Entanglement and Reference frames
Lecture I

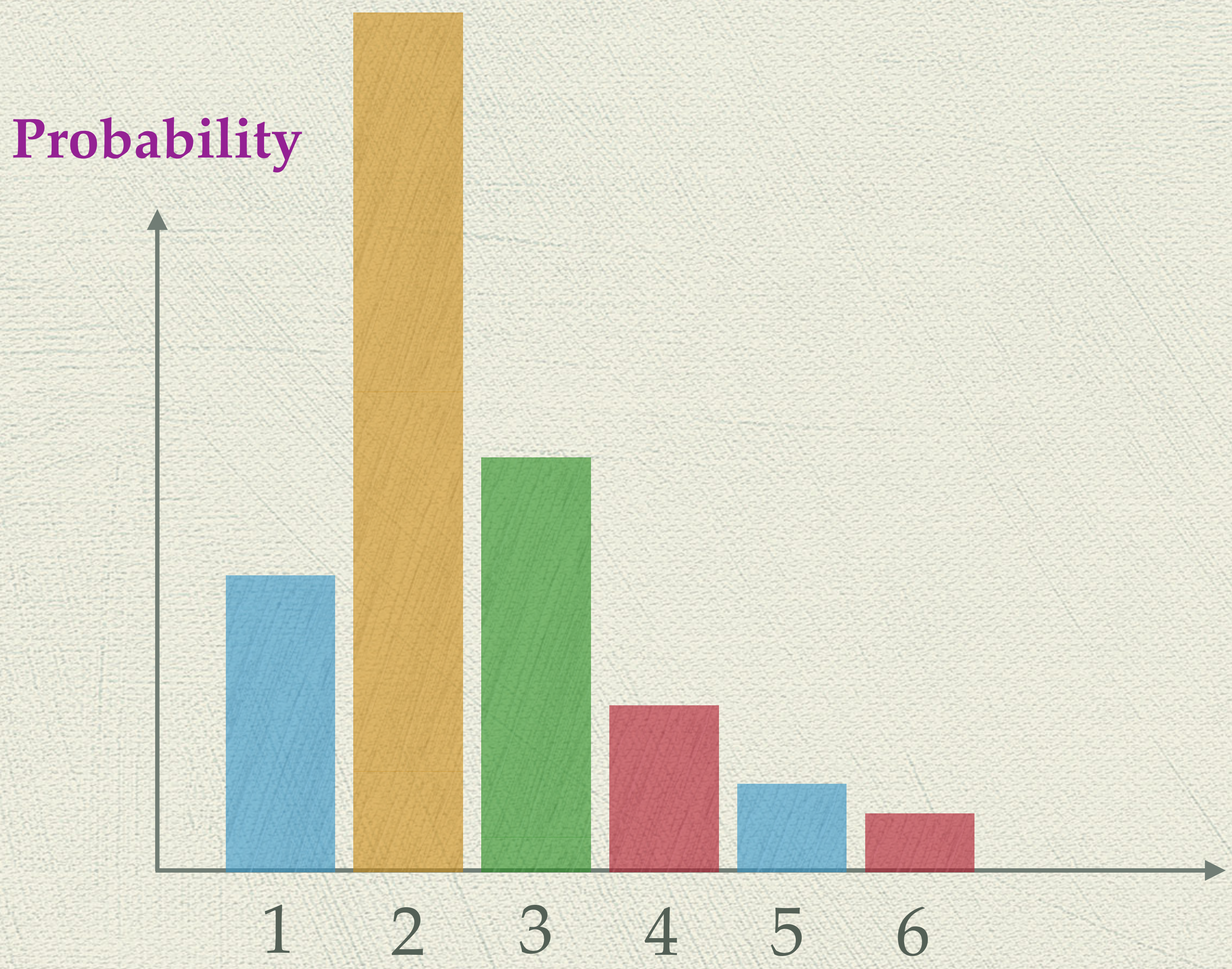
Vahid Karimipour,
Sharif University of Technology,
Tehran, Iran.

New Advances in Quantum Information Science
and
Quantum Technology

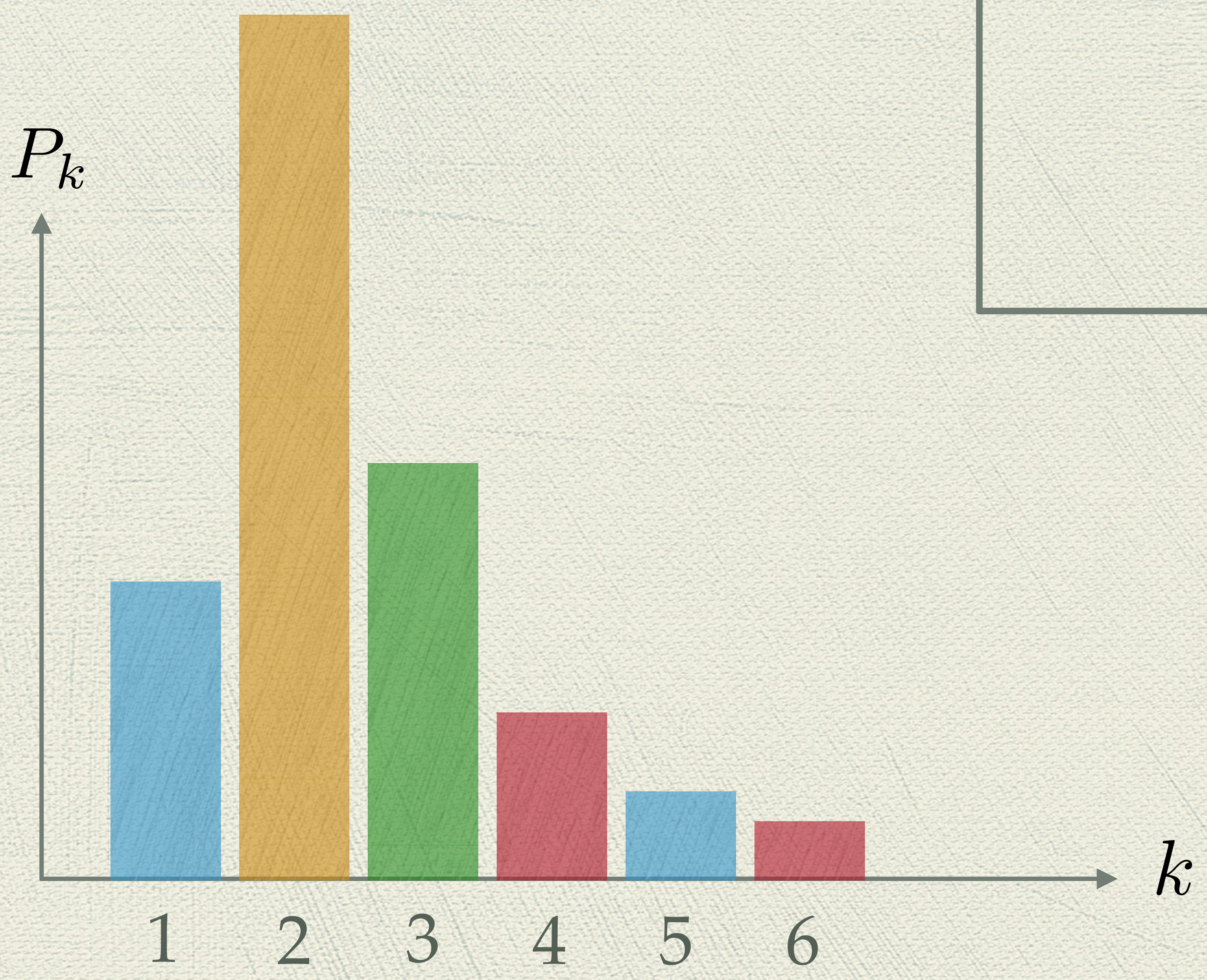
September 10-18, 2019
Samarkand, Uzbekistan,

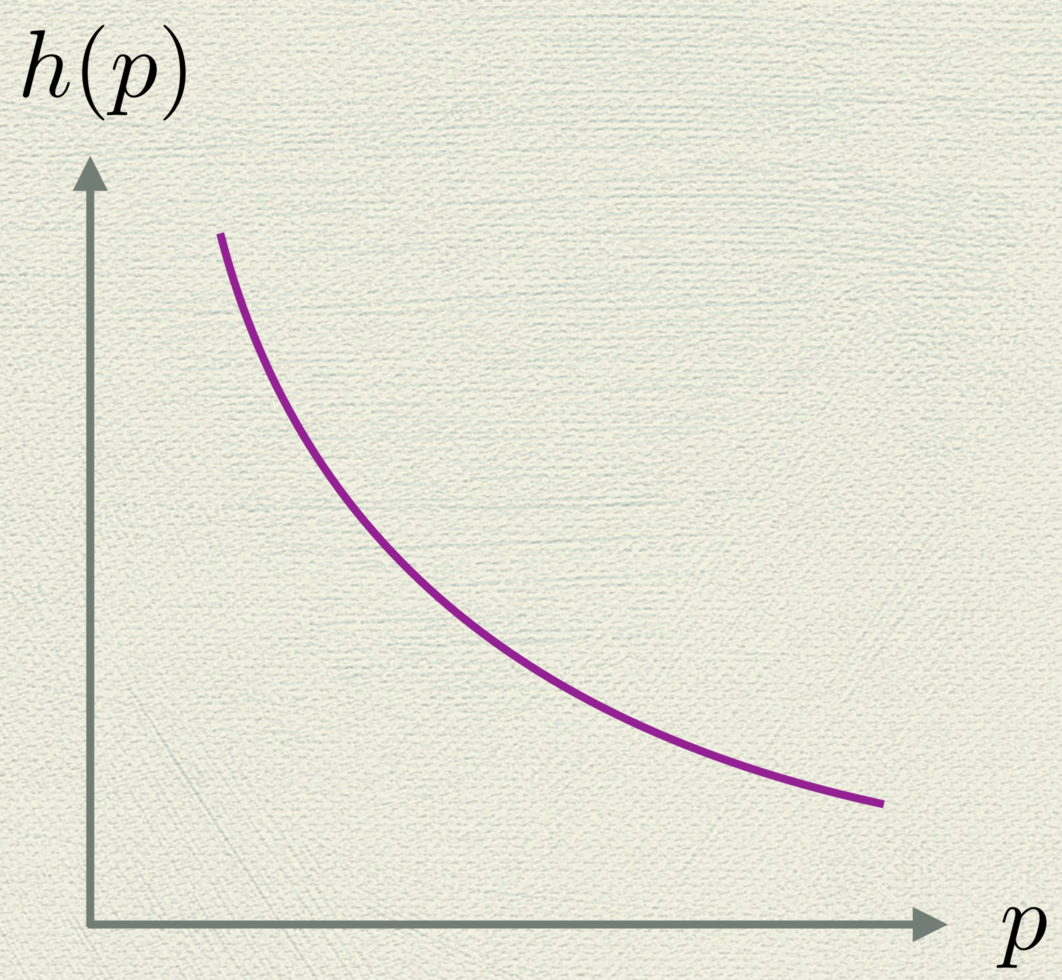
0-Elementary Concepts

The element of surprise



$$h_k = h(p_k)$$





$$h(pq) = h(p) + h(q)$$

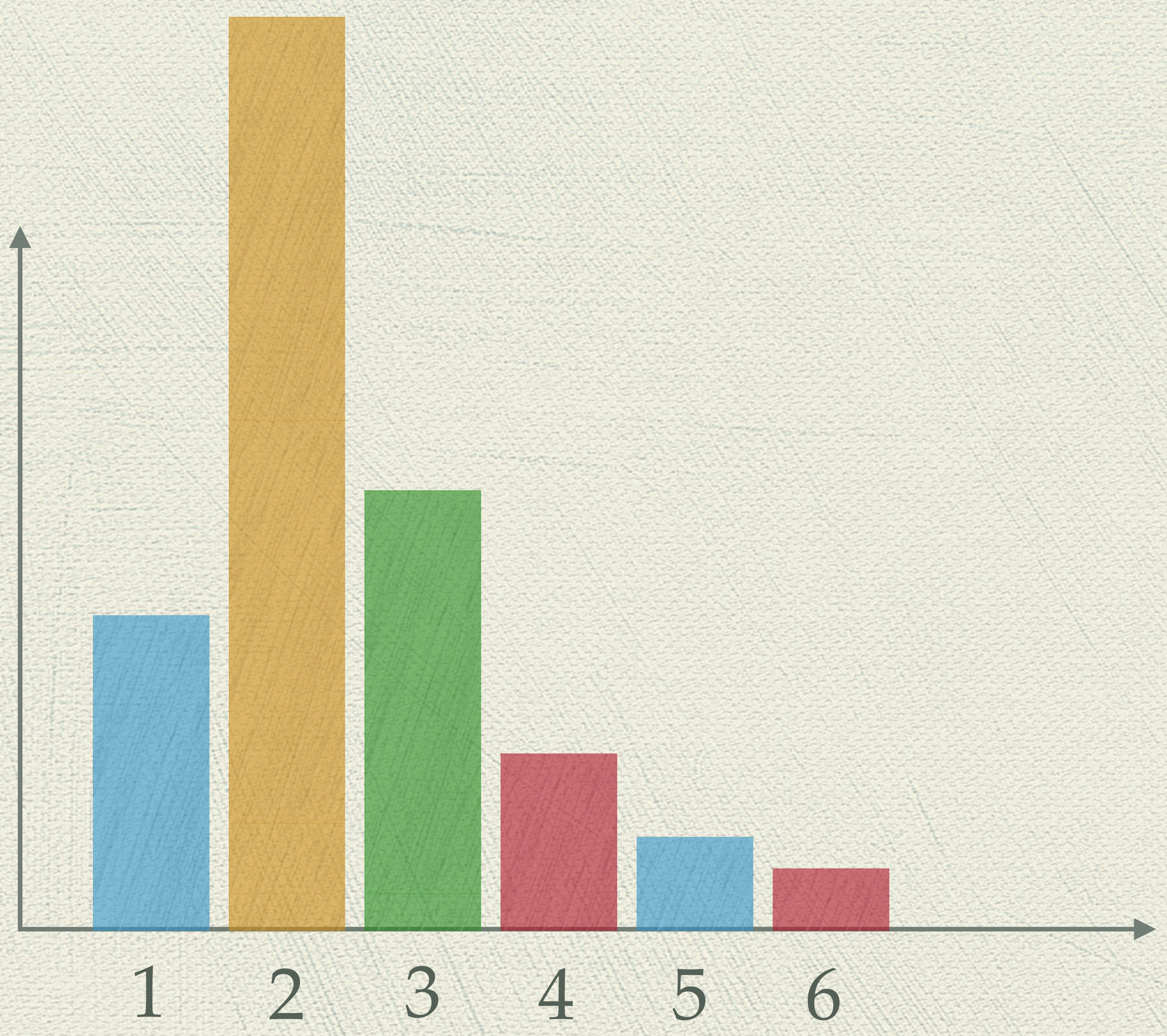
p q

$$h(p) = -a \log p$$

$$h(1/2) = 1$$

$$h(p) = -\log_2(p)$$

$$H = -\sum_k p_k \log(p_k)$$

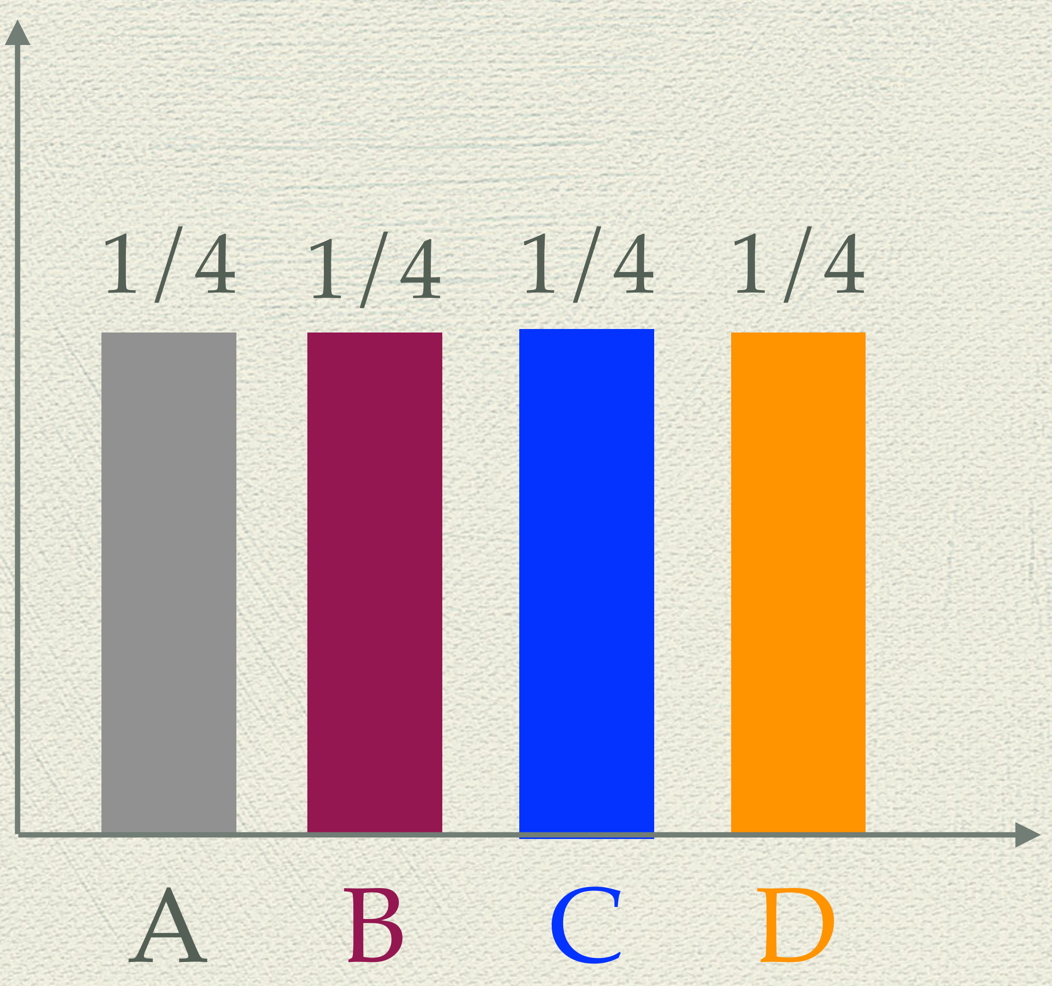


Shannon Entropy

Shannon Information

Is information physical?

