|^2 = |C_4^{\rm FF}|^2 = |C_5^{\rm FF}|^2 = |C_7^{\rm FF}|^2$ (dotted line).

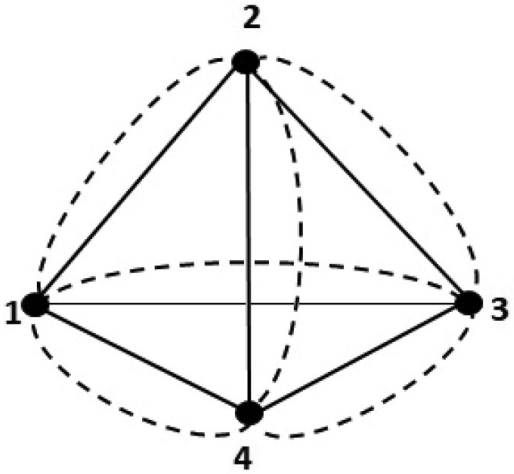
We conclude, for $N = 3$ spin clusters, that the driving interaction consists of only the geometry-dependent pairwise interactions and there is no need for the three-body interaction.

V. TRIANGULAR PYRAMID, SQUARE, STAR GRAPH, AND OPEN LINEAR FOUR-SPIN CHAINS

Now we will investigate regular spin clusters with $N = 4$ spins, namely, a triangular pyramid, square, star graph, and open linear four-spin chains in Fig. 2. Their original (reference) and regularization Hamiltonians have already been given in Eqs. (3.1) and (3.2), respectively, where we set $N = 4$.

By using the spin configuration bases $|1\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle$, $|2\rangle = |\uparrow\uparrow\uparrow\downarrow\rangle$, $|3\rangle = |\uparrow\uparrow\downarrow\uparrow\rangle$, $|4\rangle = |\uparrow\downarrow\uparrow\uparrow\rangle$, $|5\rangle = |\downarrow\uparrow\uparrow\uparrow\rangle$, $|6\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle$, $|7\rangle = |\uparrow\downarrow\downarrow\uparrow\rangle$, $|8\rangle = |\downarrow\downarrow\uparrow\uparrow\rangle$, $|9\rangle = |\downarrow\uparrow\uparrow\downarrow\rangle$, $|10\rangle = |\uparrow\downarrow\uparrow\downarrow\rangle$, $|11\rangle = |\downarrow\uparrow\downarrow\uparrow\rangle$, $|12\rangle = |\downarrow\downarrow\downarrow\uparrow\rangle$, $|13\rangle = |\downarrow\downarrow\uparrow\downarrow\rangle$, $|14\rangle = |\downarrow\uparrow\downarrow\downarrow\rangle$, $|15\rangle = |\uparrow\downarrow\downarrow\downarrow\rangle$, and $|16\rangle = |\downarrow\downarrow\downarrow\downarrow\rangle$, the matrix form for the original Hamiltonian H_0 in Eq. (3.1) can be constructed.

Triangular pyramid or Tetrahedron



Due to the symmetry of $\{C_j\}$, there are three independent equations in Eq. (2.8):

$$
\begin{aligned}
-12\tilde{W}C_2 &= \hbar\frac{\partial C_1}{\partial R}, \\
3\tilde{W}C_1 - 3\tilde{W}C_6 &= \hbar\frac{\partial C_2}{\partial R}, \\
4\tilde{W}C_2 &= \hbar\frac{\partial C_6}{\partial R}.
\end{aligned}
\tag{5.3}
$$

While one of Eqs. (5.3) is trivial due to Eq. (5.1), we need one more unknown variable to make meaningful the algebraic equations (5.3). Here we evaluate the contribution of the three-body interaction. The geometrical symmetry allows a universal three-body interaction $\tilde{Q} \equiv \tilde{Q}^{xyz}_{ijk}$, independent of all possible three-body configurations (i, j, k). The inclusion of the three-body interaction improves some matrix elements of

After the above improvements, the algebraic equations (5.3) are revised as

$$
\begin{aligned}
-12\tilde{W}C_2 - 24\tilde{Q}C_6 &= \hbar\frac{\partial C_1}{\partial R}, \\
3\tilde{W}C_1 - 3\tilde{W}C_6 &= \hbar\frac{\partial C_2}{\partial R}, \\
8\tilde{Q}C_1 + 4\tilde{W}C_2 &= \hbar\frac{\partial C_6}{\partial R},
\end{aligned}
\tag{5.5}
$$

where one of Eqs. (5.5) is again trivial because of Eq. (5.1). Equations (5.5), whose coefficient matrix has the rank 2, give

