Entanglement and Reference frames
Lecture II

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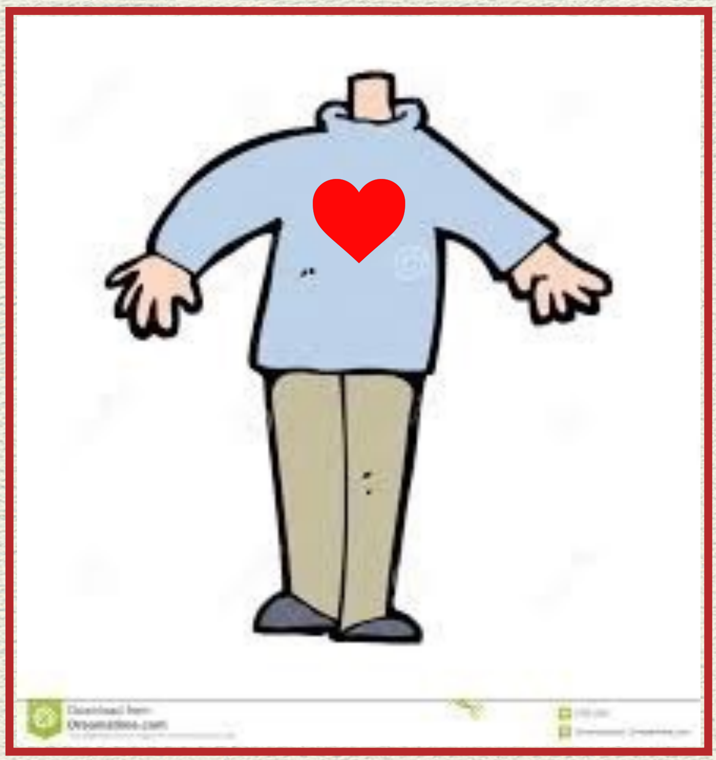
Sharif University of Technology,
Tehran, Iran.