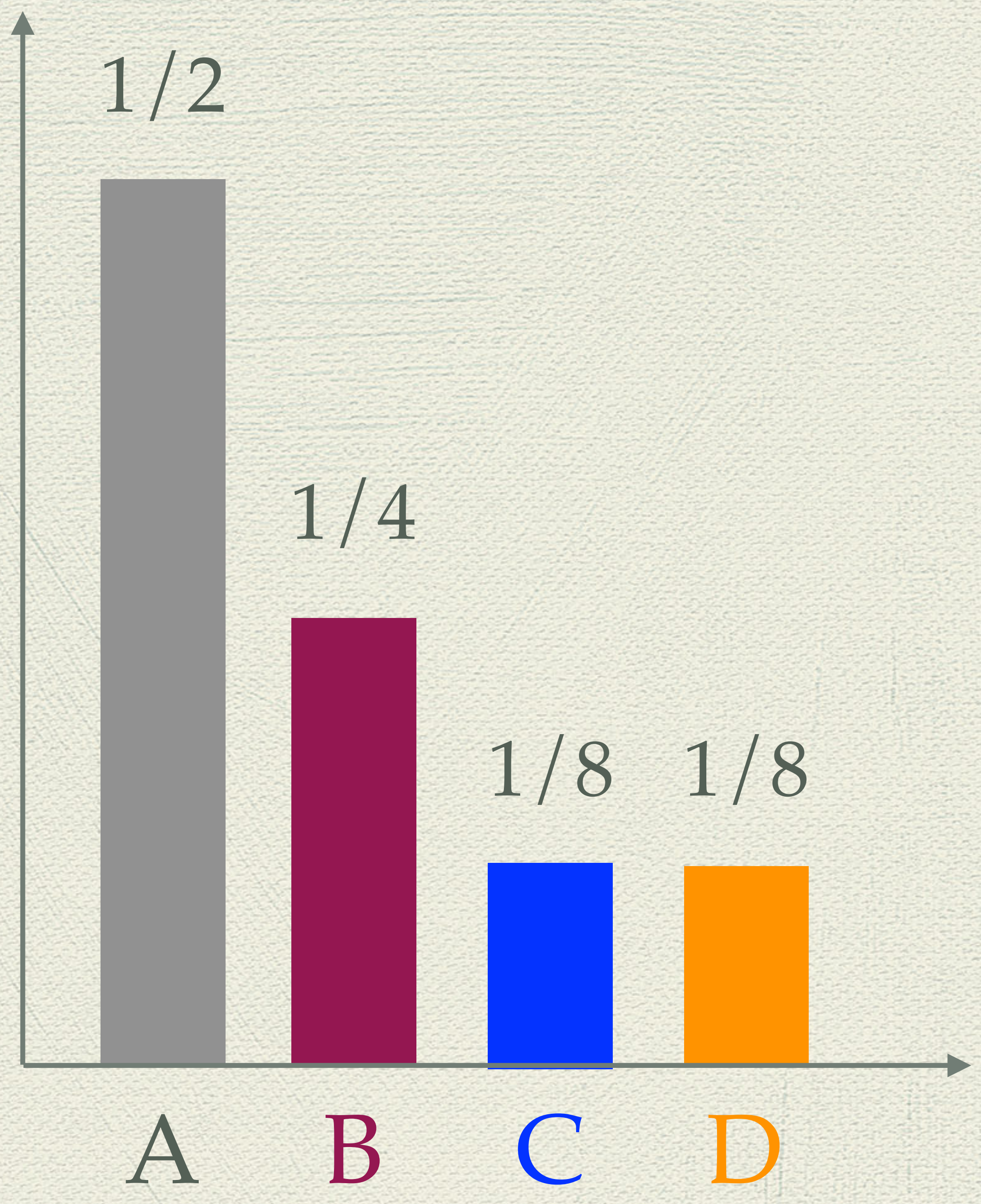
A	00
B	01
C	10
D	11



$$H = -\sum \frac{1}{4} \log \frac{1}{4} = 2$$

2 bits per letter

A	0
B	10
C	110
D	111

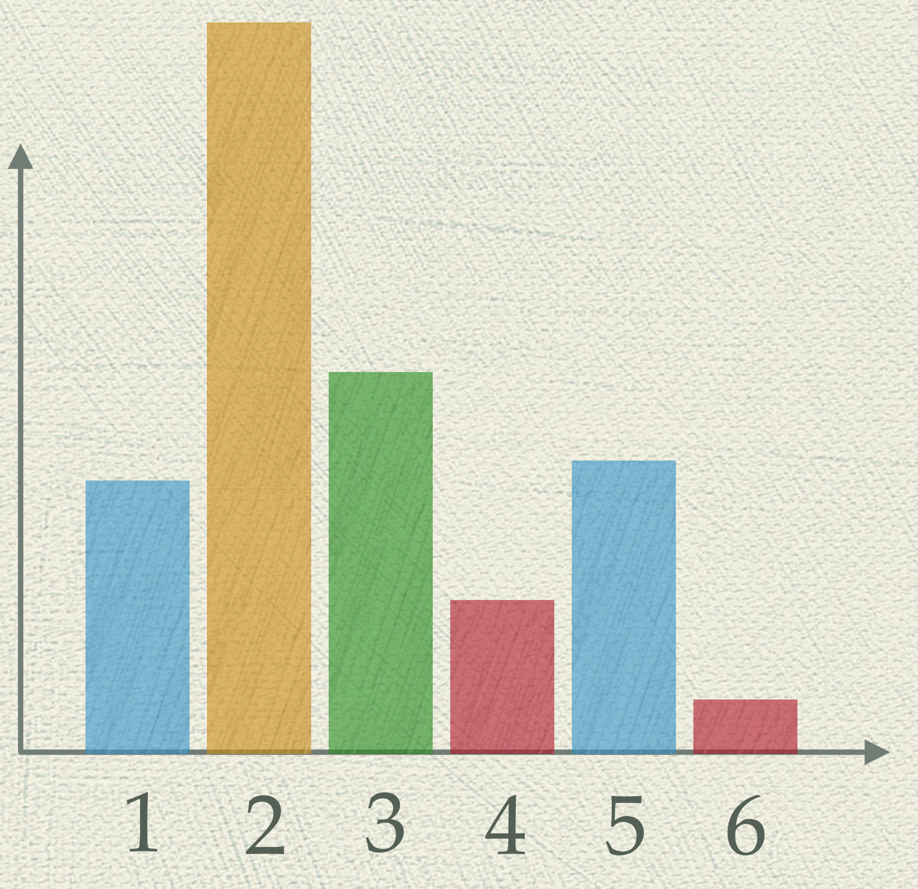


$$H = \frac{7}{4}$$

7/4 bits per letter

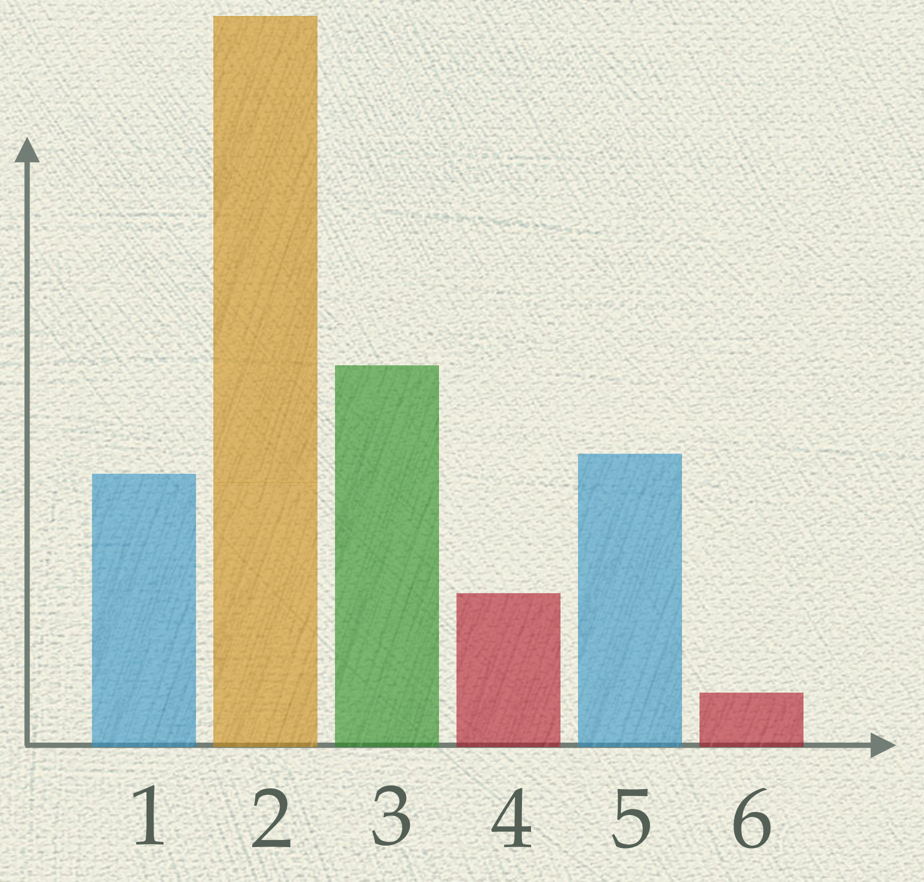
Information Gain

$H(X)$

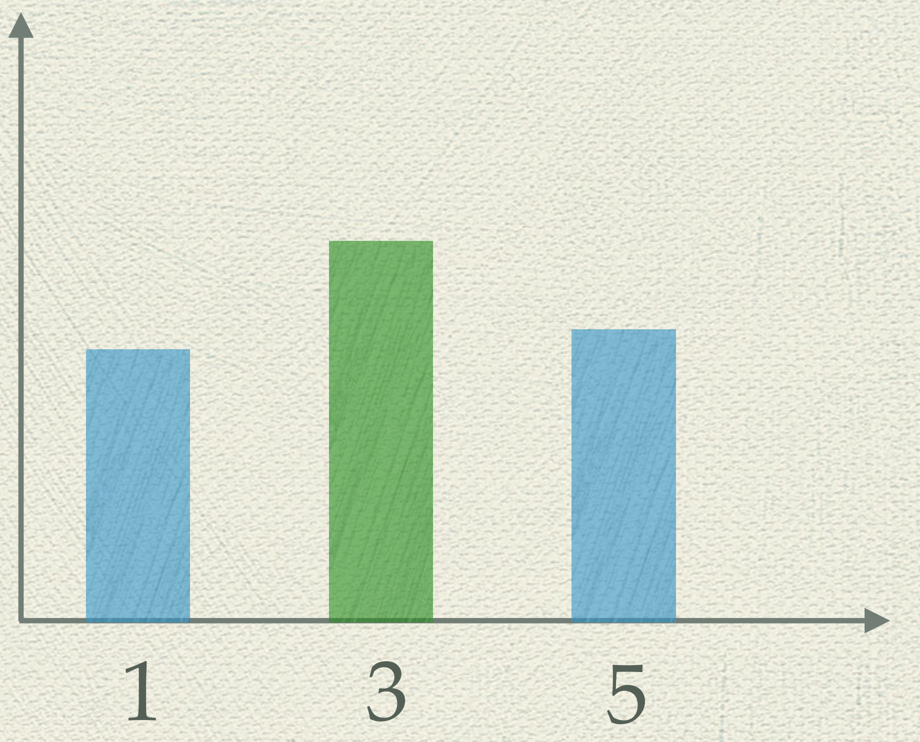


Information Gain

$H(X)$

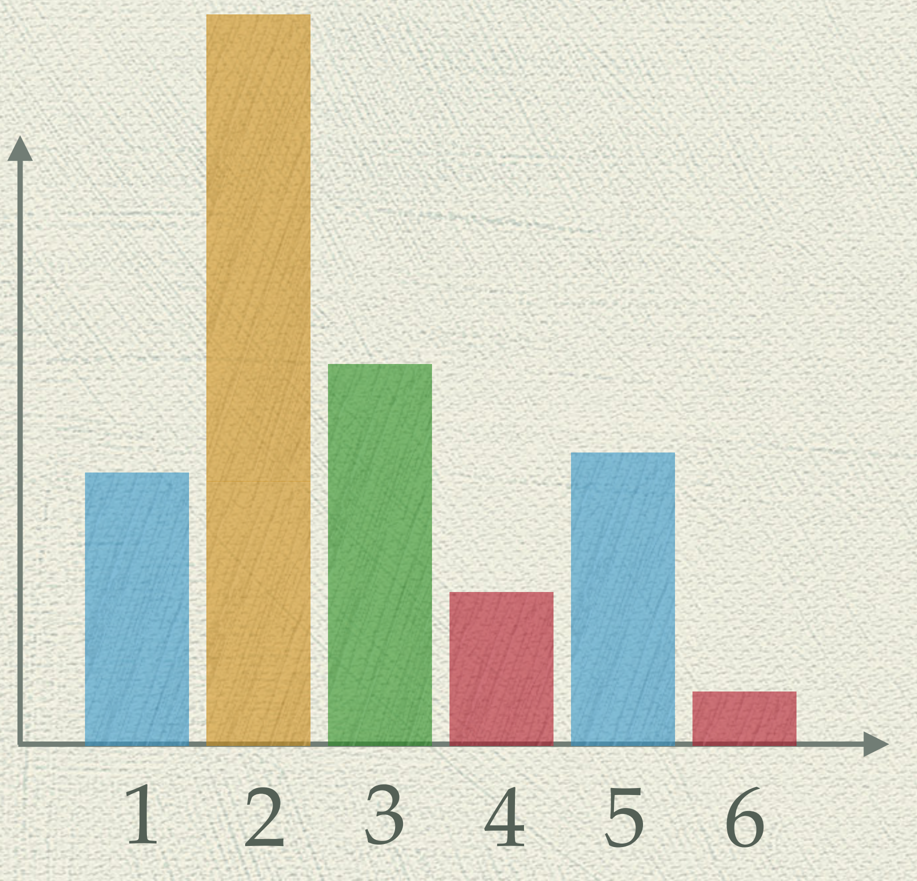


$H(X|Odd)$

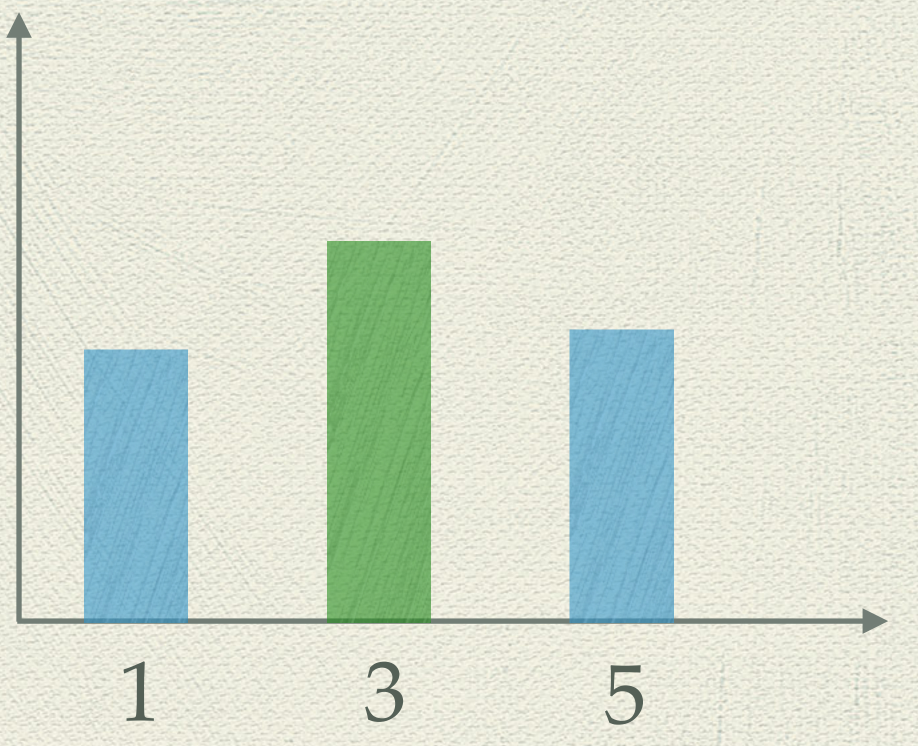


Information Gain

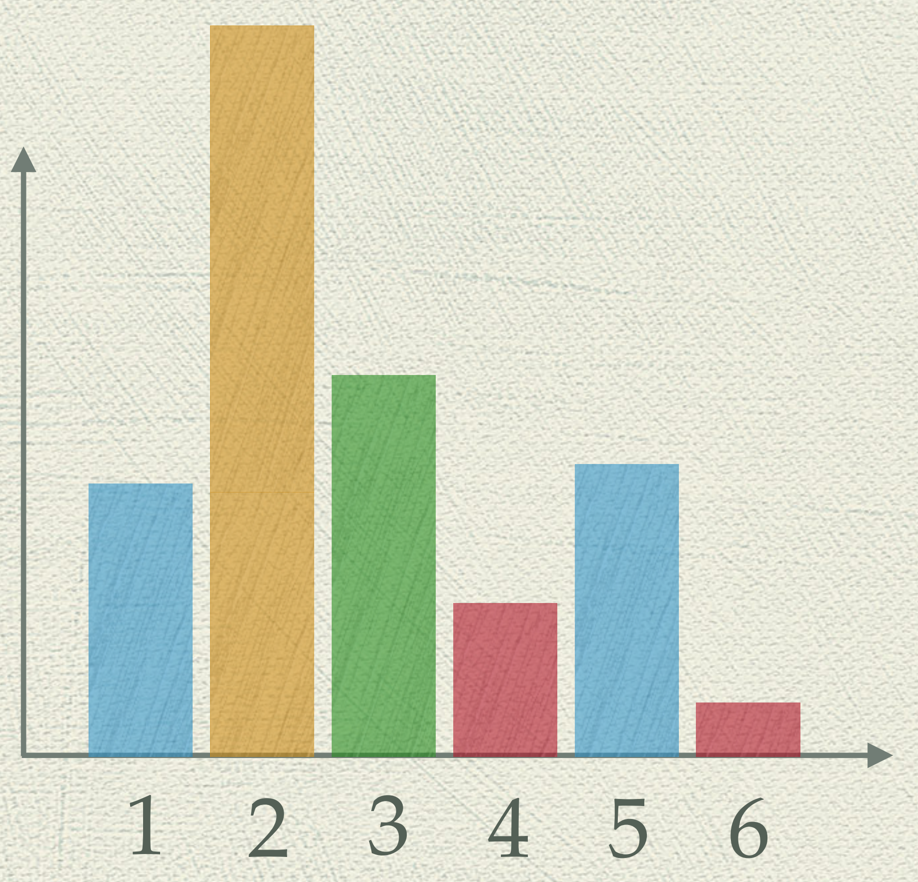
$H(X)$

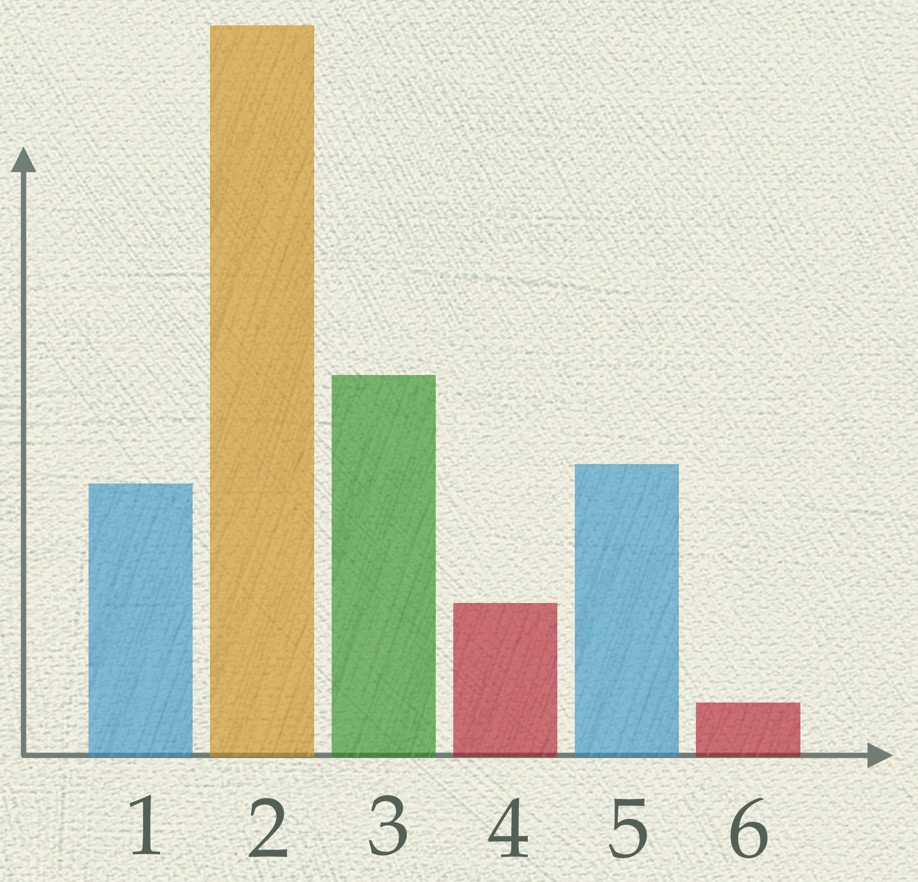


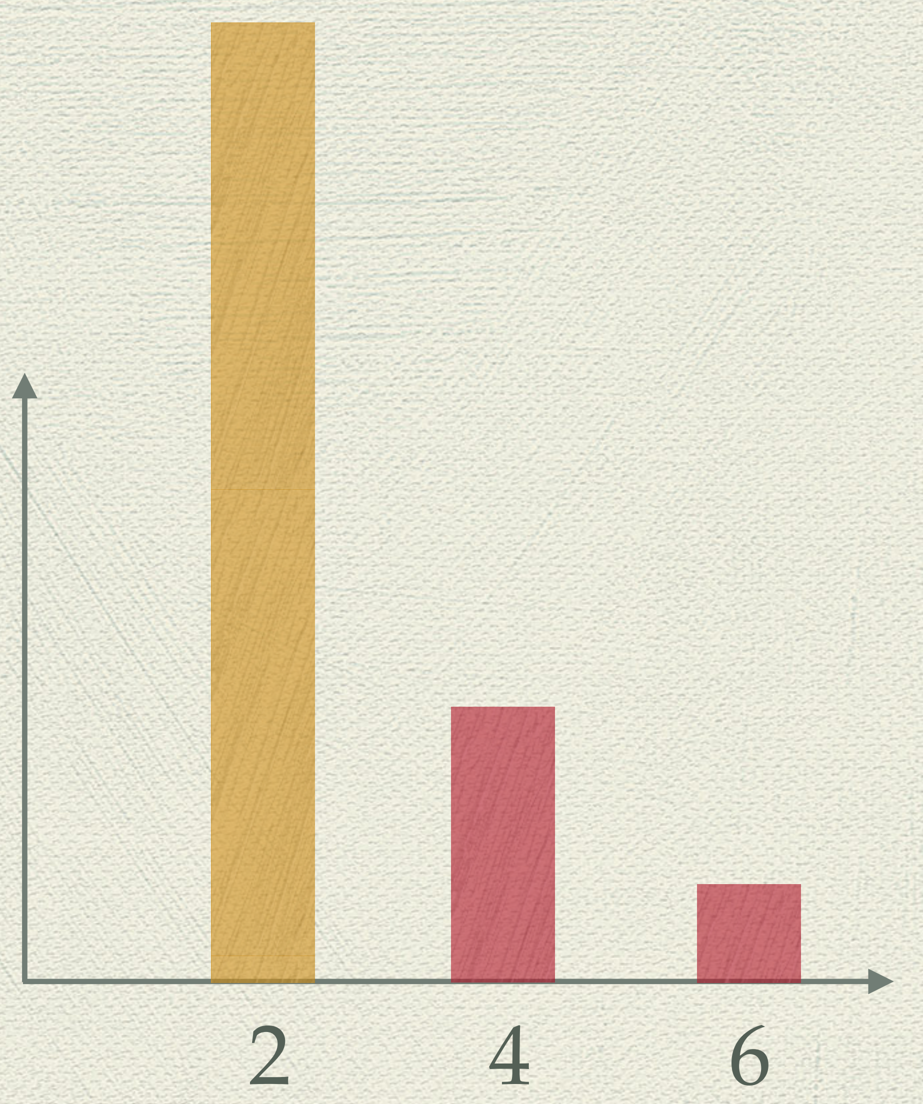
$H(X|Odd)$

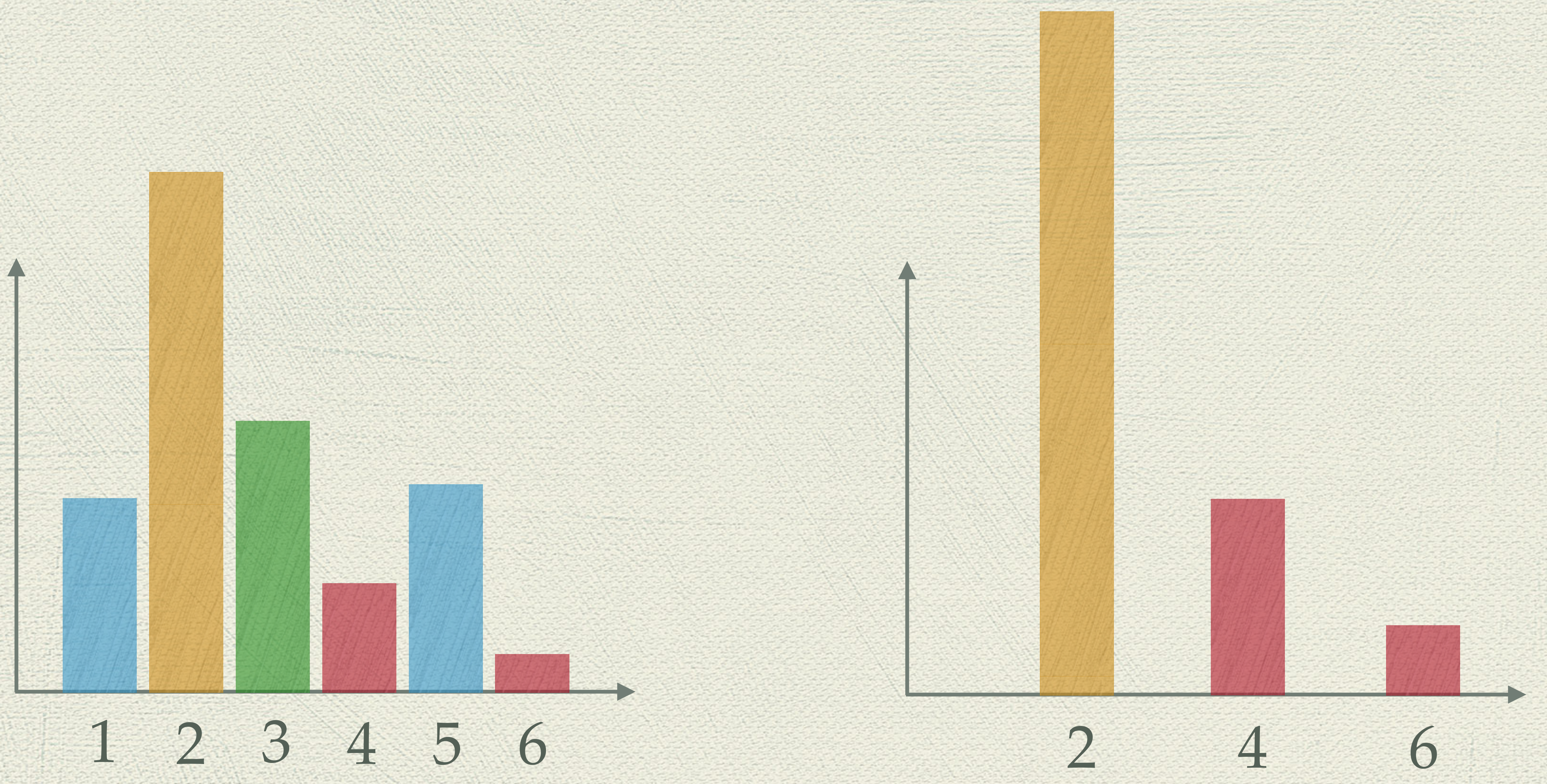


$$H(X) - H(X|Odd)$$


1 2 3 4 5 6


1 2 3 4 5 6


2 4 6



$$H(X) - H(X|Even)$$

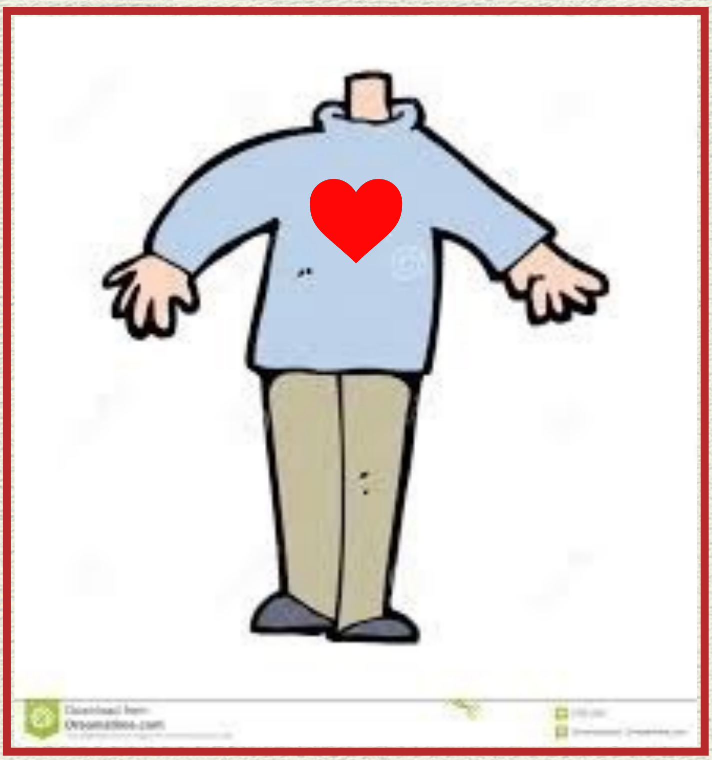
0- Unspeakable Information

Information transfer requires a common frame of reference.

10101000100001000010000

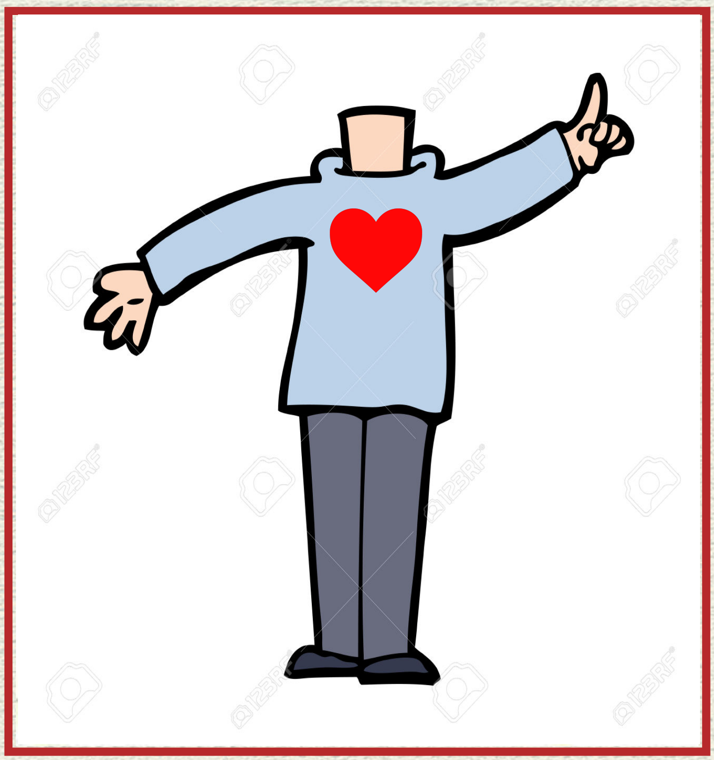
An example

Raise your LEFT hand.