the solution

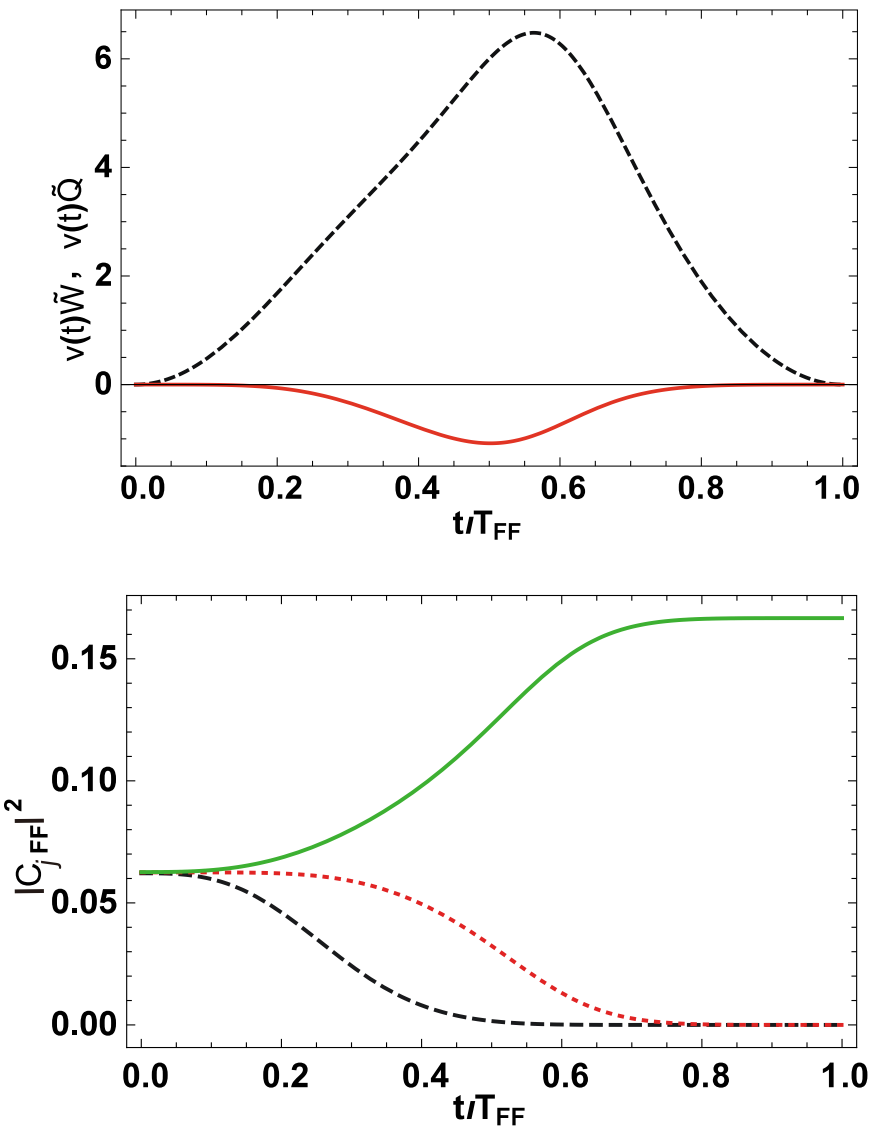
$$\tilde{W} = \frac{\hbar \partial_R C_2}{3(C_1 - C_6)}, \quad \tilde{Q} = \frac{\hbar \partial_R (C_1 + 3C_6)}{24(C_1 - C_6)}. \tag{5.6}$$

The fast-forward Hamiltonian is given by

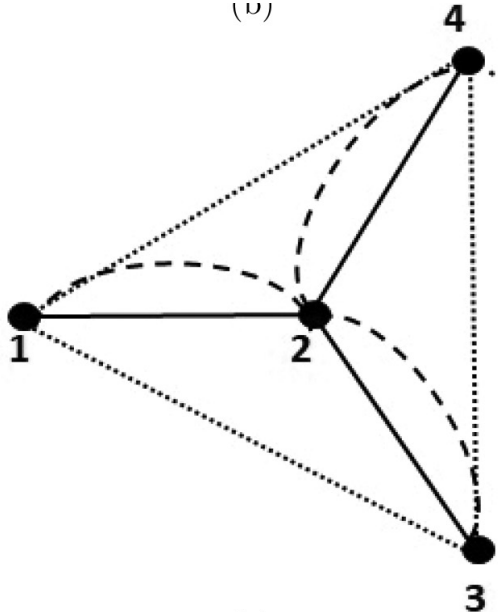
$$\begin{aligned} H_{\mathrm{FF}} = {} & J(R(\Lambda(t))) \sum_{(i,j)\in \mathrm{NN}} \sigma_i^z \sigma_j^z - \tfrac{1}{2} B_x(R(\Lambda(t))) \sum_{i=1}^{4} \sigma_i^x \\ & + v(t) \tilde{\mathcal{H}}(R(\Lambda(t))), \end{aligned} \tag{5.7}$$

with

$$\begin{aligned} v\tilde{\mathcal{H}} = {} & \sum_{(i,j)\in \mathrm{all}} v(t) \tilde{W}(R(\Lambda(t))) \left(\sigma_i^y \sigma_j^z + \sigma_i^z \sigma_j^y\right) \\ & + \sum_{(i,j,k)\in \mathrm{all}} v(t) \tilde{Q}(R(\Lambda(t))) \left(\sigma_i^x \sigma_j^y + \sigma_i^y \sigma_j^x\right) \cdot \sigma_k^z. \end{aligned} \tag{5.8}$$


6
4
2
0
v(t)W̃ , v(t)Q̃
0.0
0.2
0.4
0.6
0.8
1.0
t/T_FF
0.15
0.10
0.05
0.00
|C_j,FF|^2
0.0
0.2
0.4
0.6
0.8
1.0
t/T_FF

Primary star graph



equations in Eq. (2.8) and cannot be acceptable. Due to the symmetry of $\{C_j\}$, there are four independent equations

$$(-6\tilde{W}_2 - 3\tilde{W}_1)C_2 + (-3\tilde{W}_1)C_4 - 24\tilde{Q}C_6 = \hbar\frac{\partial C_1}{\partial R}, \quad (5.14a)$$

$$(2\tilde{W}_2 + \tilde{W}_1)C_1 + (-3\tilde{W}_1)C_6 = \hbar\frac{\partial C_2}{\partial R}, \quad (5.14b)$$

$$(3\tilde{W}_1)C_1 + (3\tilde{W}_1 - 6\tilde{W}_2)C_6 = \hbar\frac{\partial C_4}{\partial R}, \quad (5.14c)$$

$$8\tilde{Q}C_1 + (3\tilde{W}_1)C_2 + (-\tilde{W}_1 + 2\tilde{W}_2)C_4 = \hbar\frac{\partial C_6}{\partial R}. \quad (5.14d)$$

Because of Eq. (5.13), one of Eqs. (5.14) becomes trivial. Ignoring Eq. (5.14a), for example, Eqs. (5.14b)–(5.14d), whose coefficient matrix has the rank 3, give the solution

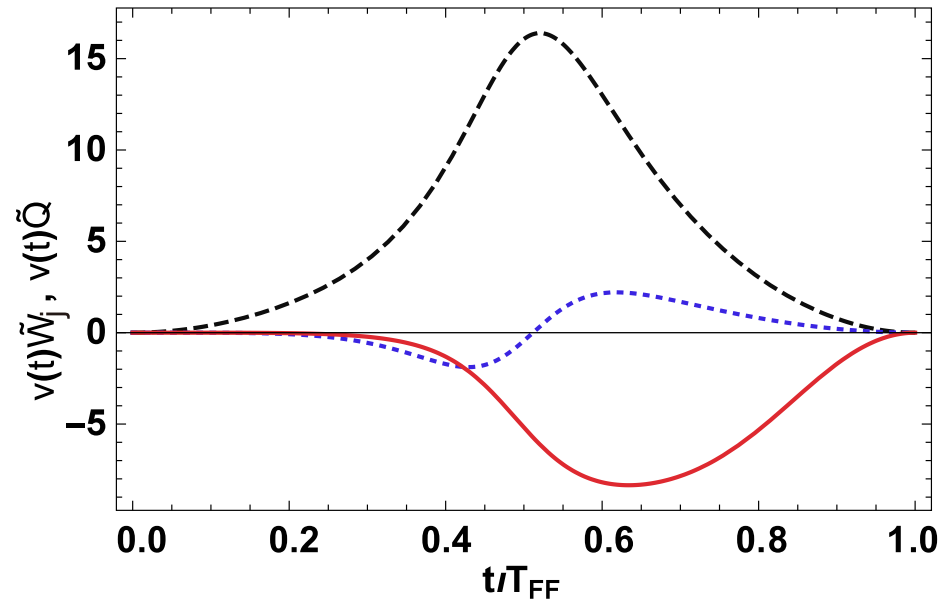
$$\tilde{W}_1 = \hbar \frac{C_1 \partial_R C_4 + 3C_6 \partial_R C_2}{3(C_1 - C_6)(C_1 + 3C_6)},$$