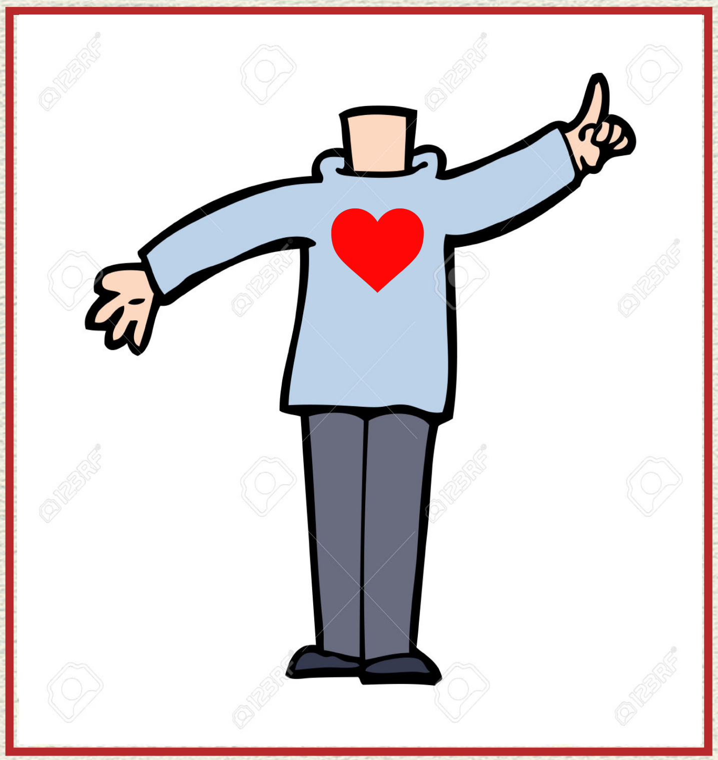


An example

Raise your LEFT hand.



An example



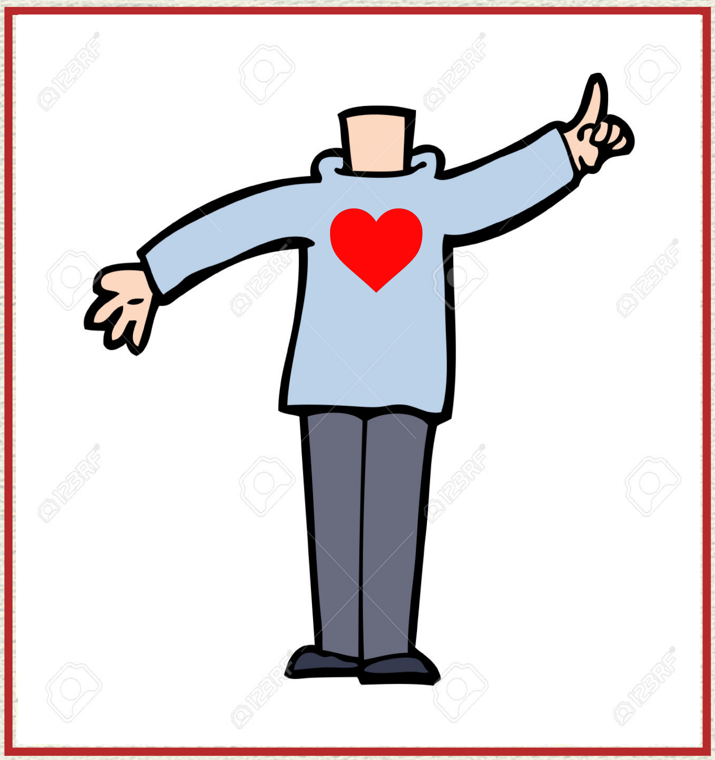
Raise your LEFT hand.

We cannot communicate the word
LEFT
with any string of bits

0 1 0 1 1 1 0 0 0 1 1 1 0 1 0 1 1 0

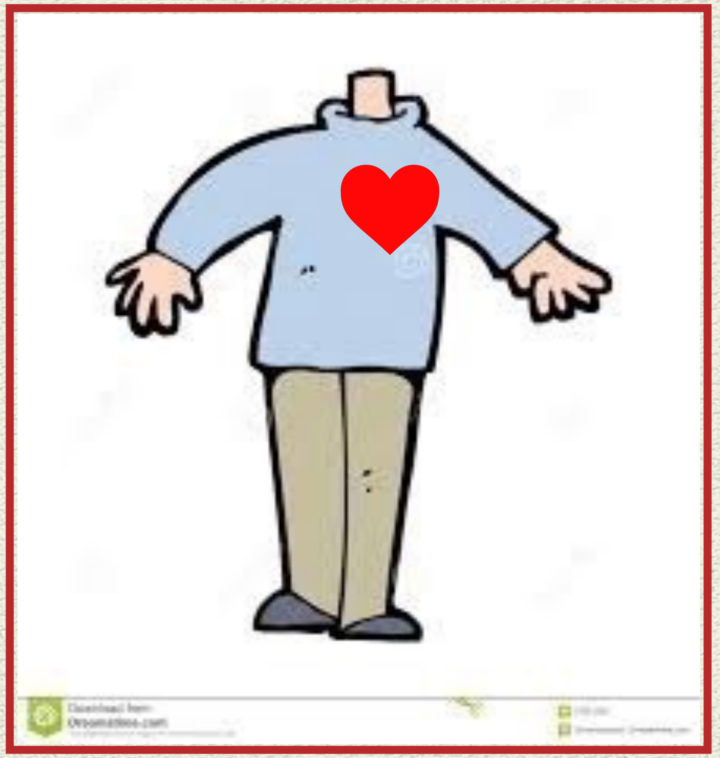
An example

Raise your LEFT hand.

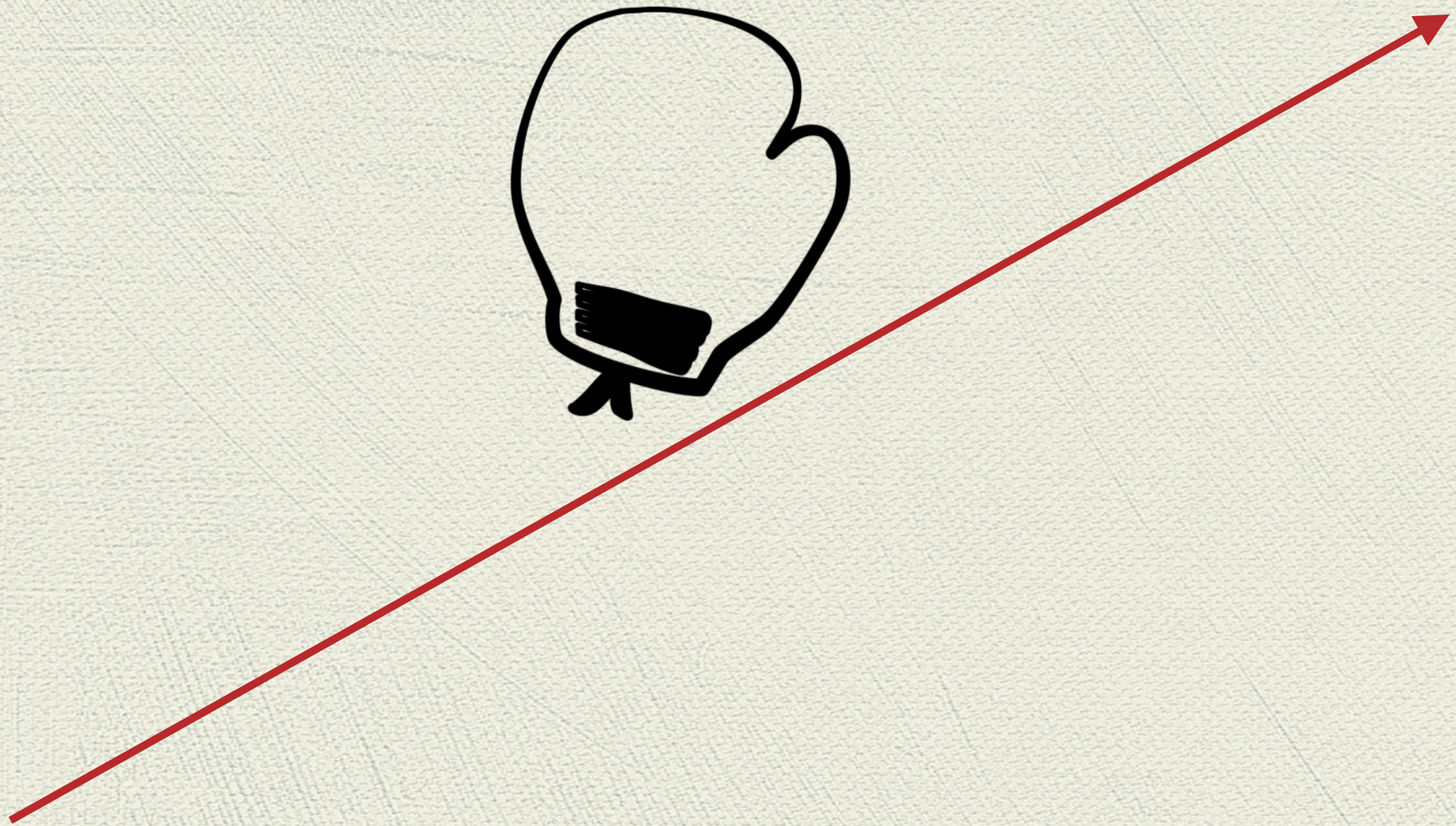


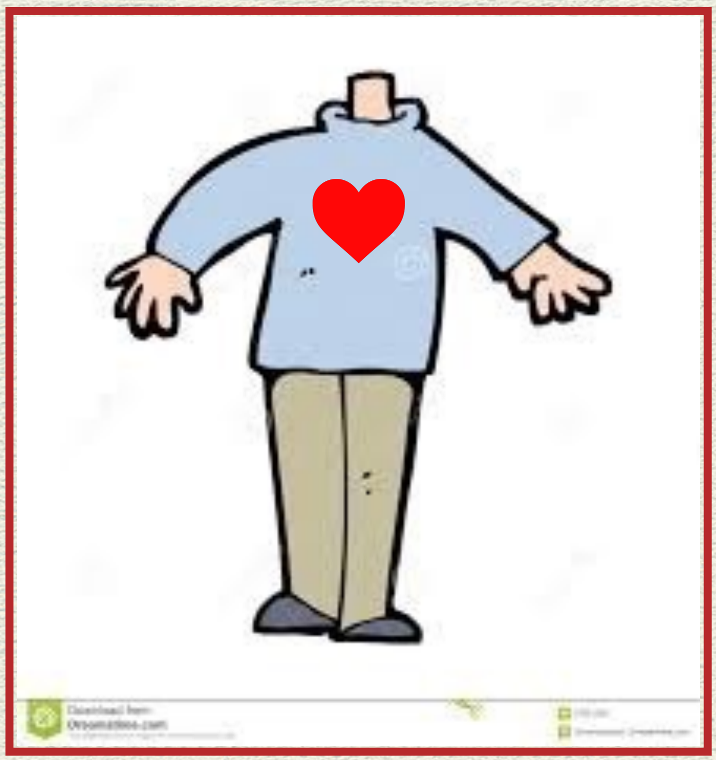
We cannot communicate the word
LEFT
with any string of bits

0 1 0 1 1 1 0 0 0 1 1 1 0 1 0 1 1 0

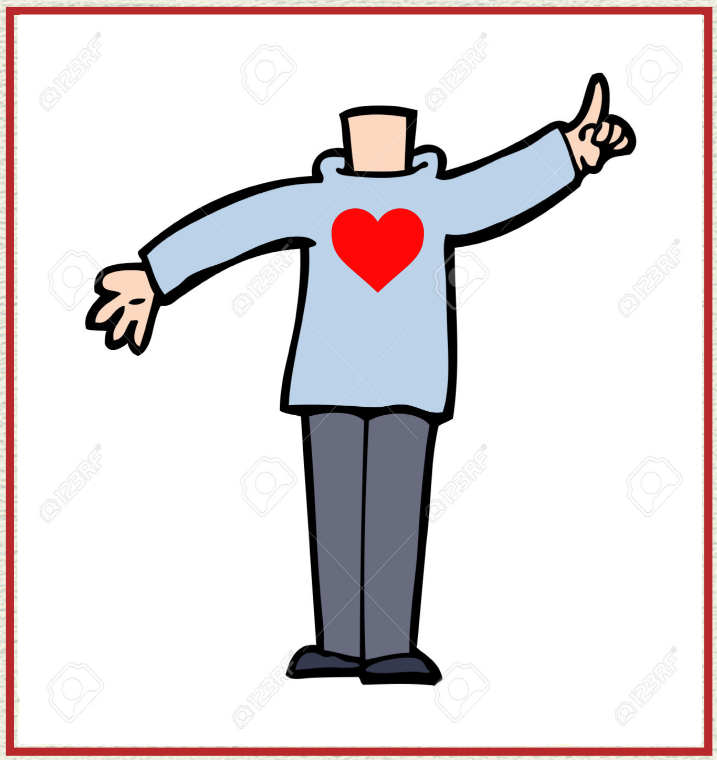


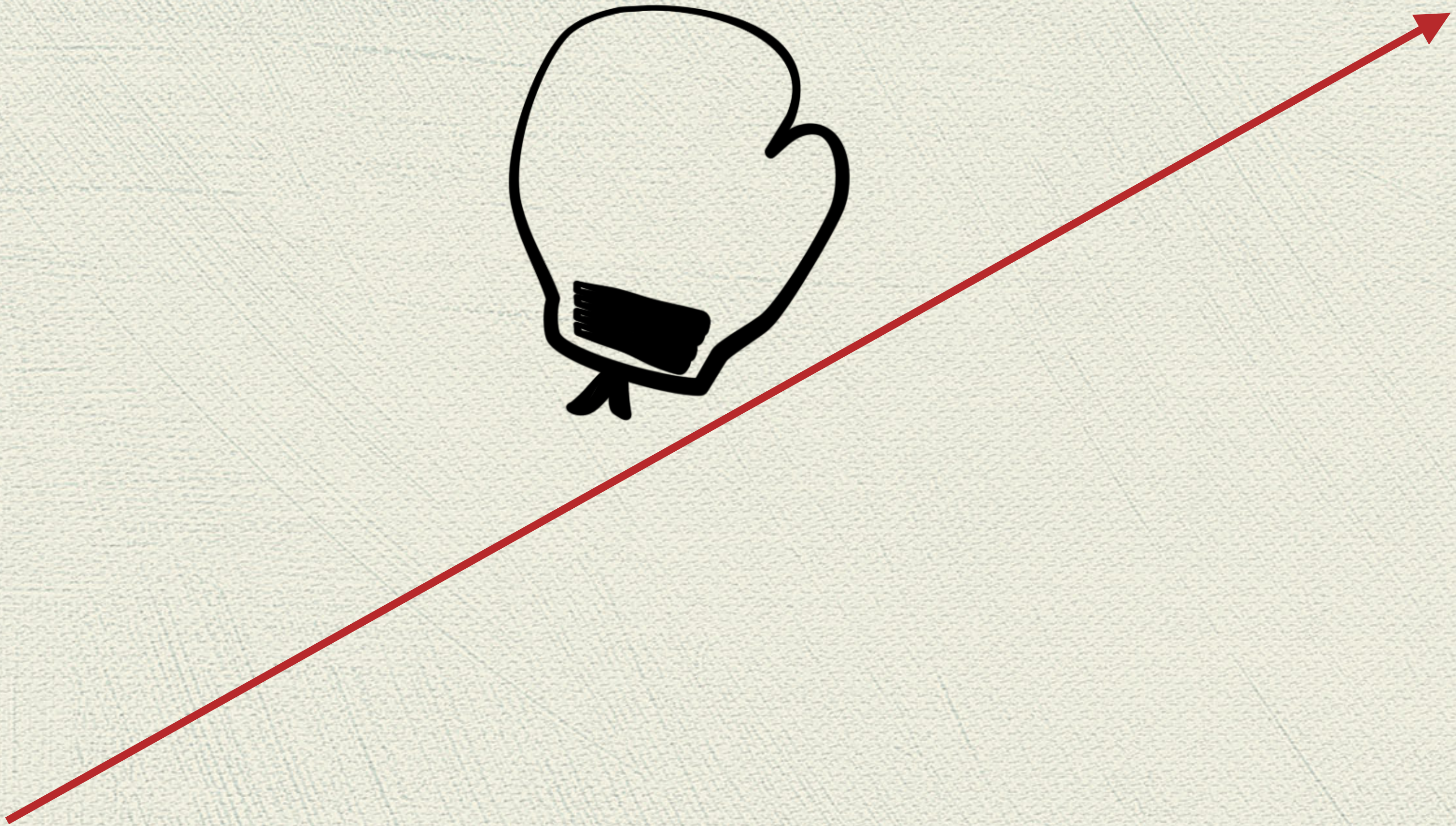
We should send a physical object.





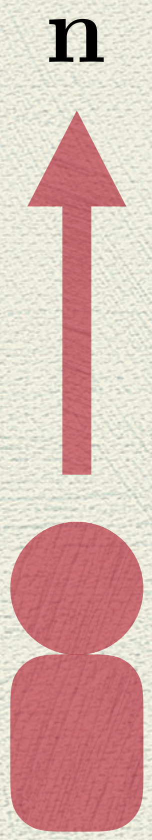
We should send a physical object.





Estimating a direction

How good is our guess?

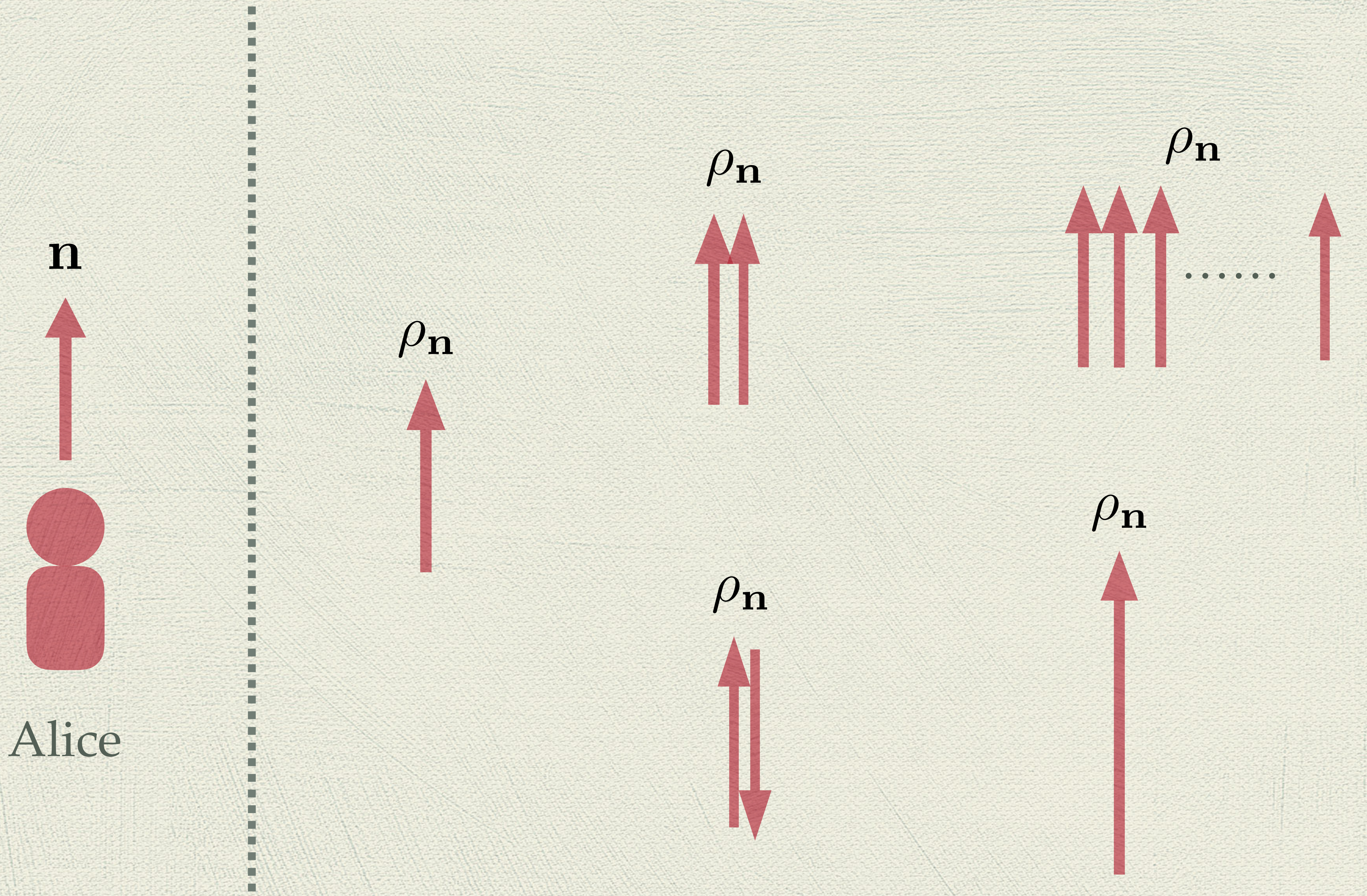


Alice

Bob

$$F(\mathbf{n}, \mathbf{n}_g) = \frac{1}{2}(1 + \mathbf{n} \cdot \mathbf{n_g})$$

Encoding a direction



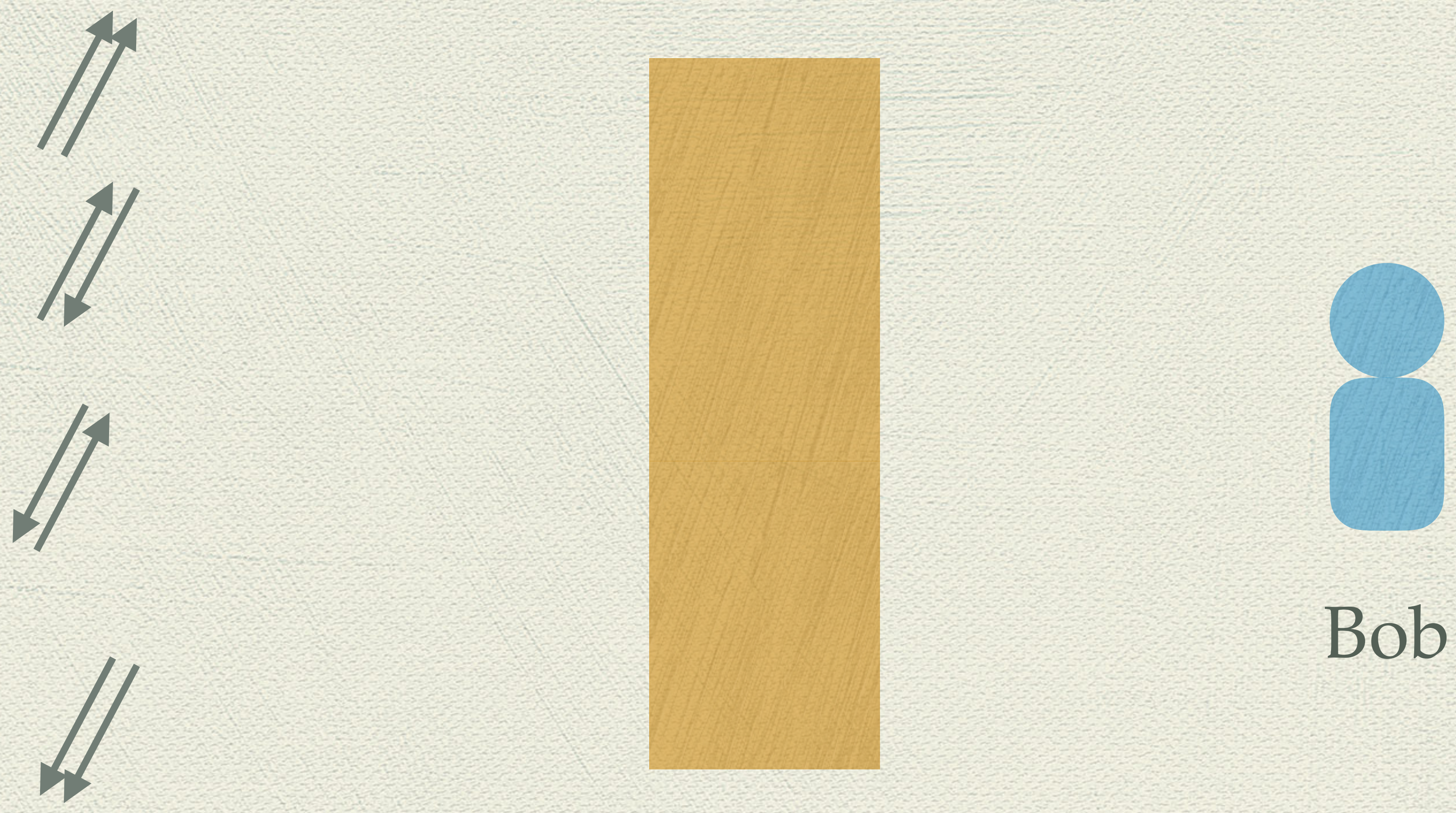
Measurements

$|n_g, n_g\rangle$

$|n_g, -n_g\rangle$

$|-n_g, n_g\rangle$

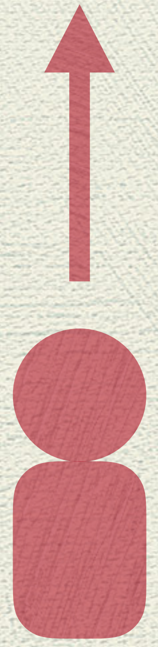
$|-n_g, -n_g\rangle$



$$\{E_1, E_2, \cdots E_g, \cdots\}$$

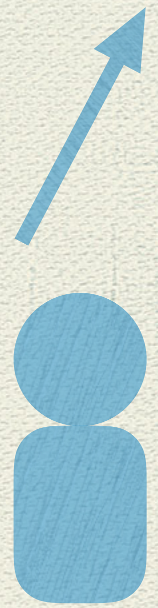
Estimating a direction

$\mathbf{n}$



Alice

$\mathbf{n}_g$



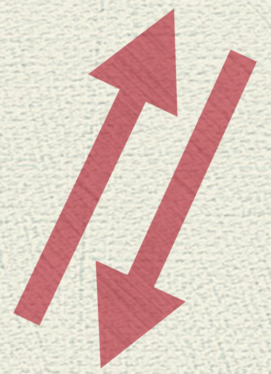
Bob

$$P(\mathbf{n}_g|\mathbf{n}) = Tr(E_g \rho_{\mathbf{n}})$$

$$F(\mathbf{n}, \mathbf{n}_g) = \frac{1}{2}(1 + \mathbf{n} \cdot \mathbf{n_g})$$

An interesting question

OR



Which pair is better?