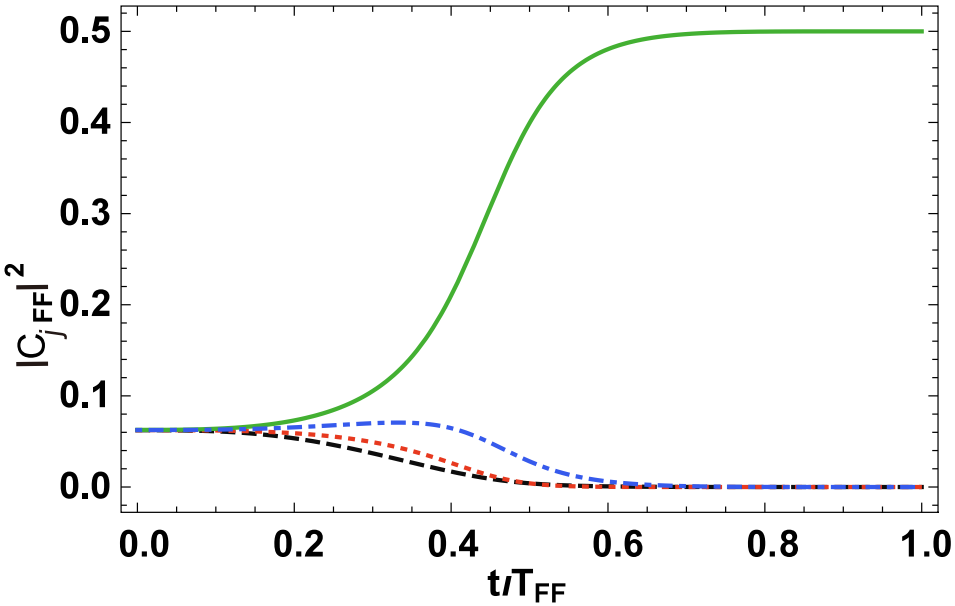
$$\tilde{W}_2 = \hbar \frac{3(C_1 + C_6)\partial_R C_2 - (C_1 - 3C_6)\partial_R C_4}{6(C_1 - C_6)(C_1 + 3C_6)},$$

$$\begin{aligned}\tilde{Q} = {} & \frac{\hbar}{24C_1(C_1 - C_6)(C_1 + 3C_6)} \\ & \times \left[3\left(C_1^2 + 2C_1C_6 - 3C_6^2\right)\partial_R C_6\right. \\ & - 3(3C_2C_6 + C_1C_4)\partial_R C_2 \\ & \left. - (3C_1C_2 - 2C_1C_4 + 3C_4C_6)\partial_R C_4\right]. \end{aligned} \tag{5.15}$$

The fast-forward Hamiltonian is given by Eq. (5.7), where $v(t)\tilde{\mathcal{H}}(R(\Lambda(t)))$ is replaced by

$$\begin{aligned} v\tilde{\mathcal{H}} = {} & \sum_{(i,j)=(1,2),(2,3),(2,4)} v(t)\tilde{W}_1(R(\Lambda(t)))\left(\sigma_i^y\sigma_j^z + \sigma_i^z\sigma_j^y\right) + \sum_{(i,j)=(1,4),(1,3),(3,4)} v(t)\tilde{W}_2(R(\Lambda(t)))\left(\sigma_i^y\sigma_j^z + \sigma_i^z\sigma_j^y\right) \\ & + \sum_{(i,j,k)\in \text{all}} v(t)\tilde{Q}(R(\Lambda(t)))\left(\sigma_i^x\sigma_j^y + \sigma_i^y\sigma_j^x\right)\cdot\sigma_k^z. \end{aligned}$$


15
10
5
0
−5
v(t)W̃j , v(t)Q̃
0.0
0.2
0.4
0.6
0.8
1.0
t/TFF


0.5
0.4
0.3
0.2
0.1
0.0
|Cj FF|2
0.0
0.2
0.4
0.6
0.8
1.0
t/TFF

We find for $N = 4$ spin clusters, that the geometry-dependent pairwise interactions again constitute a major part of the driving interaction, whereas the universal three-body interaction free from the geometry is necessary but plays a subsidiary role.

Summary

The fast forward (FF) of adiabatic dynamics of coupled spins can generate entangled states from the initial product state very quickly, leaving neither residual oscillations nor disturbances.

Geometric symmetry reduces the complexity of the driving interactions.

Broad range of choosing the driving pair interactions and magnetic field will make flexible the experimental design of reproducing the adiabatic quantum spin dynamics or quantum computation in an extremely short time.