Gisin and Popescu, PRL(1999).

N

N

$$\overline{F} = \frac{N+1}{N+2}$$

N

$$\overline{F} = \frac{N+1}{N+2}$$

N

Massar and Popescu, PRL (1995).

Existence of Continuous Optimal measurement

N

$$\overline{F} = \frac{N+1}{N+2}$$

N

Massar and Popescu, PRL (1995).

Existence of Continuous Optimal measurement

Derka, Buzek, and Ekert, PRL (1998)

Construction of finite Optimal measurement

$$\overline{F} = \frac{N+1}{N+2}$$

N

N

Massar and Popescu, PRL (1995).

Existence of Continuous Optimal measurement

Derka, Buzek, and Ekert, PRL (1998)

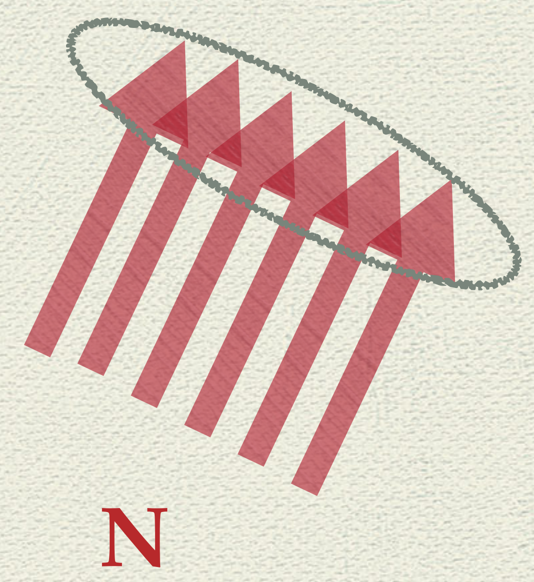
Construction of finite Optimal measurement

Latorre, Pascual, and Tarrach (1998)

Construction of minimal Optimal measurement for N<7

Using N spins

Optimal measurement



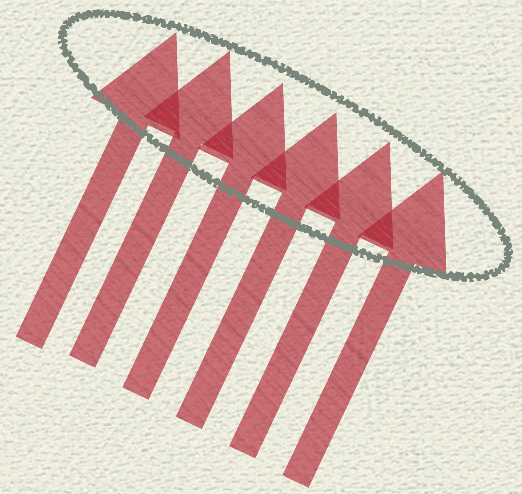
Massar and Popescu, PRL (1995).

Using N spins

Optimal measurement

N

$$\overline{F} = \frac{N+1}{N+2}$$



N

Massar and Popescu, PRL (1995).

The problem of security

Eve can do measurement on half of the spins

The problem of security

Eve can do measurement on half of the spins

I- Using Entangled States for setting up an SRF

F. Rezazadeh, A. Mani, V. Karimipour, Phys. Rev. A, 96, 022310 (2017)

Using entanglement

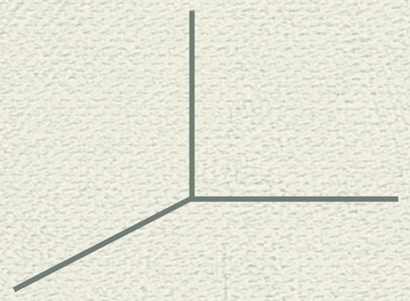
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

The idea of QKD:

Alice

QKD: Publicly announce bases

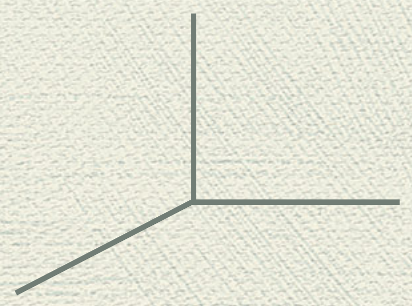
Bob



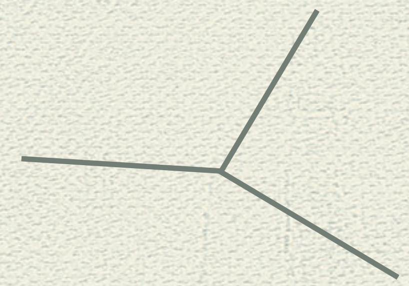
Keep the results for yourself.

The idea of Direction Sharing

Alice



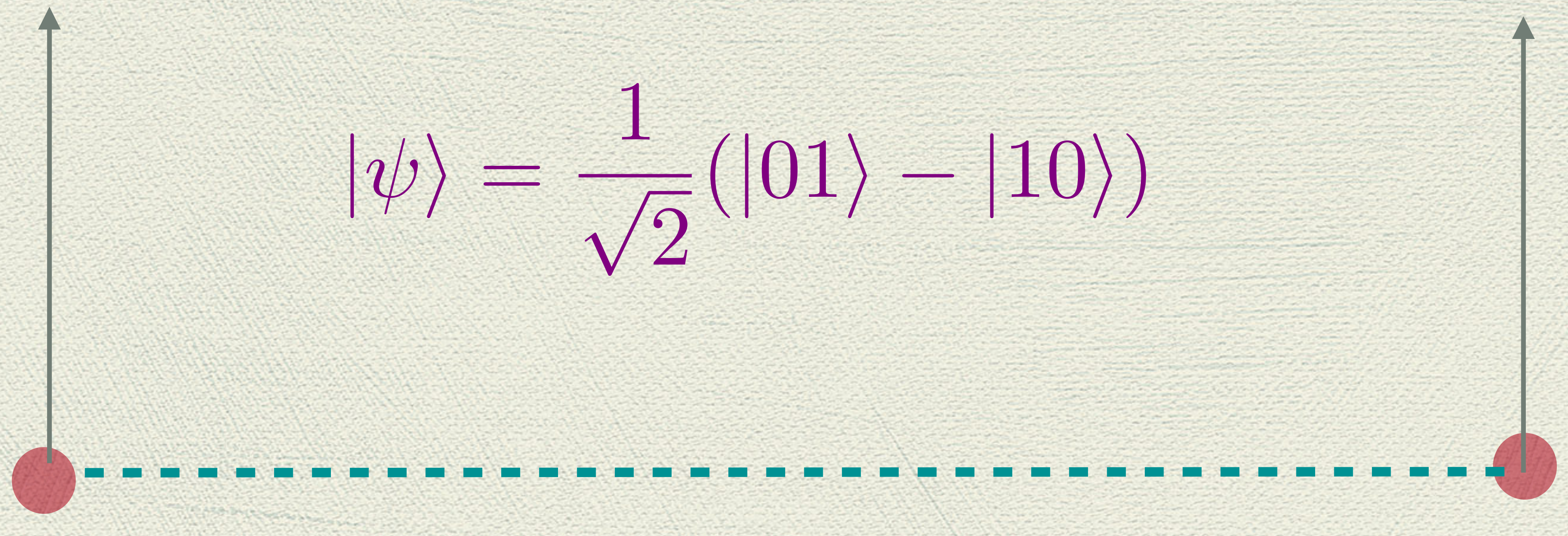
Bob



Publicly announce the results

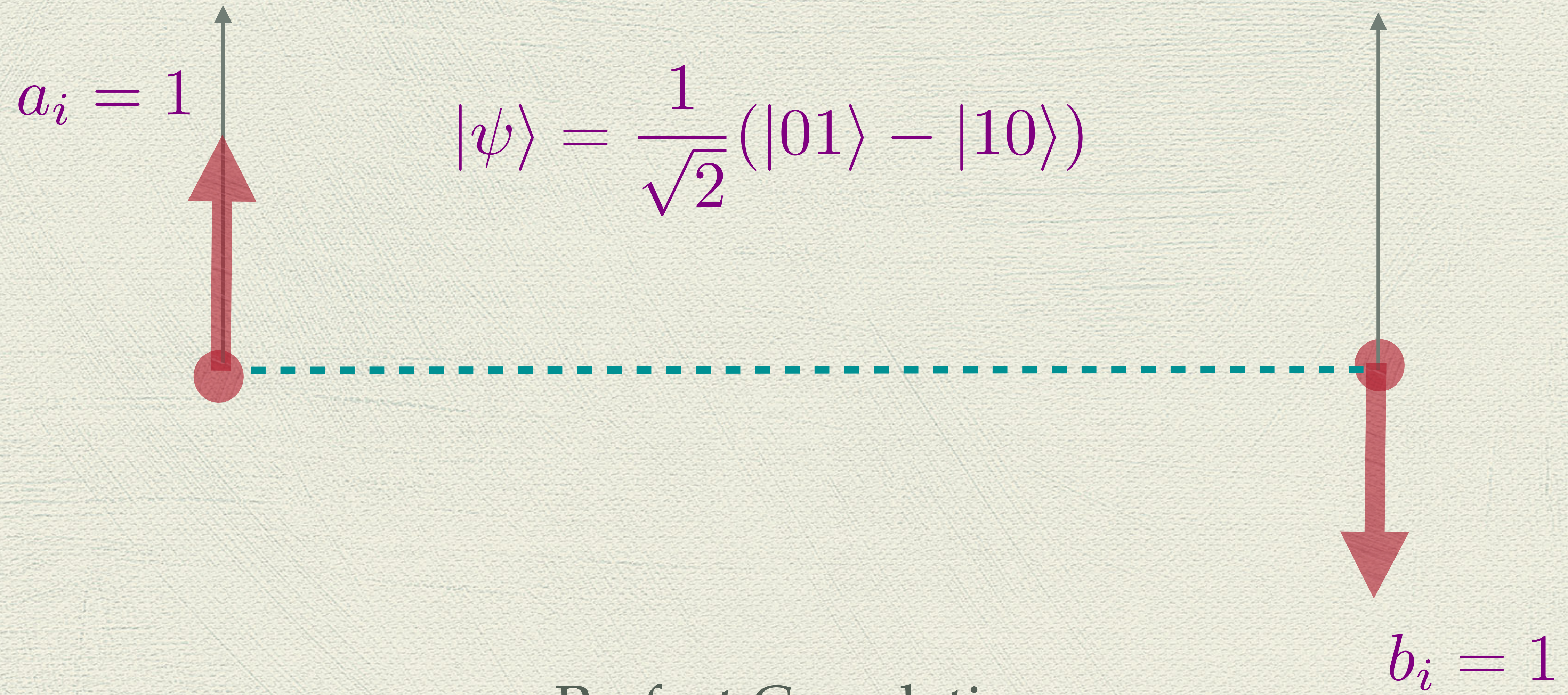
And use the correlations to align the bases

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

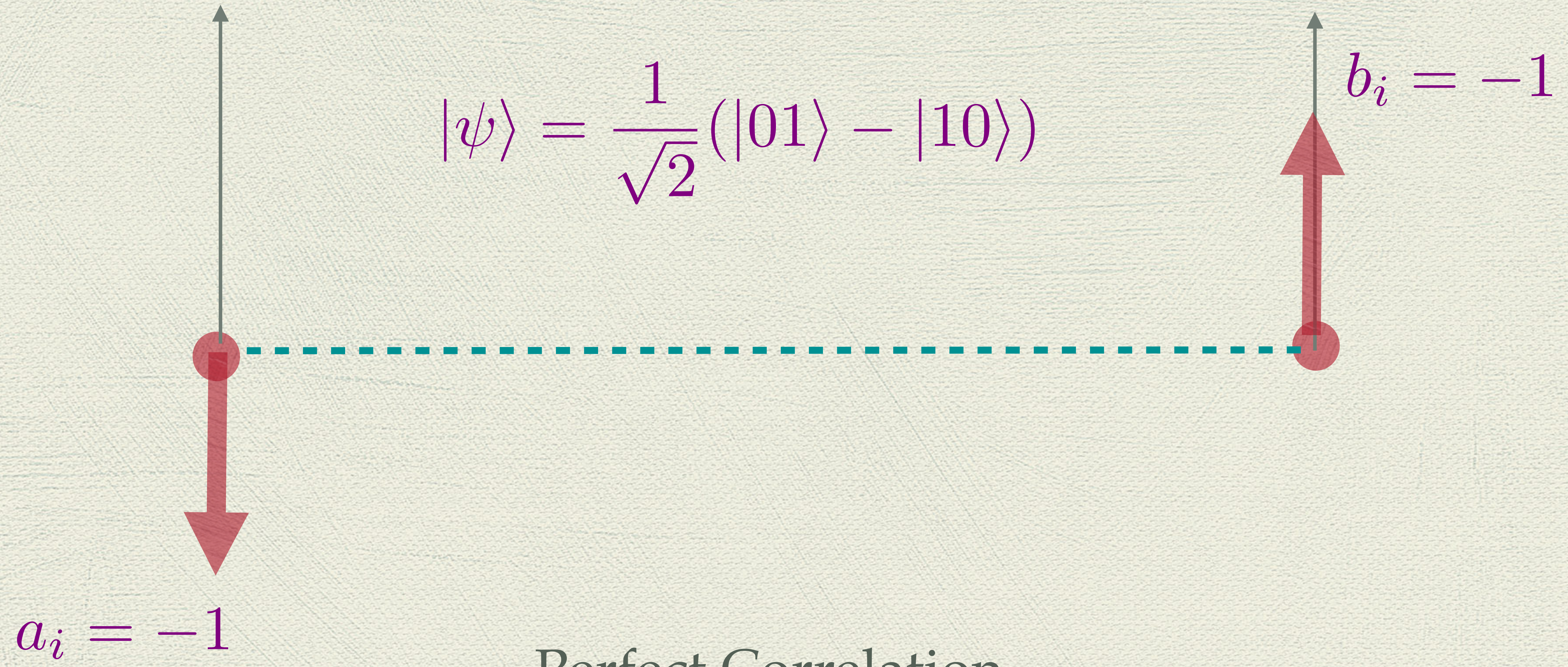


Perfect Correlation

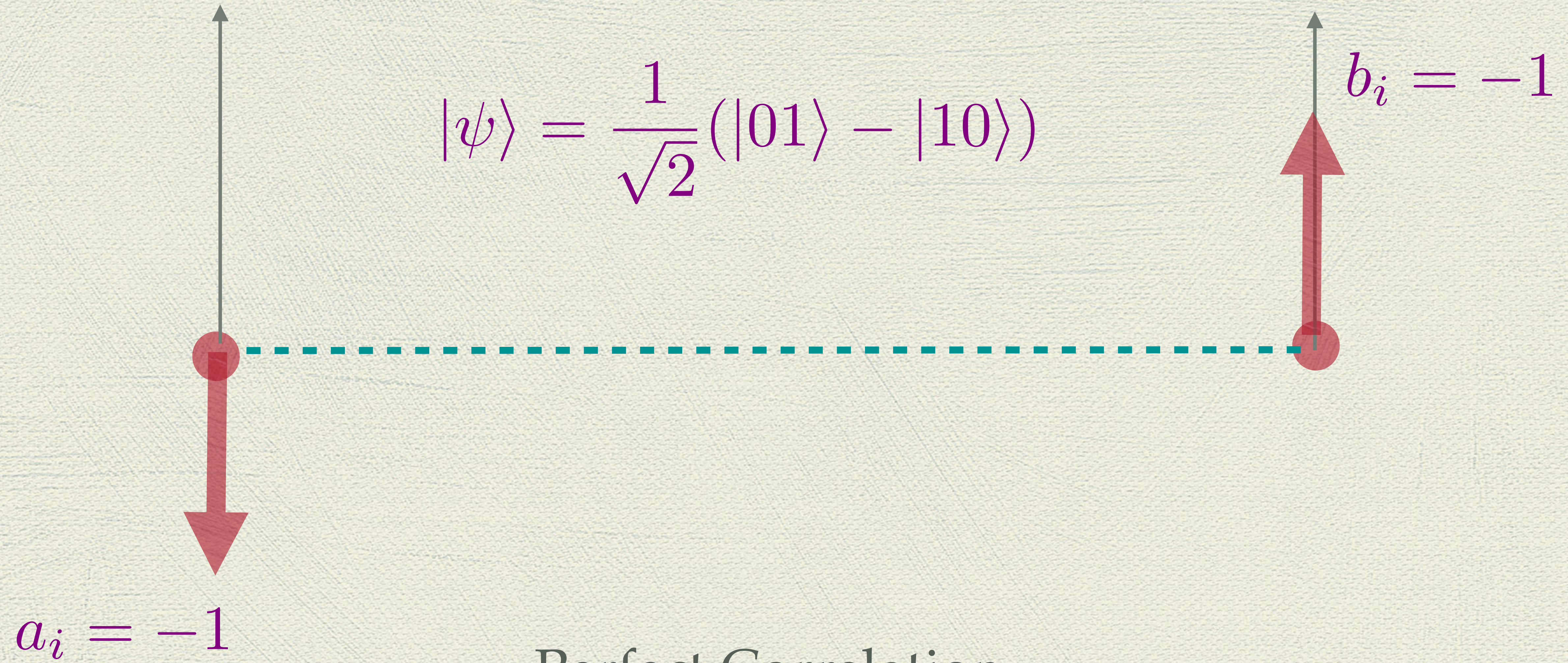
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



Perfect Correlation

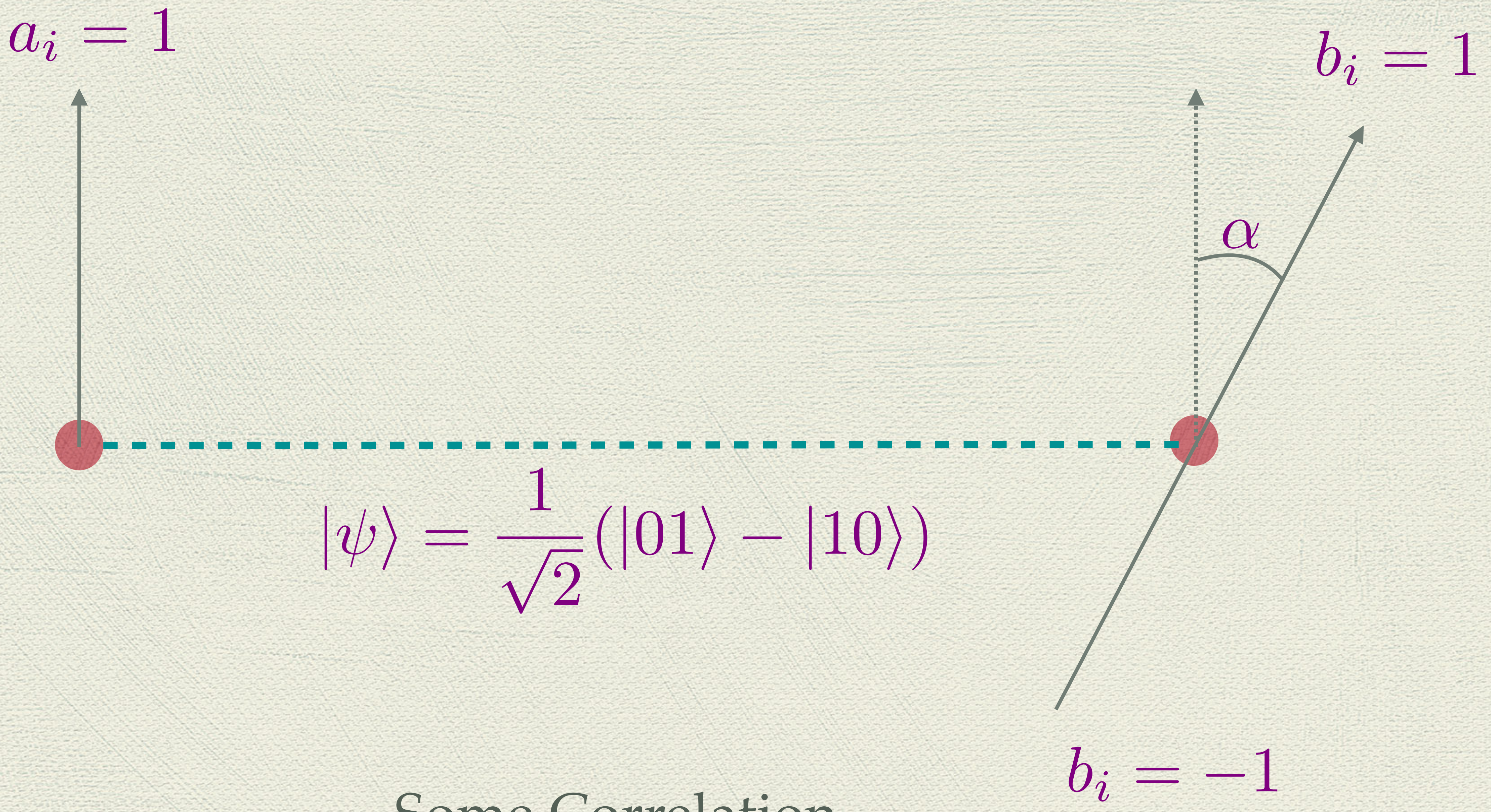


Perfect Correlation

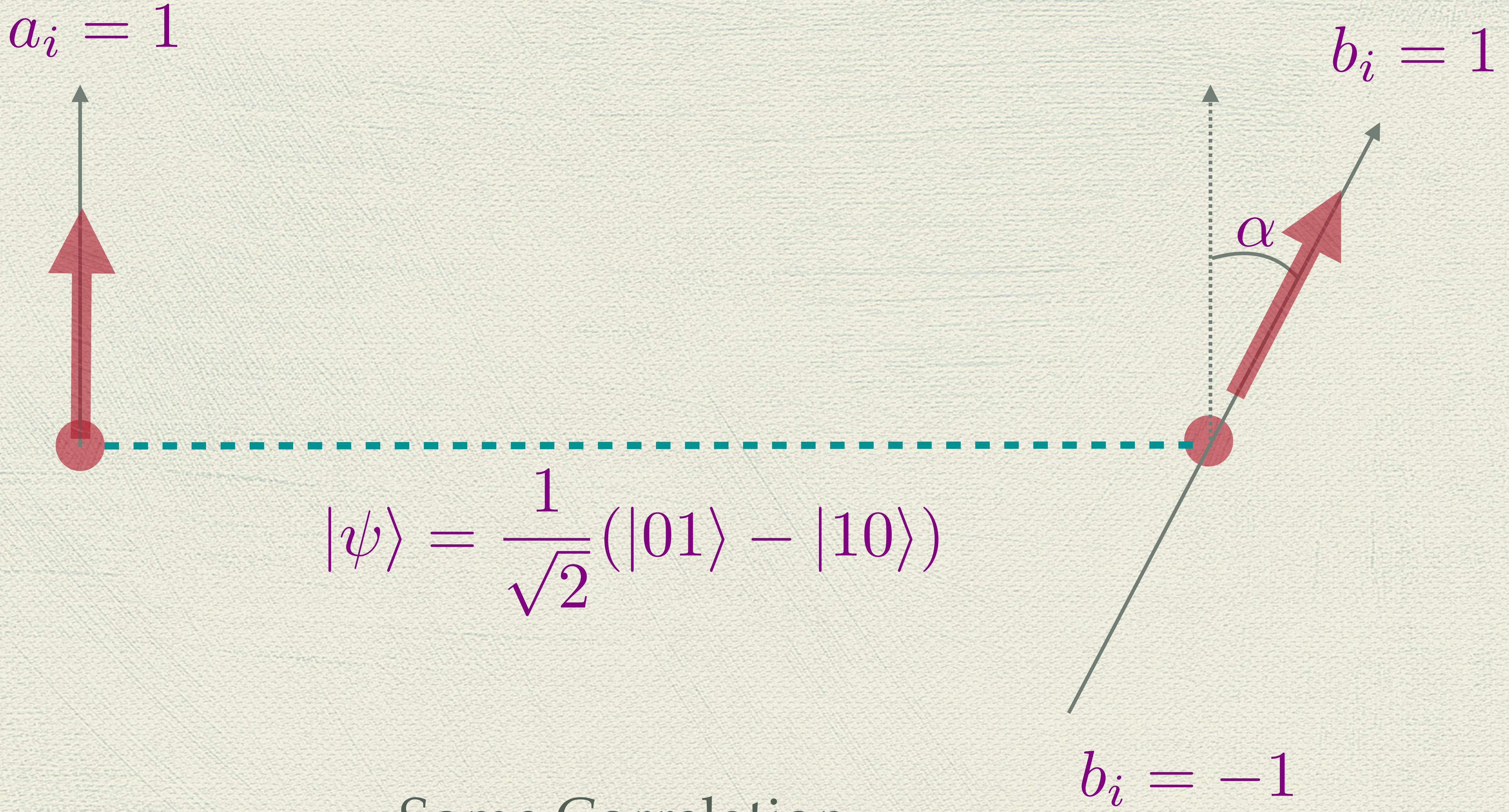


Perfect Correlation

$$q_N = \frac{1}{N}\sum_i a_i b_i = 1$$

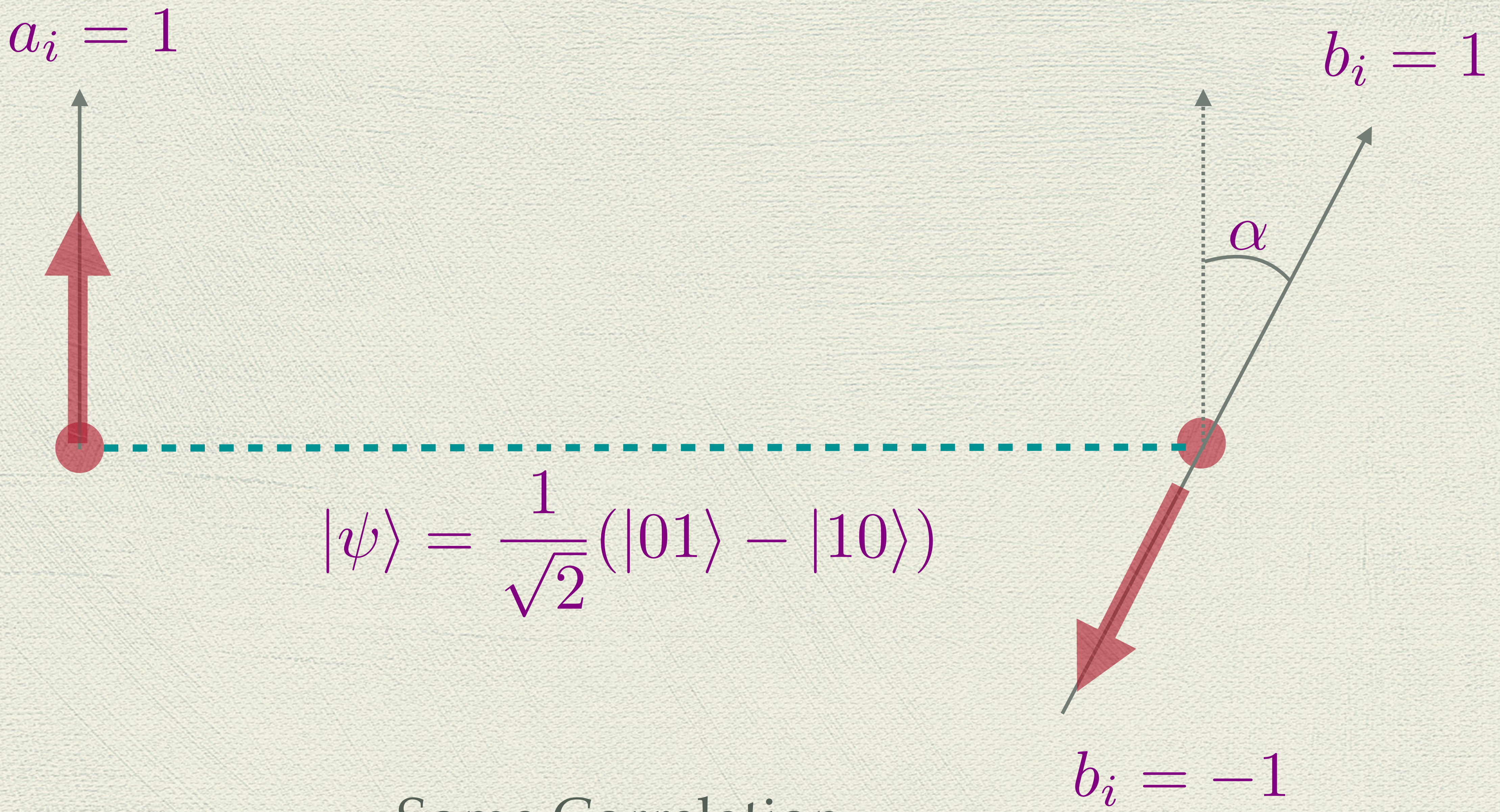


Some Correlation

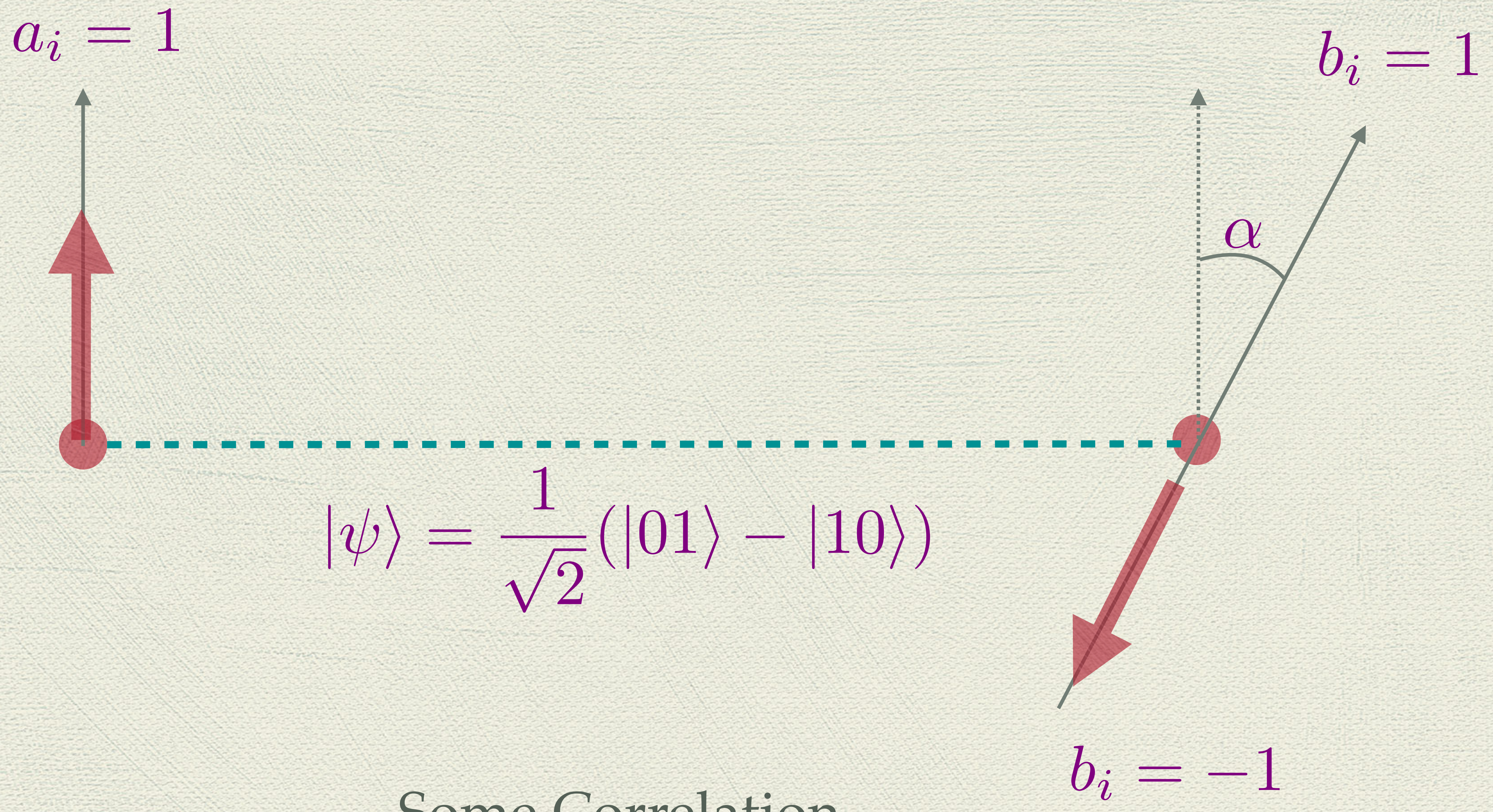


$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Some Correlation



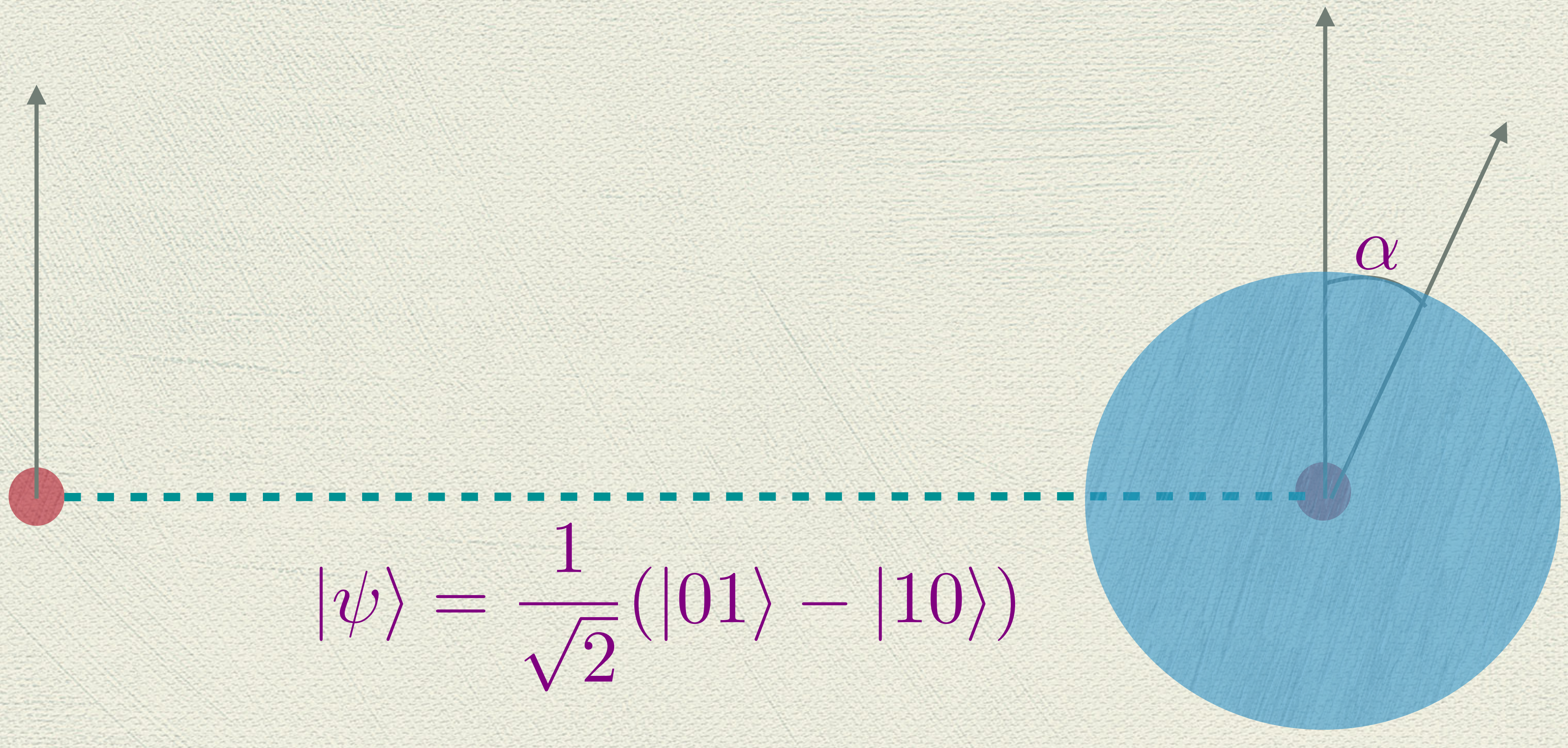
Some Correlation



Some Correlation

$$q_N = \frac{1}{N}\sum_i a_i b_i$$

A naive method: Brute force search

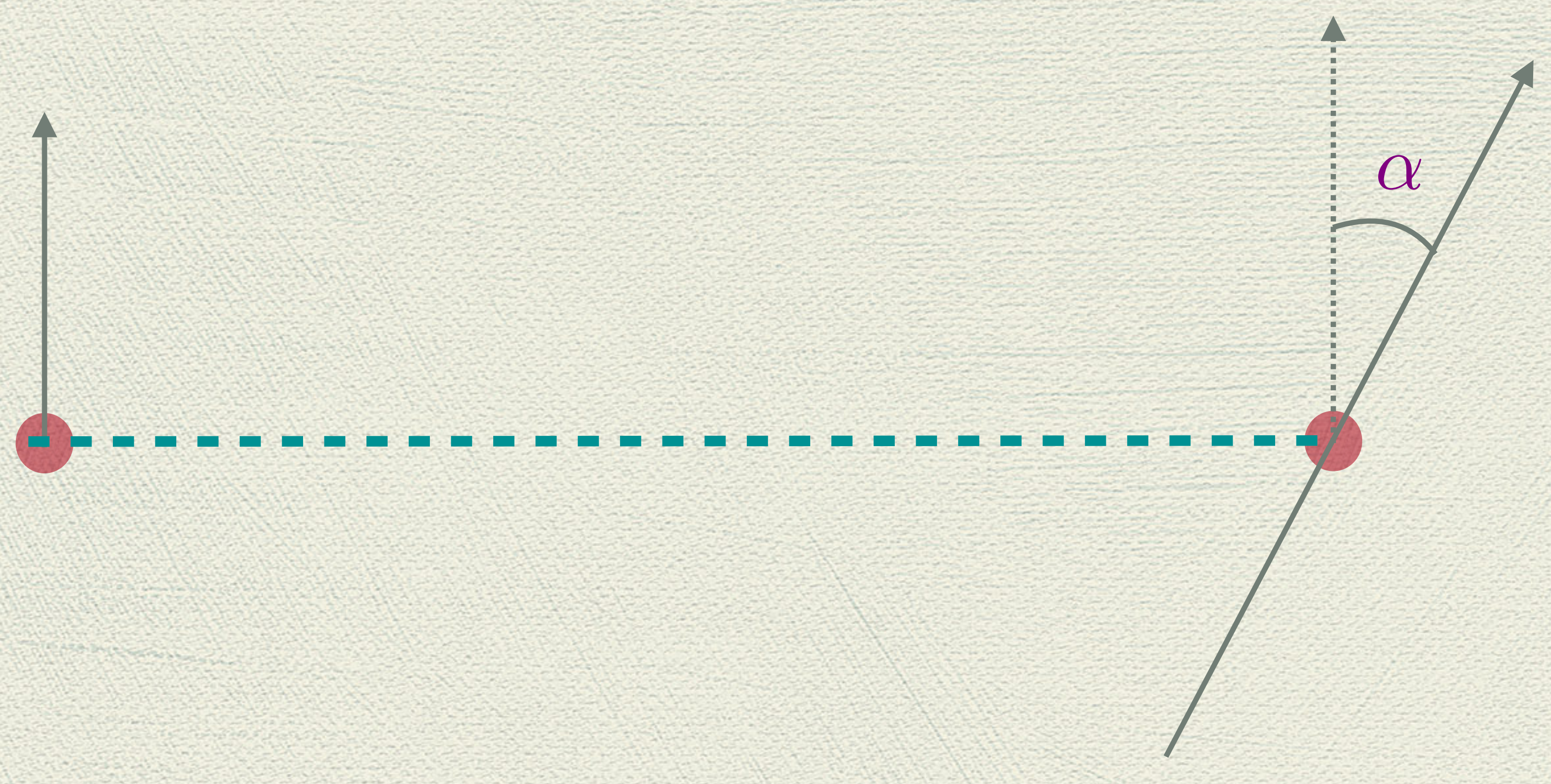


$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

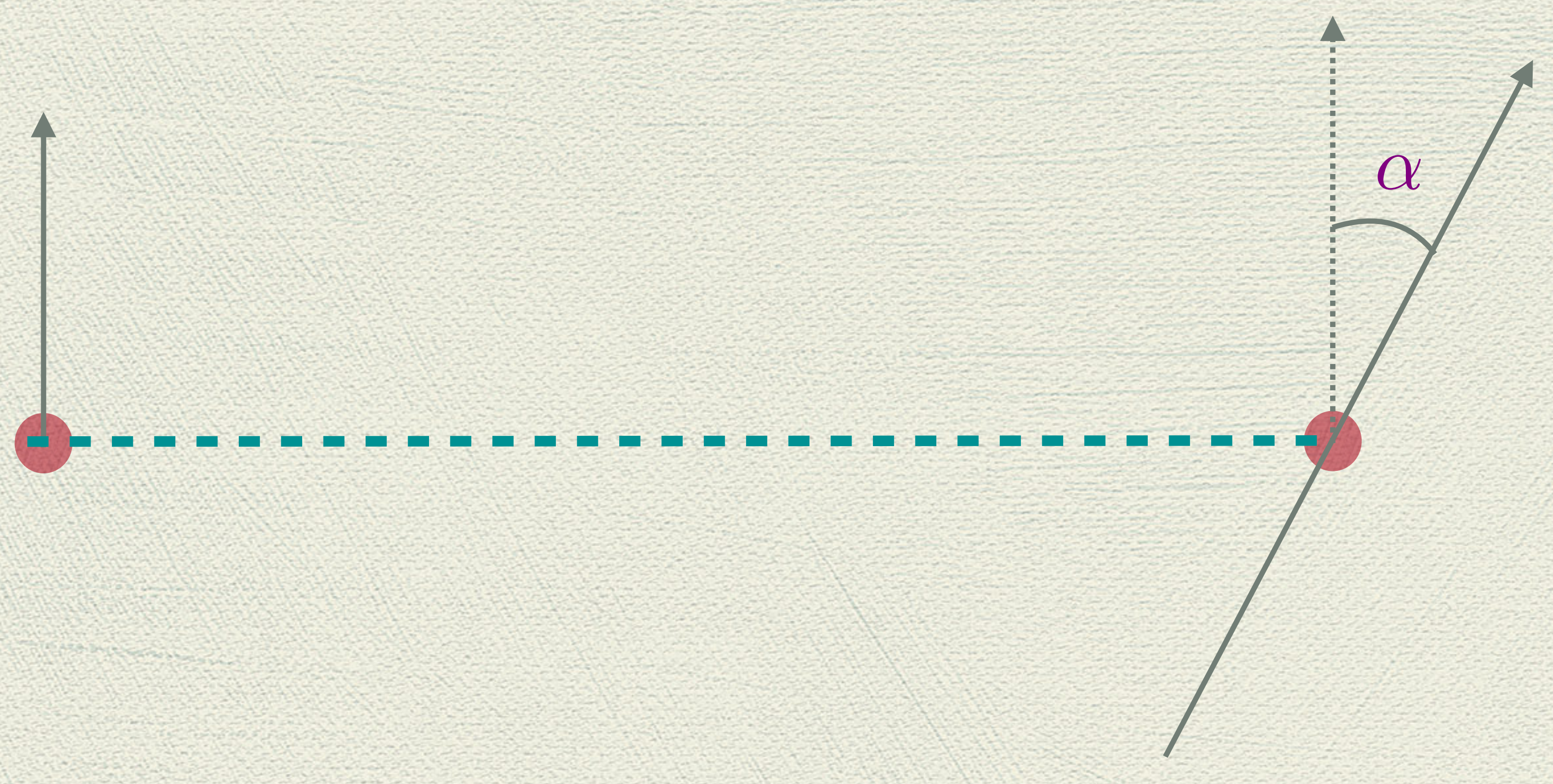
However

The number of pairs is not infinite!

So we have to estimate the angle from a correlation which has fluctuations.



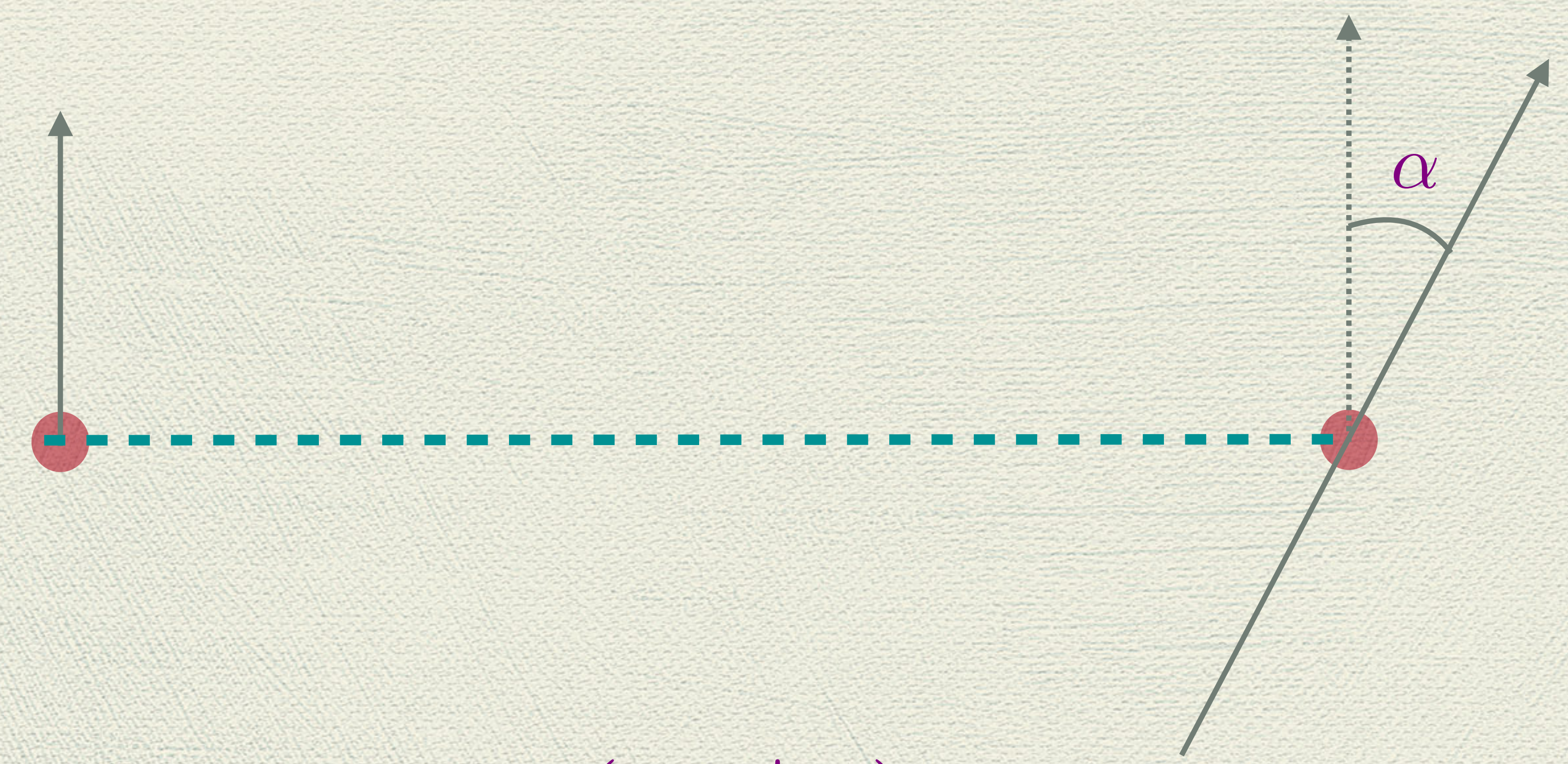
The probability that the correlation is q_N if the angle is α



$$P(q_N|\alpha)$$

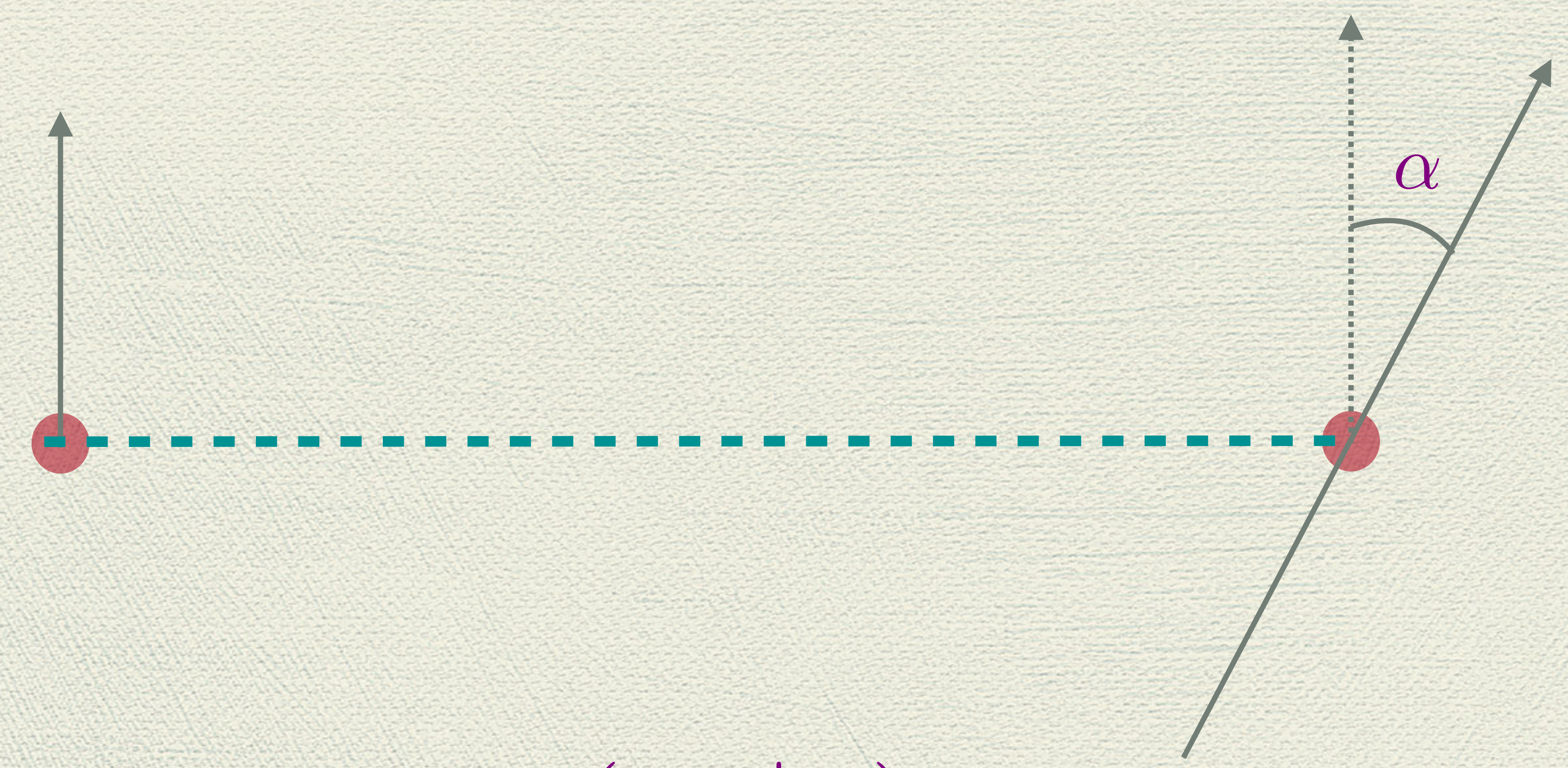
The probability that the correlation is q_N if the angle is α

$$P(q_N|\alpha)$$



$$P(q_N|\alpha)$$

$$\langle q_N \rangle = \cos\alpha$$



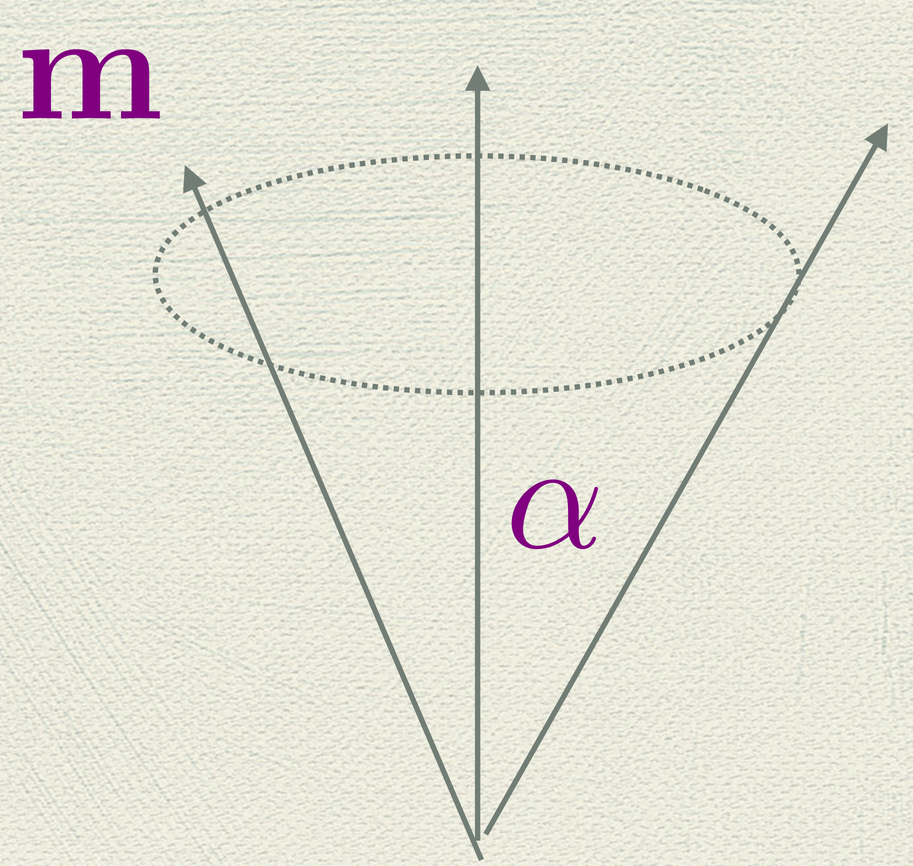
$$P(q_N|\alpha)$$

$$\langle q_N \rangle = \cos\alpha$$

$$\langle q_N^2 \rangle = \cos^2\alpha + \frac{1}{N}\sin^2\alpha$$

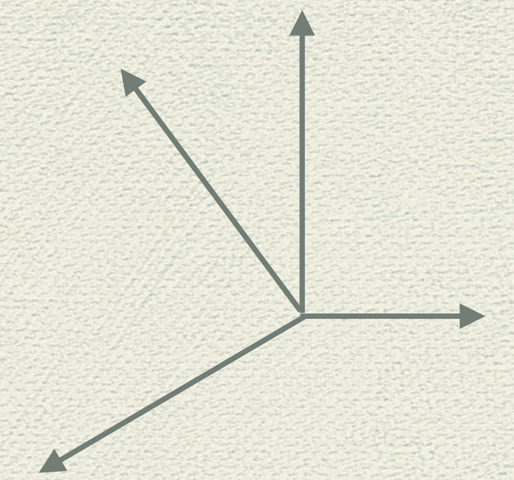
The Baeysian Approach

$$P(\alpha|q_N)$$



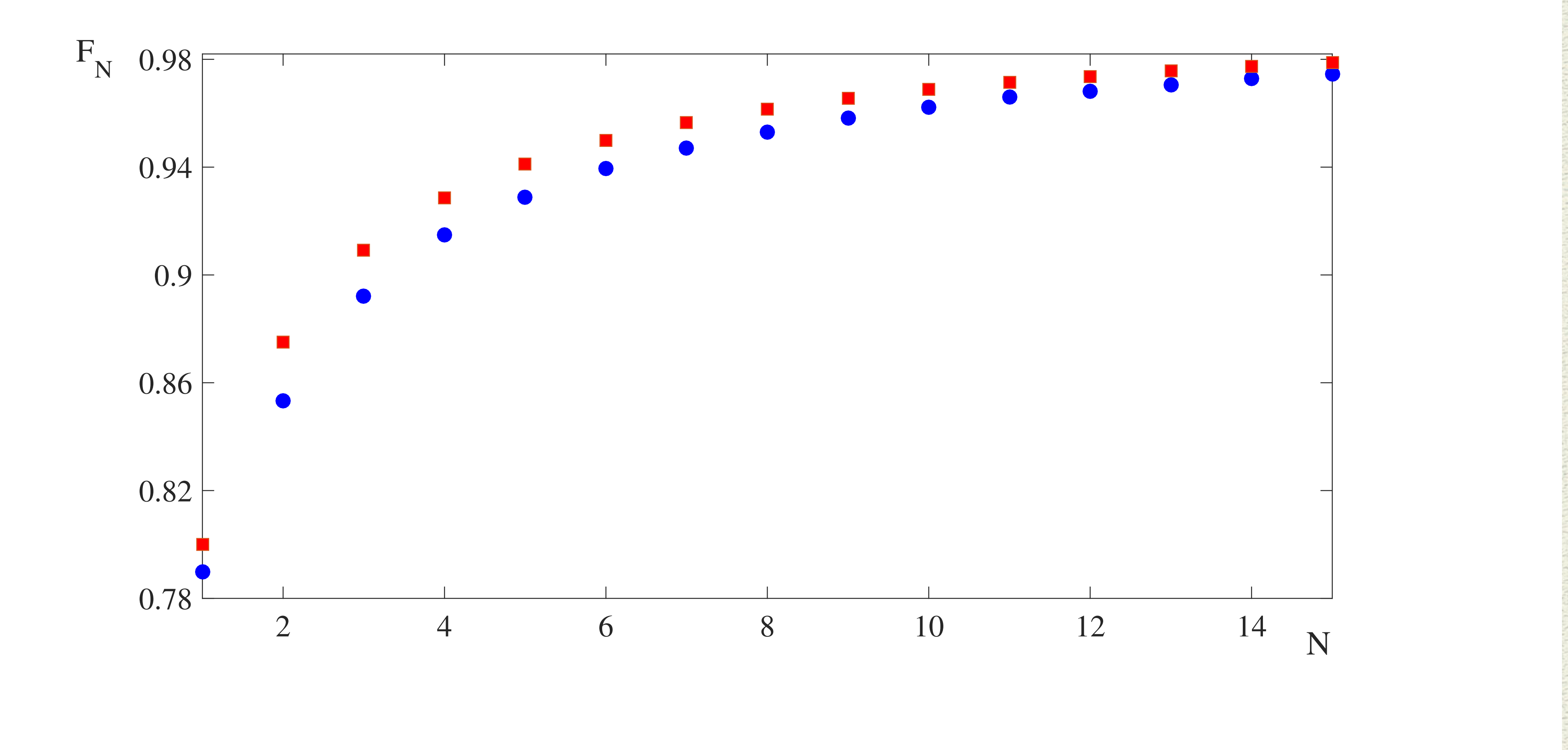
What is the probability that the angle is α if the correlation is q_N

A good estimate with three measurements



$$\mathbf{m}_e = \frac{1}{\sqrt{q_x^2 + q_y^2 + q_z^2}} \left(q_x \mathbf{x} + q_y \mathbf{y} + q_z \mathbf{z} \right)$$

Comparison with previous methods



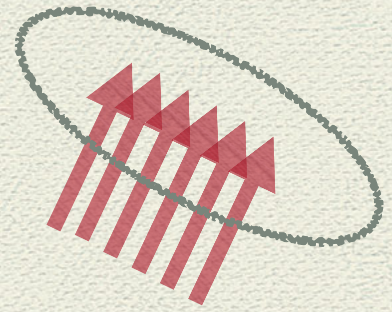
- Our method
- Other methods

$$\overline{F}_N = \frac{3N+1}{3N+2}$$

Advantages of our method-1

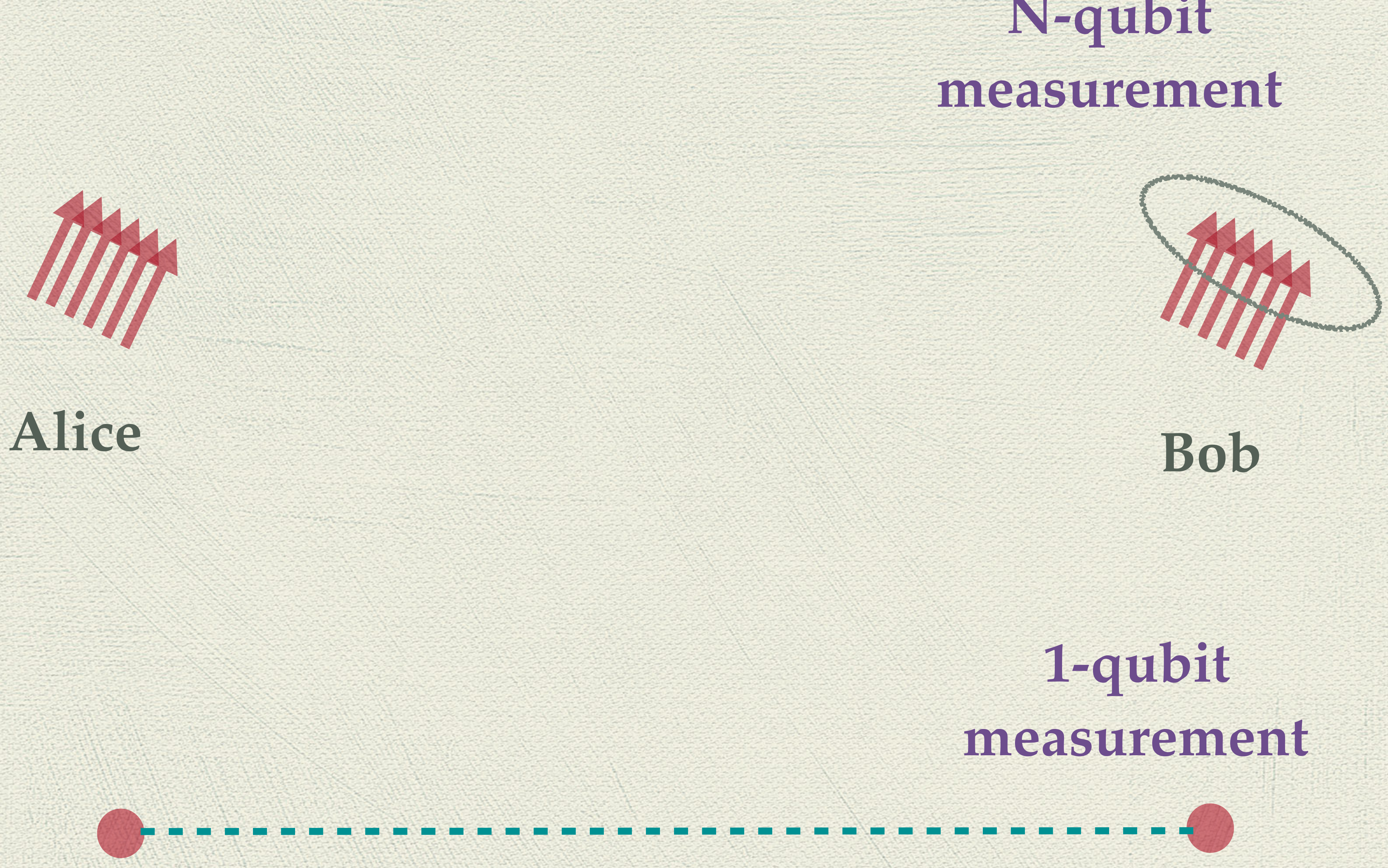
N-qubit measurement

Alice



Bob

Advantages of our method-1



2- The problem of security

Eve cannot unravel the shared direction, since only

unspeakable

information is being communicated.

1 0 1 0 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0

II- Power of a shared singlet state

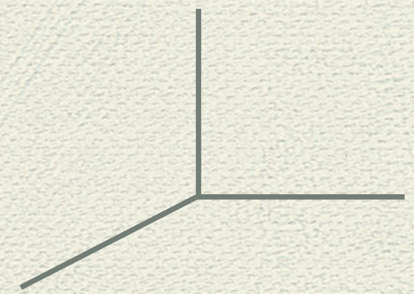
F. Rezazadeh, A. Mani, V. Karimipour, PRA, 100, (2019).

Which one is better?

Alice

Bob

Shared Reference Frame (SRF)



$|\psi^-\rangle$

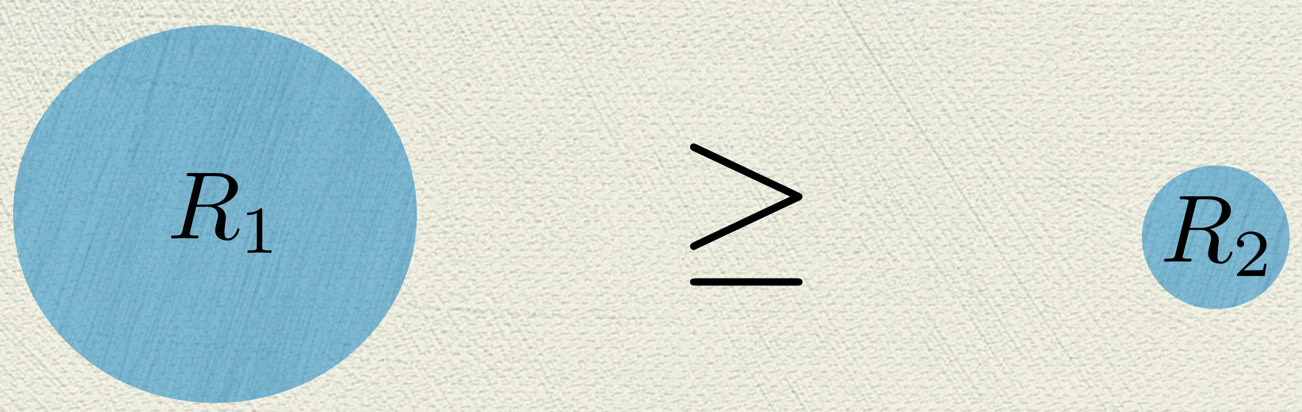
Alice

Bob

Shared Singlet State (SRF)


FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
J 00000007 A
WASHINGTON, D.C.
10
TEN
SERIES 1950
HAMILTON
WILL PAY TO THE BEARER ON DEMAND
TEN DOLLARS

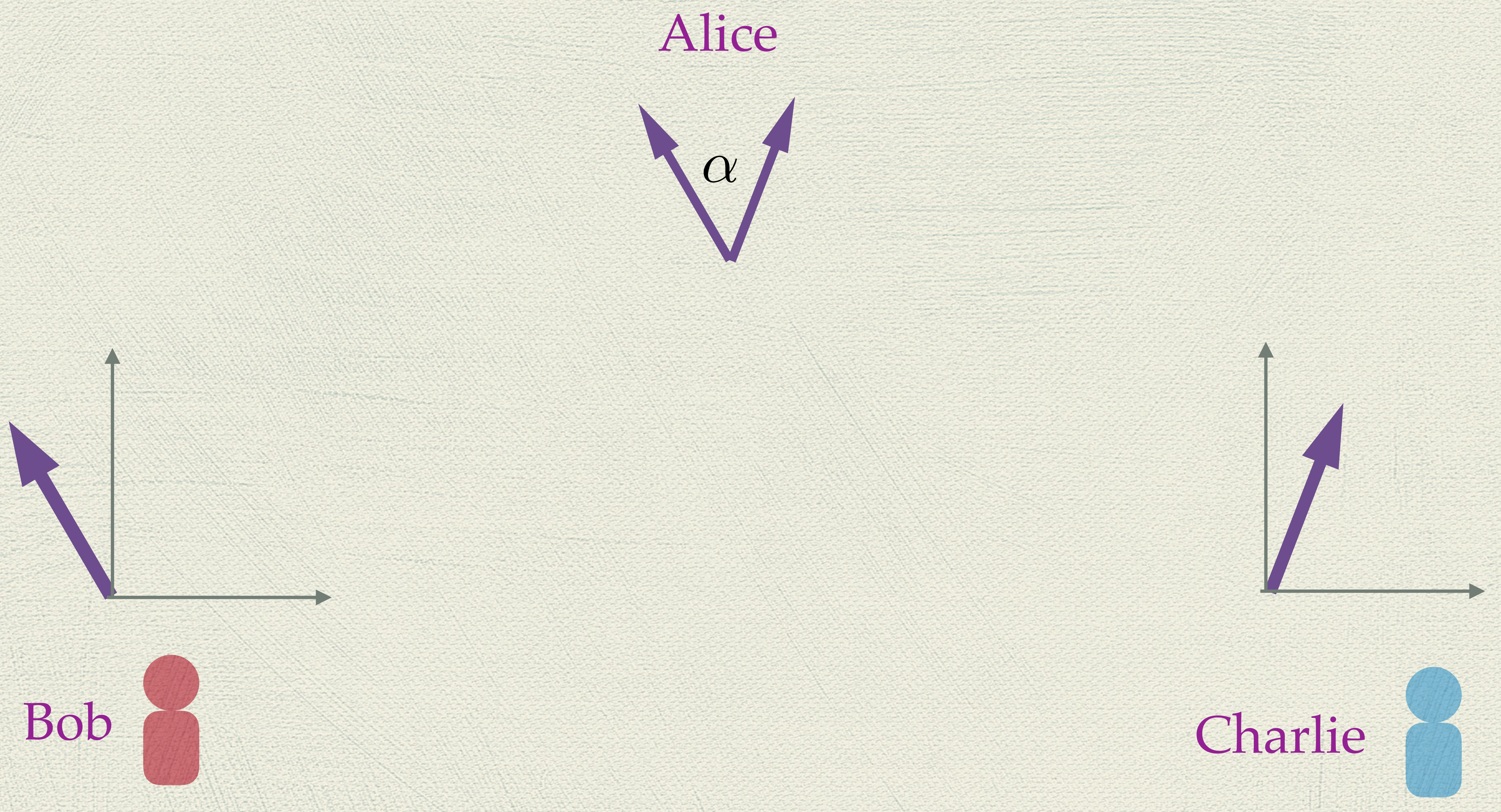
The main idea of resource theory:



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad \geq \qquad a|00\rangle + b|11\rangle$$

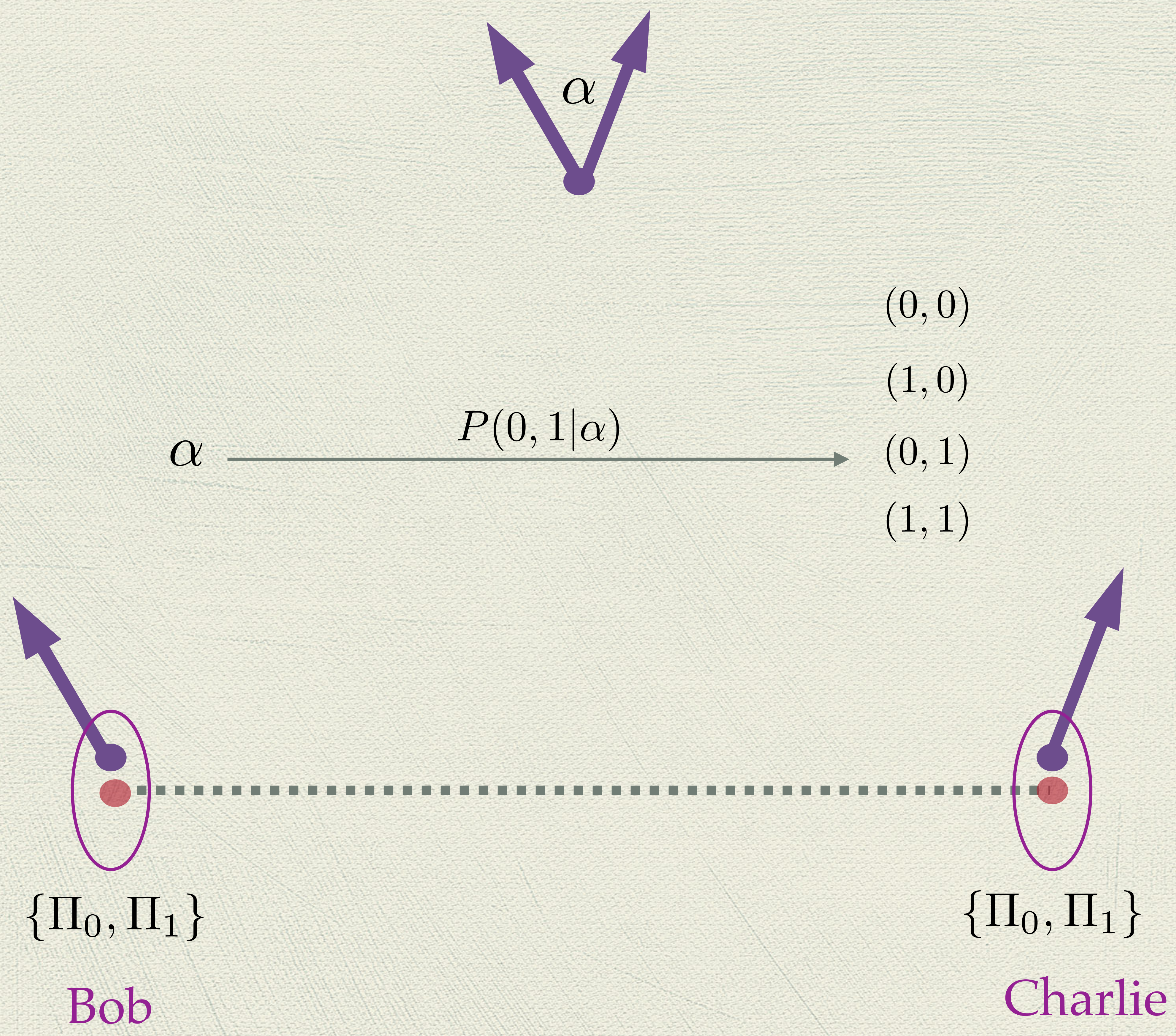
$\leq$

An example: Estimation of an angle

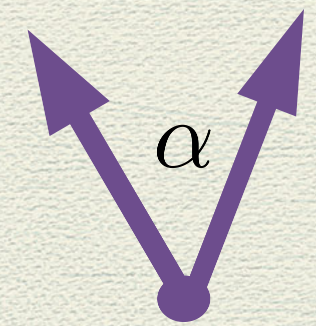


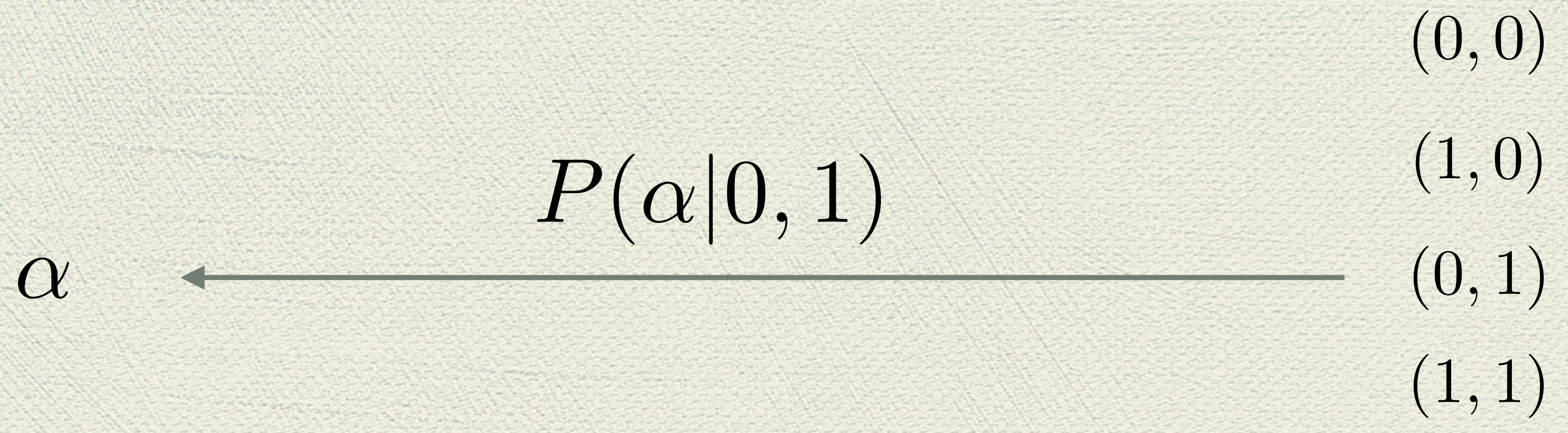
Information Gain= 0.0270

S. D. Bartlett, T. Rudolph and R. W. Spekkens, PRA (2004).



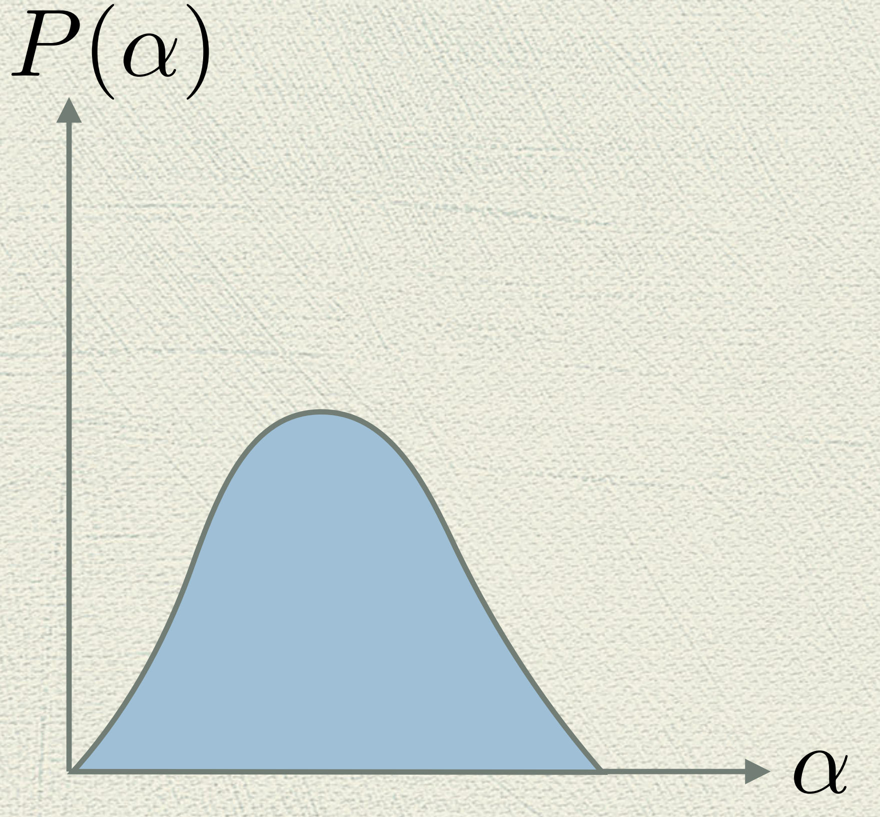
α
$(0,0)$
$(1,0)$
$P(0,1|\alpha)$
α
$(0,1)$
$(1,1)$
$\{\Pi_0, \Pi_1\}$
$\{\Pi_0, \Pi_1\}$
Bob
Charlie

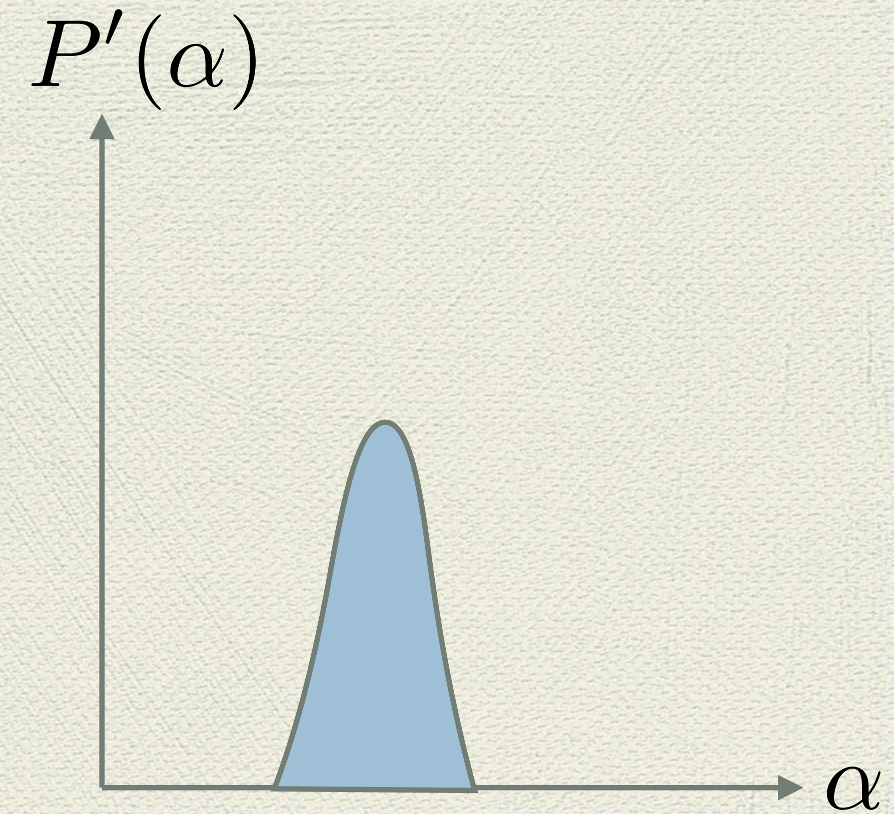

α


$(0,0)$
$(1,0)$
$P(\alpha|0,1)$
α
$(0,1)$
$(1,1)$

How do we judge our success?

$$S = -\int P(\alpha) \log P(\alpha) d\alpha$$

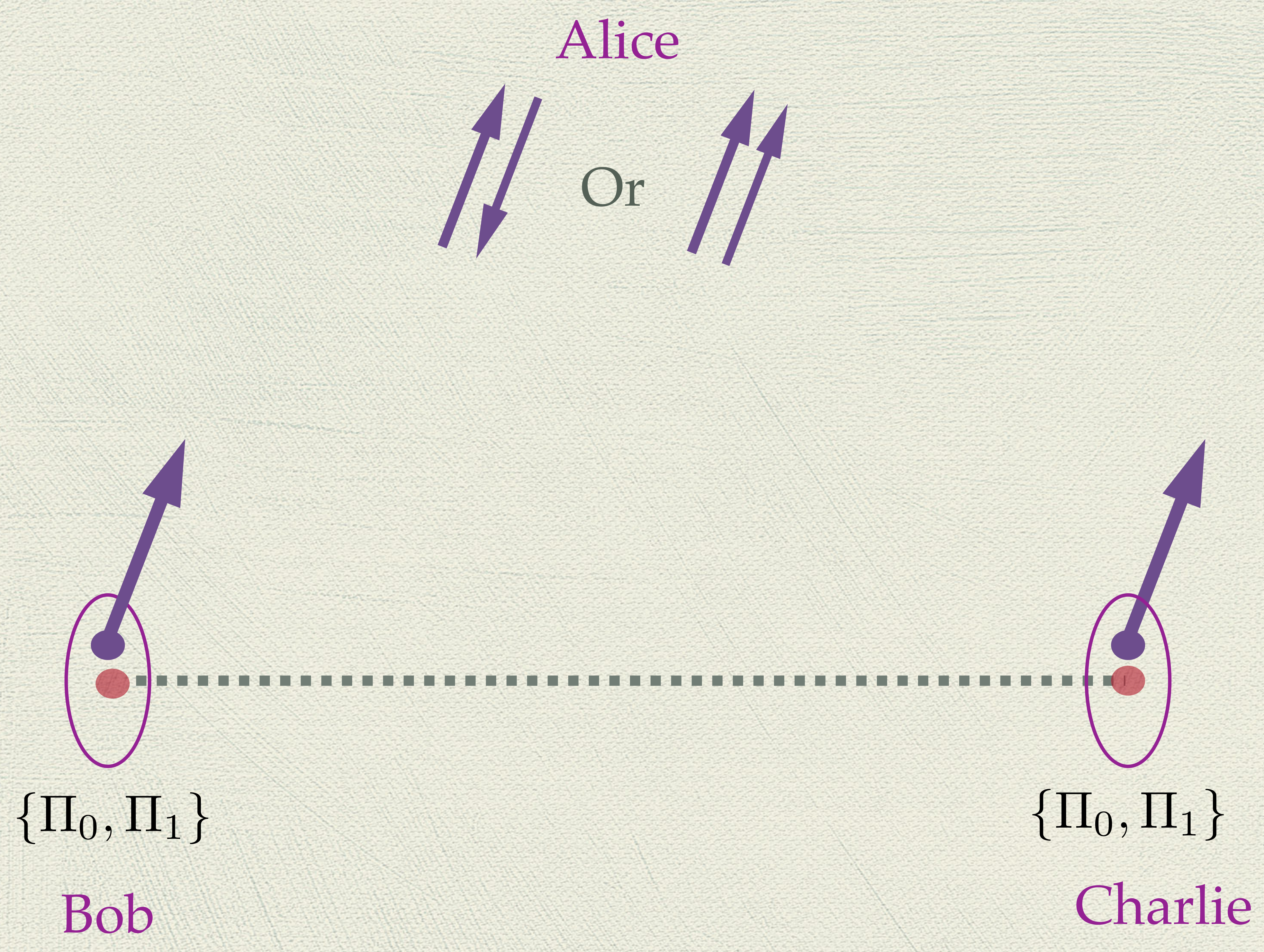




Information Gain= 0.0284

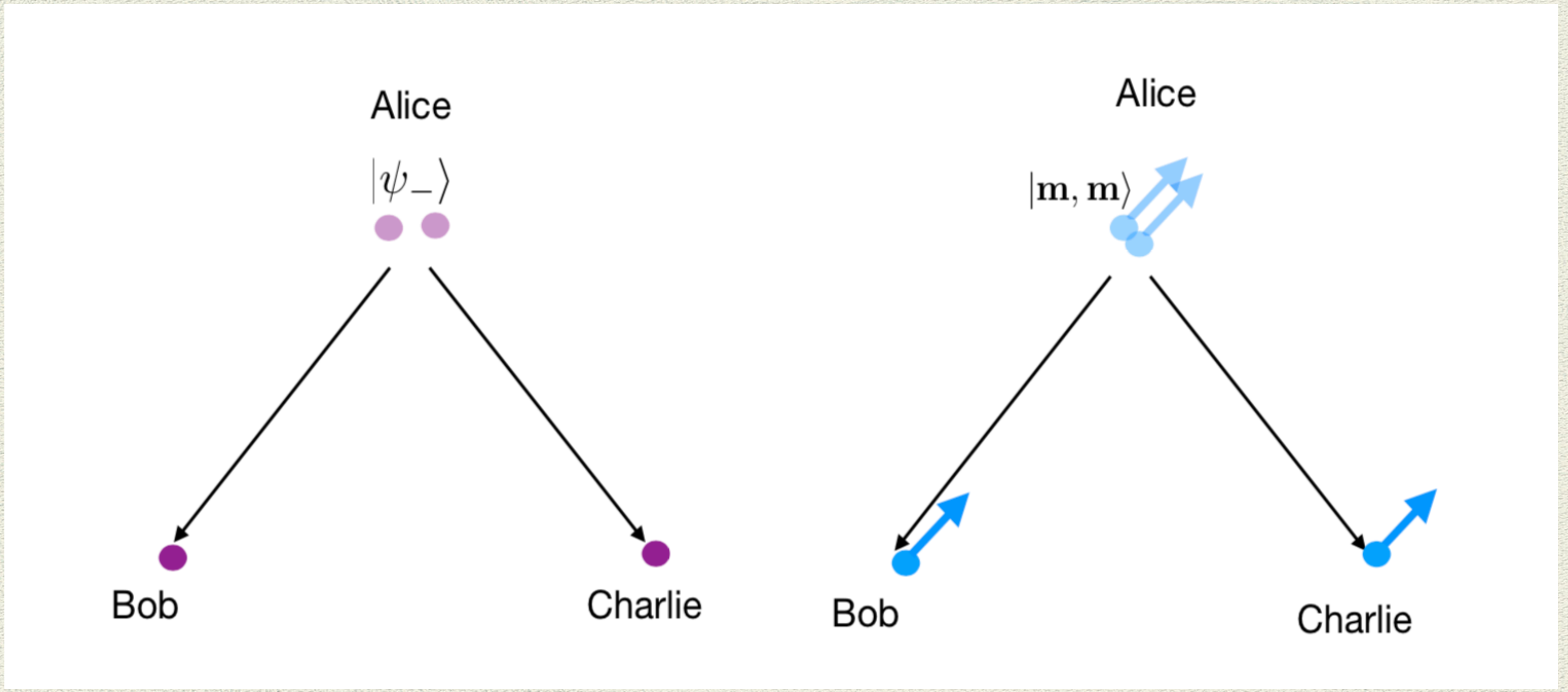
F. Rezazadeh, A. Mani, V. Karimipour, PRA, 100, (2019).

resource / task	SRF	SSS
angle estimation	0.0270	0.0284


Alice
Or
$\{\Pi_0, \Pi_1\}$
$\{\Pi_0, \Pi_1\}$
Bob
Charlie

task \ resource	SRF	SSS $j = \frac{1}{2}$	SSS $j = 1$	SSS $j \longrightarrow \infty$
Discriminating parallel and anti-parallel spins	0.0817	0.0981	0.0841	0.0817

Task b: Discrimination between Parallel spins and a singlet

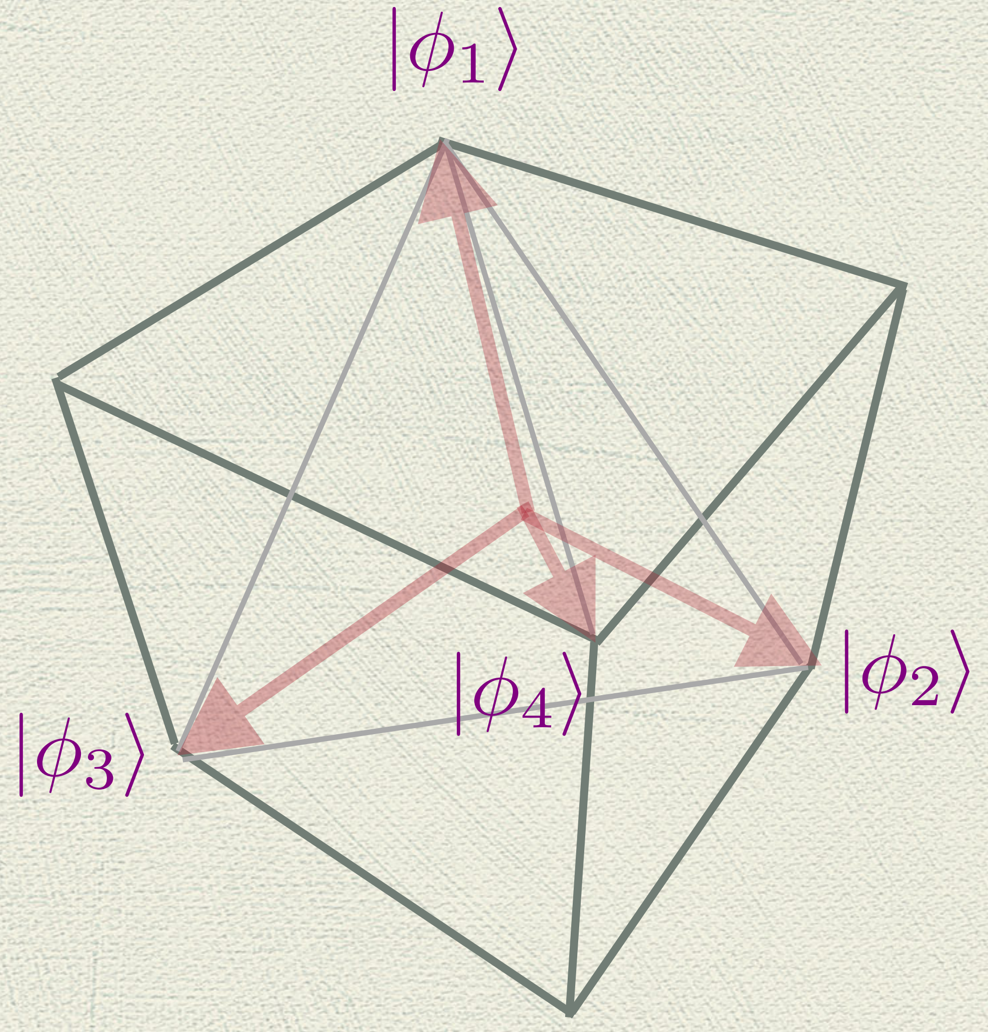


resource	SRF	SSS	refbit
probability of conclusive result	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{24}$

Continued in lecture 2

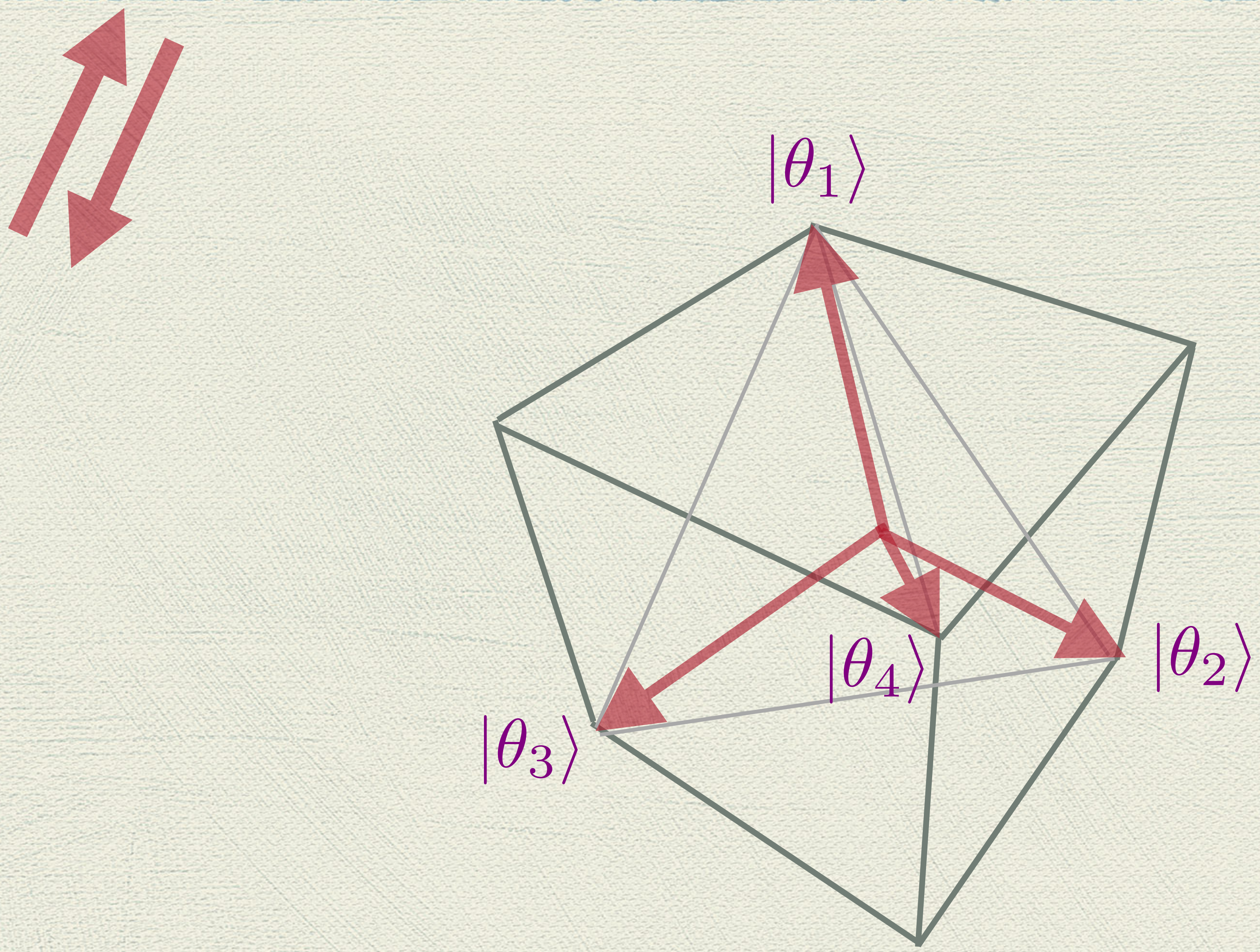
Thank you for your attention

Measurements

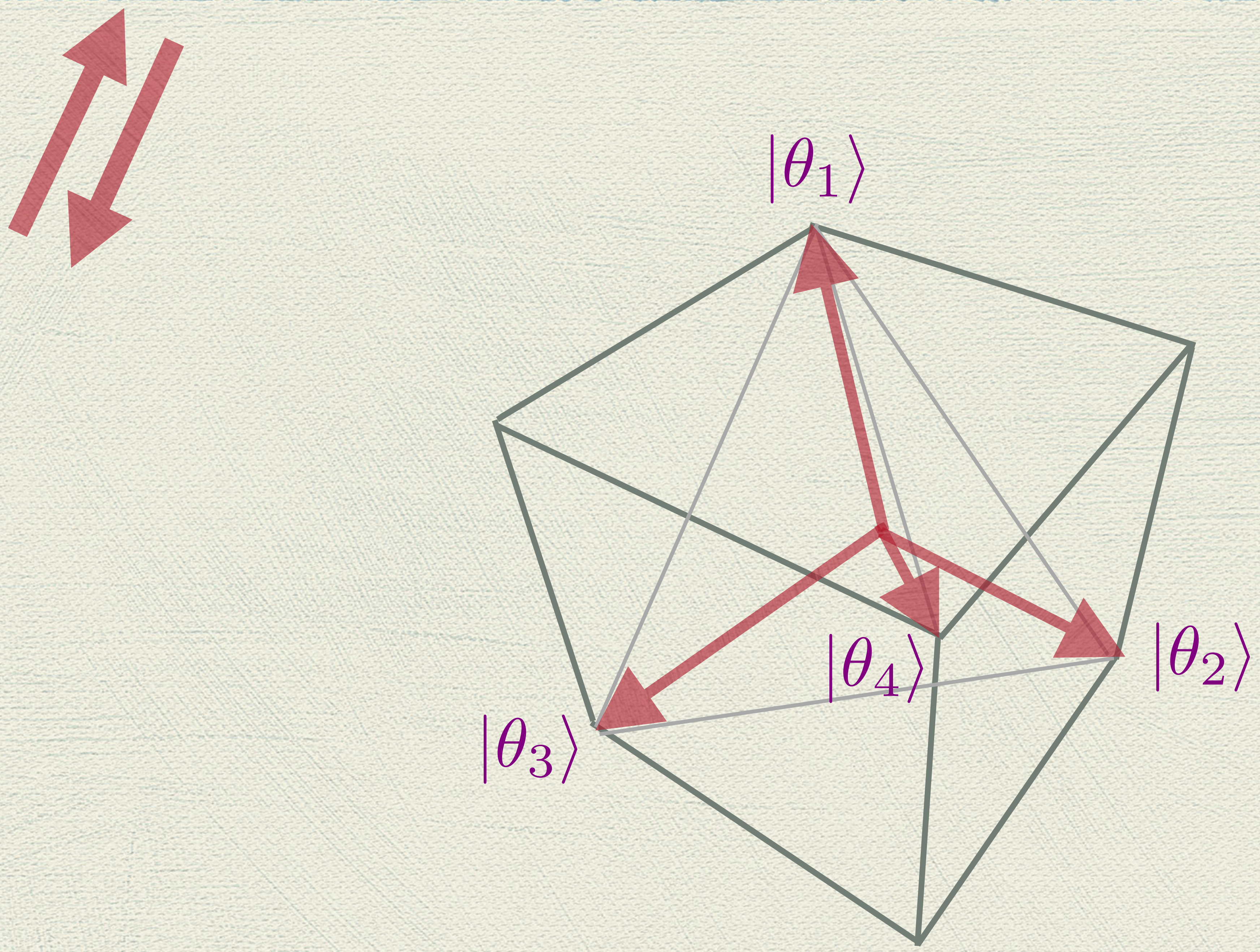


$$|\phi_j\rangle = \frac{\sqrt{3}}{2}|\mathbf{n}_j, \mathbf{n}_j\rangle + \frac{1}{2}|\psi^-\rangle$$

$$P(\mathbf{n}_g|\mathbf{n}) = Tr(E_g \rho_{\mathbf{n}})$$



$$|\theta_i\rangle = \alpha|\mathbf{n}_i, -\mathbf{n}_i\rangle + \beta|\omega\rangle$$



$\overline{F} = 0.79$

$$|\theta_i\rangle = \alpha|\mathbf{n}_i, -\mathbf{n}_i\rangle + \beta|\omega\rangle$$

Bayes Method

$$(p, 1 - p)$$

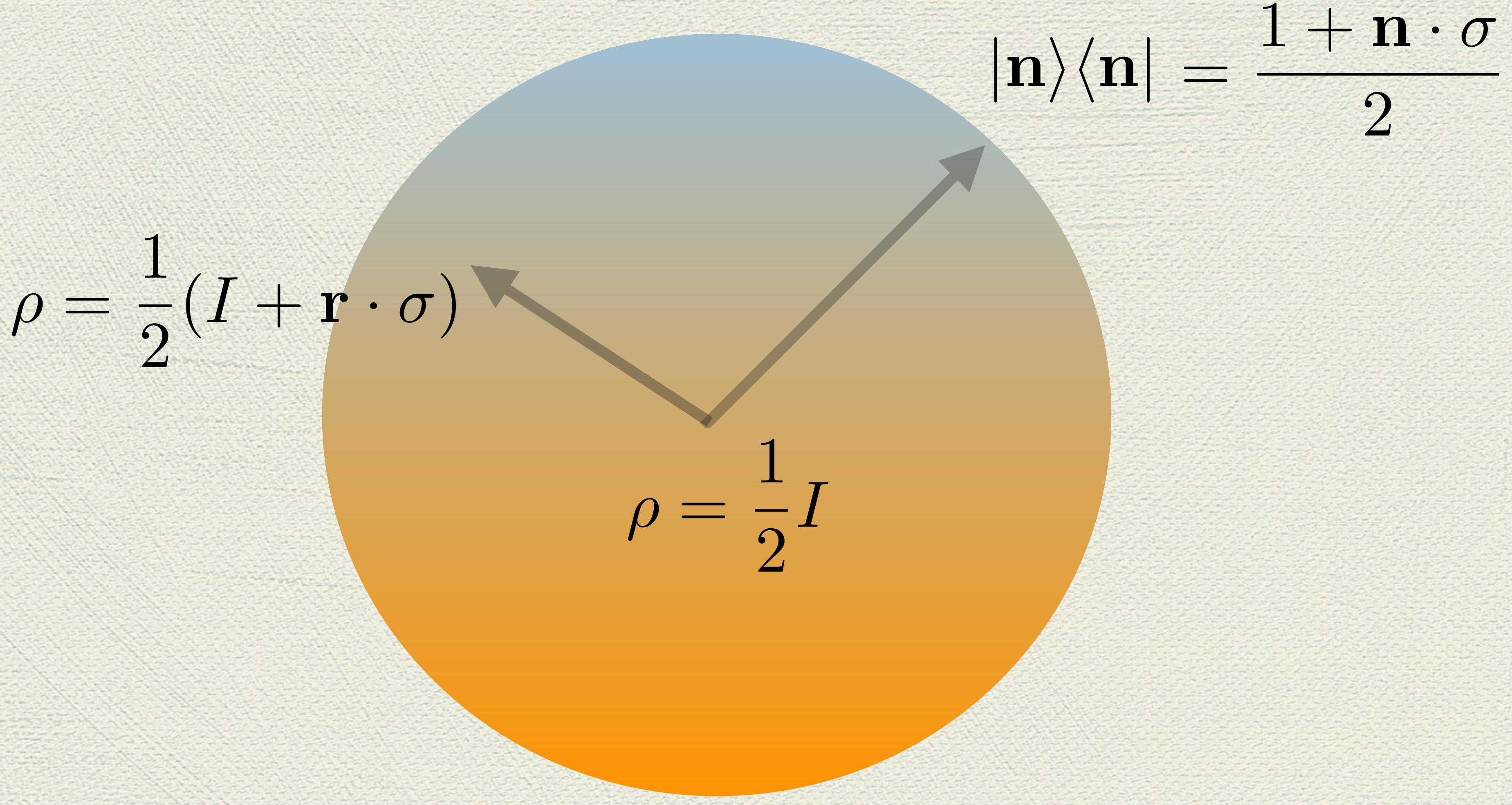
$$(q, 1 - q)$$

9 Tails

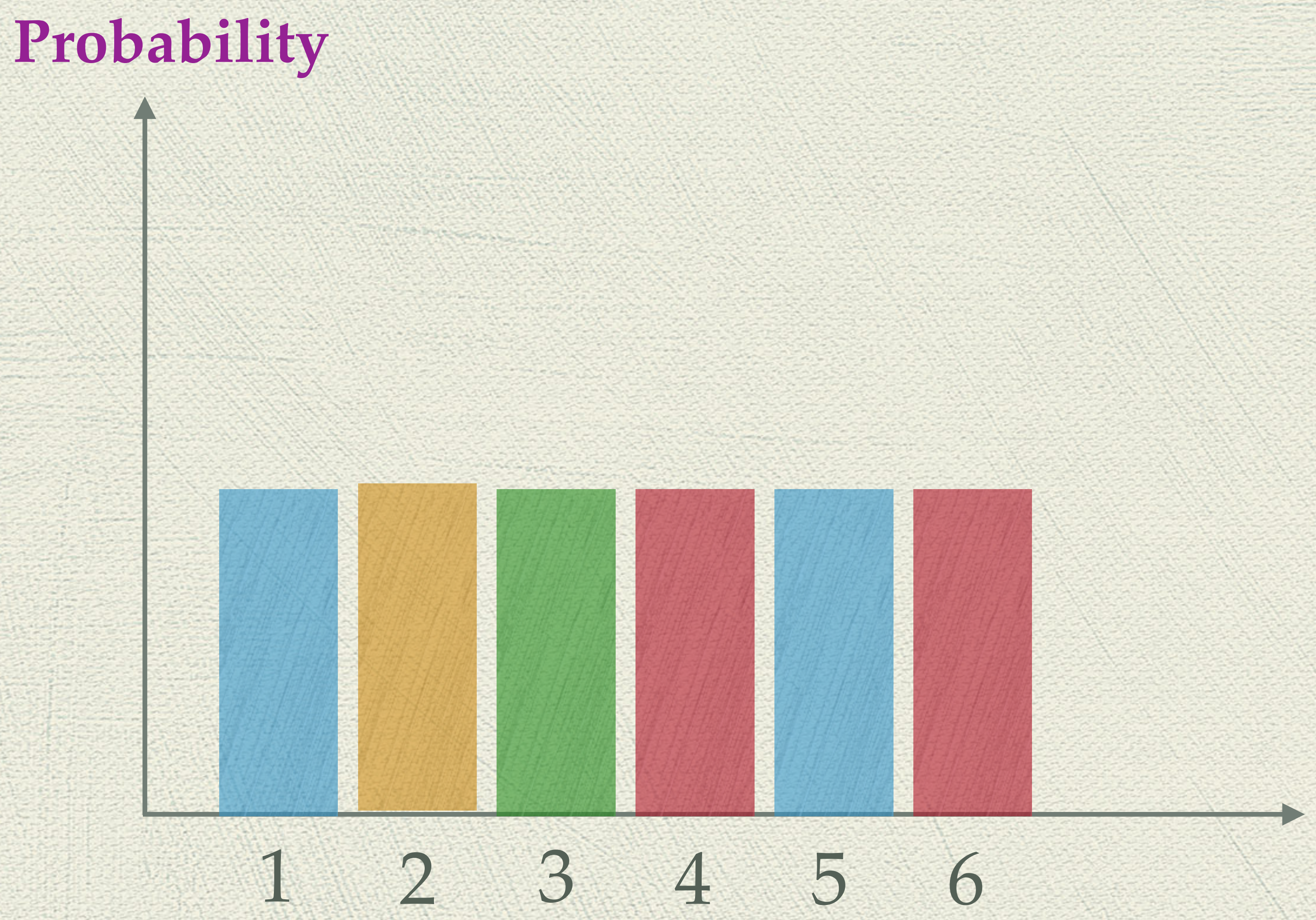
7 Heads

What is the probability that a **p** coin has been given to you?

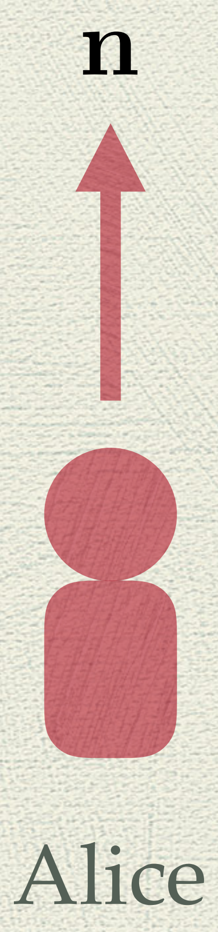
Qubits and Bloch Sphere



The amount of information



Estimating a direction



$$P_{Success}(\mathbf{n}) = \sum_g P(\mathbf{n}_g|\mathbf{n}) \frac{1 + \mathbf{n}_g \cdot \mathbf{n}}{2}$$

$$P_{Success} = \int d\mathbf{n} \sum_g P(\mathbf{n}_g|\mathbf{n}) \frac{1 + \mathbf{n}_g \cdot \mathbf{n}}{2}$$

$$Pr(inadmissible) < (\frac{N}{N+2})^2(\frac{2}{3} + \frac{4}{3N})$$

$$Pr(inadmissible) < (\frac{N}{N+2})^2(\frac{2}{3} + \frac{4}{3N})$$

A rough estimate $$Pr(inadmissible) < \frac{2}{3}$$

$$Pr(inadmissible) < (\frac{N}{N+2})^2(\frac{2}{3} + \frac{4}{3N})$$

A rough estimate $$Pr(inadmissible) < \frac{2}{3}$$

Exact calculation $$Pr(inadmissible) \approx \frac{1}{3}$$

When we have infinite pairs

$$q_N = \frac{1}{N}\sum_i a_i b_i$$

When we have infinite pairs

$$q_N = \frac{1}{N} \sum_i a_i b_i$$

$N \longrightarrow \infty$

$$q_\infty = P_{++} + P_{--} - P_{+-} - P_{-+}$$

When we have infinite pairs

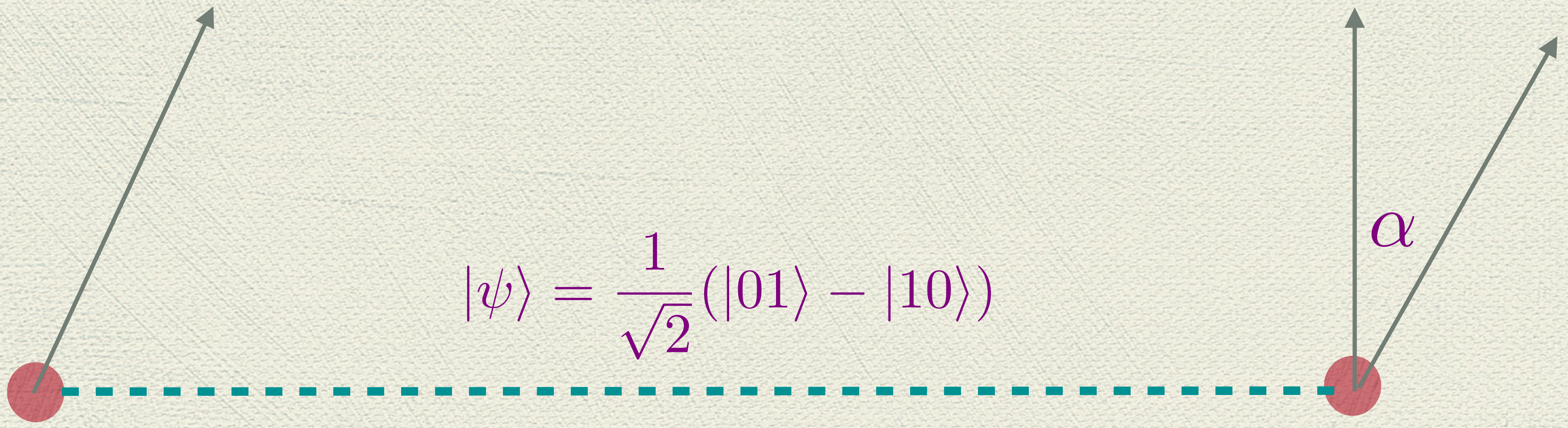
$$q_N = \frac{1}{N} \sum_i a_i b_i$$

$N \longrightarrow \infty$

$$q_\infty = P_{++} + P_{--} - P_{+-} - P_{-+}$$

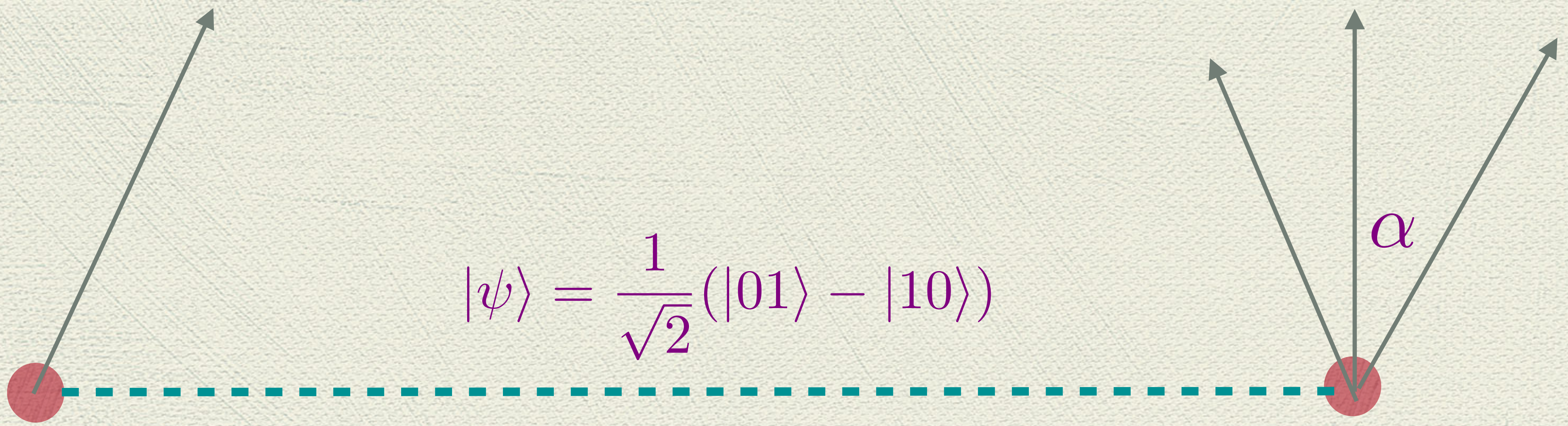
$$q_\infty = \cos \alpha$$

One measurement is not enough!



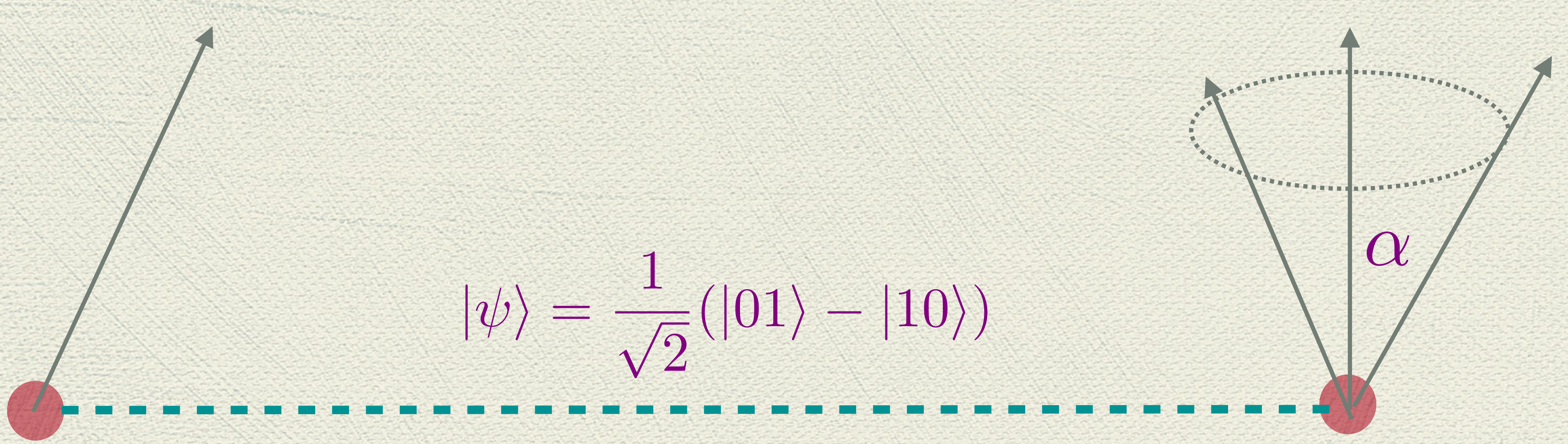
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

One measurement is not enough!

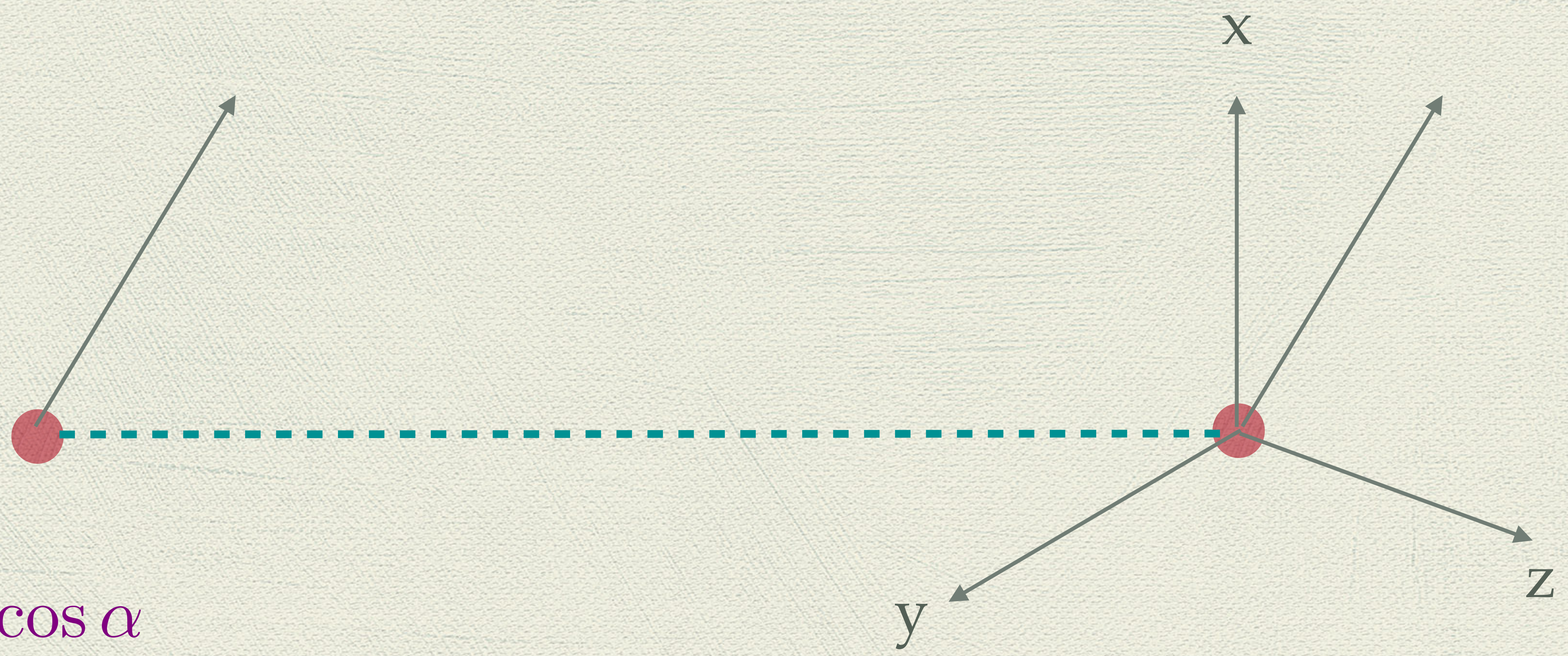


$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

One measurement is not enough!



With three measurements:

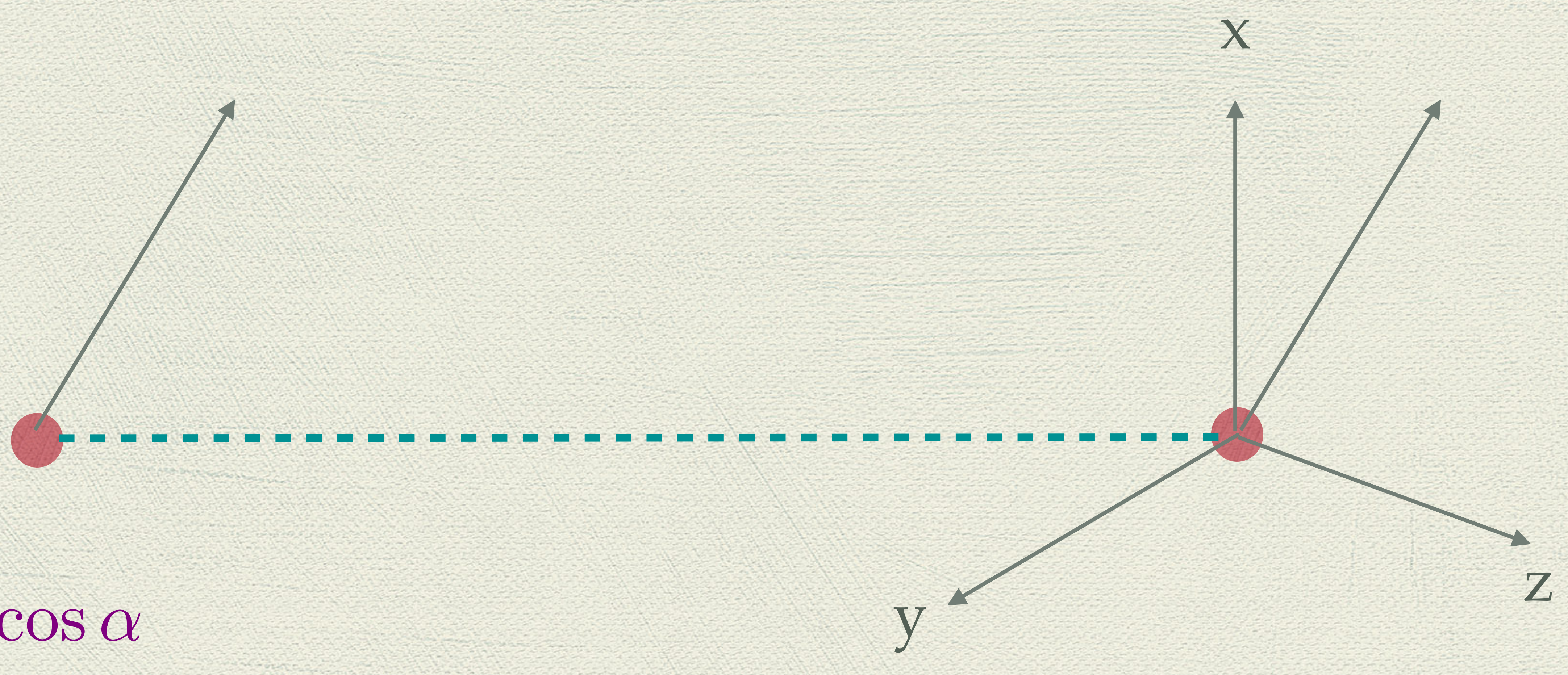


$$q_x = \cos\alpha$$

$$q_y = \cos\beta$$

$$q_z = \cos\gamma$$

With three measurements:



$$q_x = \cos\alpha$$

$$q_y = \cos\beta$$

$$q_z = \cos\gamma$$

$$\mathbf{m} = q_x\ \mathbf{x} + q_y\ \mathbf{y} + q_z\ \mathbf{z}$$

A first estimate

$$\mathbf{m}_e = \cos\alpha_e \;\mathbf{x} + \cos\beta_e \;\mathbf{y} + \cos\gamma_e \;\mathbf{z}$$

A first estimate

$$\mathbf{m}_e = \cos\alpha_e\ \mathbf{x} + \cos\beta_e\ \mathbf{y} + \cos\gamma_e\ \mathbf{z}$$

However the vector is not normalized:

A first estimate

$$\mathbf{m}_e = \cos\alpha_e\ \mathbf{x} + \cos\beta_e\ \mathbf{y} + \cos\gamma_e\ \mathbf{z}$$

However the vector is not normalized:

$$\cos^2\alpha_e + \cos^2\beta_e + \cos^2\gamma_e \neq 1$$

A first estimate

$$\cos\alpha_e = \frac{N}{N+2} q_N$$

$$\mathbf{m}_e = \cos\alpha_e \ \mathbf{x} + \cos\beta_e \ \mathbf{y} + \cos\gamma_e \ \mathbf{z}$$

However the vector is not normalized:

$$\cos^2\alpha_e + \cos^2\beta_e + \cos^2\gamma_e \neq 1$$