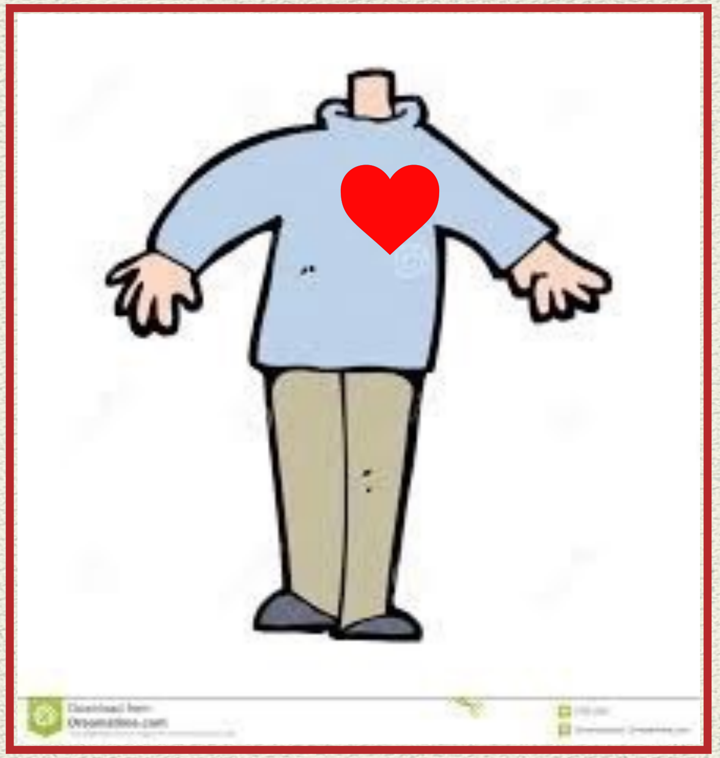
Summary of the first lecture

Unspeakable Information



We cannot communicate the word
LEFT
with any string of bits

0 1 0 1 1 1 0 0 0 1 1 1 0 1 0 1 1 0



We should send physical objects

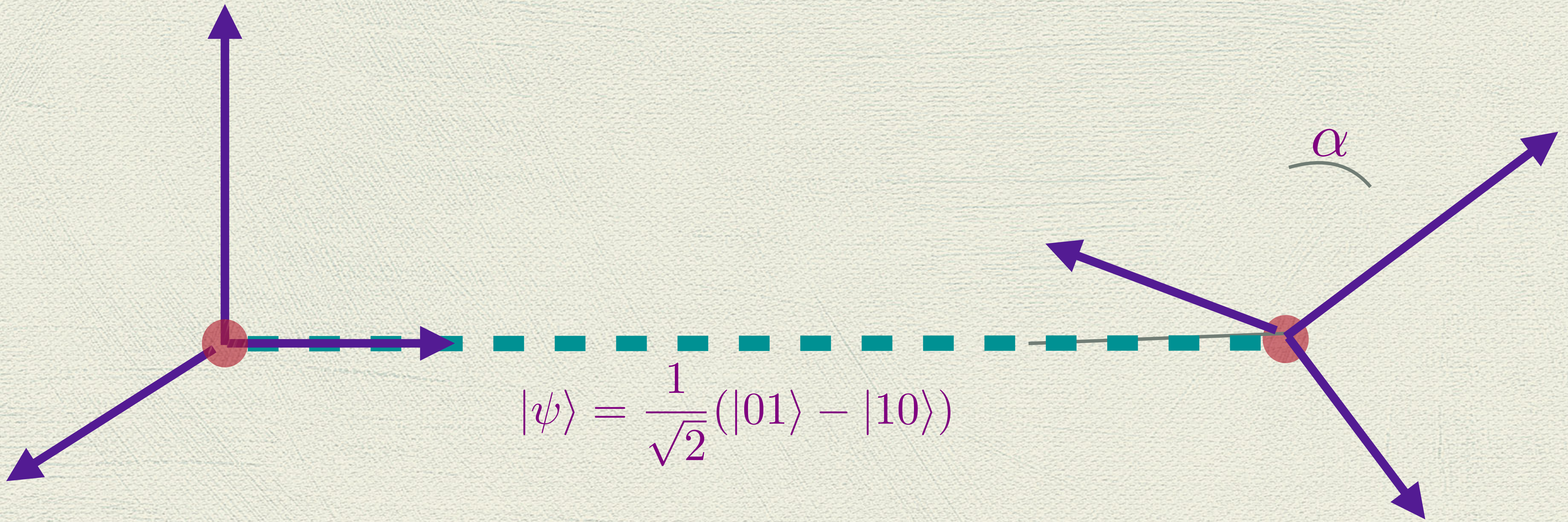
I- Using Entangled States for setting up an SRF

F. Rezazadeh, A. Mani, V. Karimipour,

Secure alignment of coordinate systems using quantum correlations

Phys. Rev. A, 96, 022310 (2017)

The idea of Direction Sharing



By calculating the correlations in their measurement result, Alice and Bob share a **Reference Frame which is Secret**.

II- Task Dependent resource theory

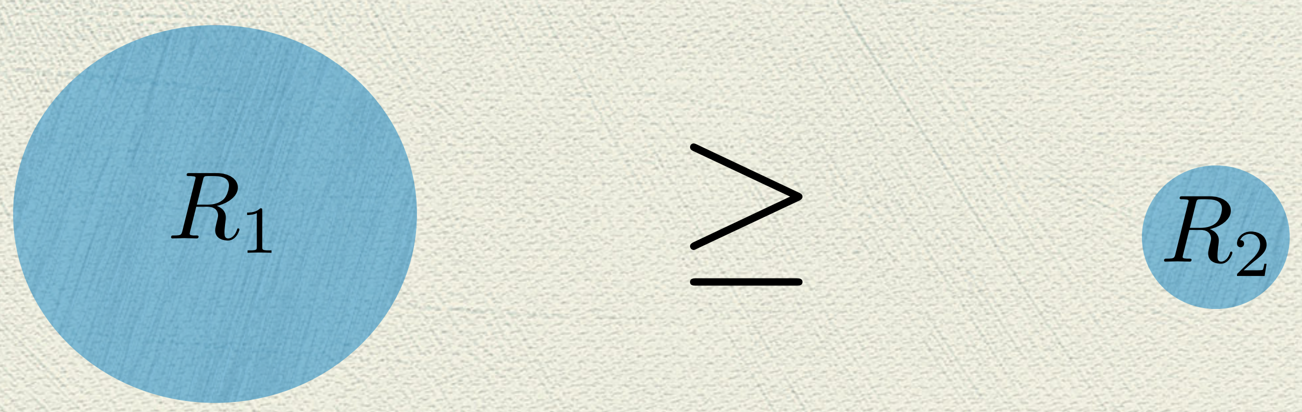
F. Rezazadeh, A. Mani, V. Karimipour,

Power of a shared singlet state in comparison to a shared reference frame

Physical Review A, 100, 022329, 2019


FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
J 00000007 A
WASHINGTON, D.C.
10
TEN
SERIES 1950
WILL PAY TO THE BEARER ON DEMAND
TEN DOLLARS

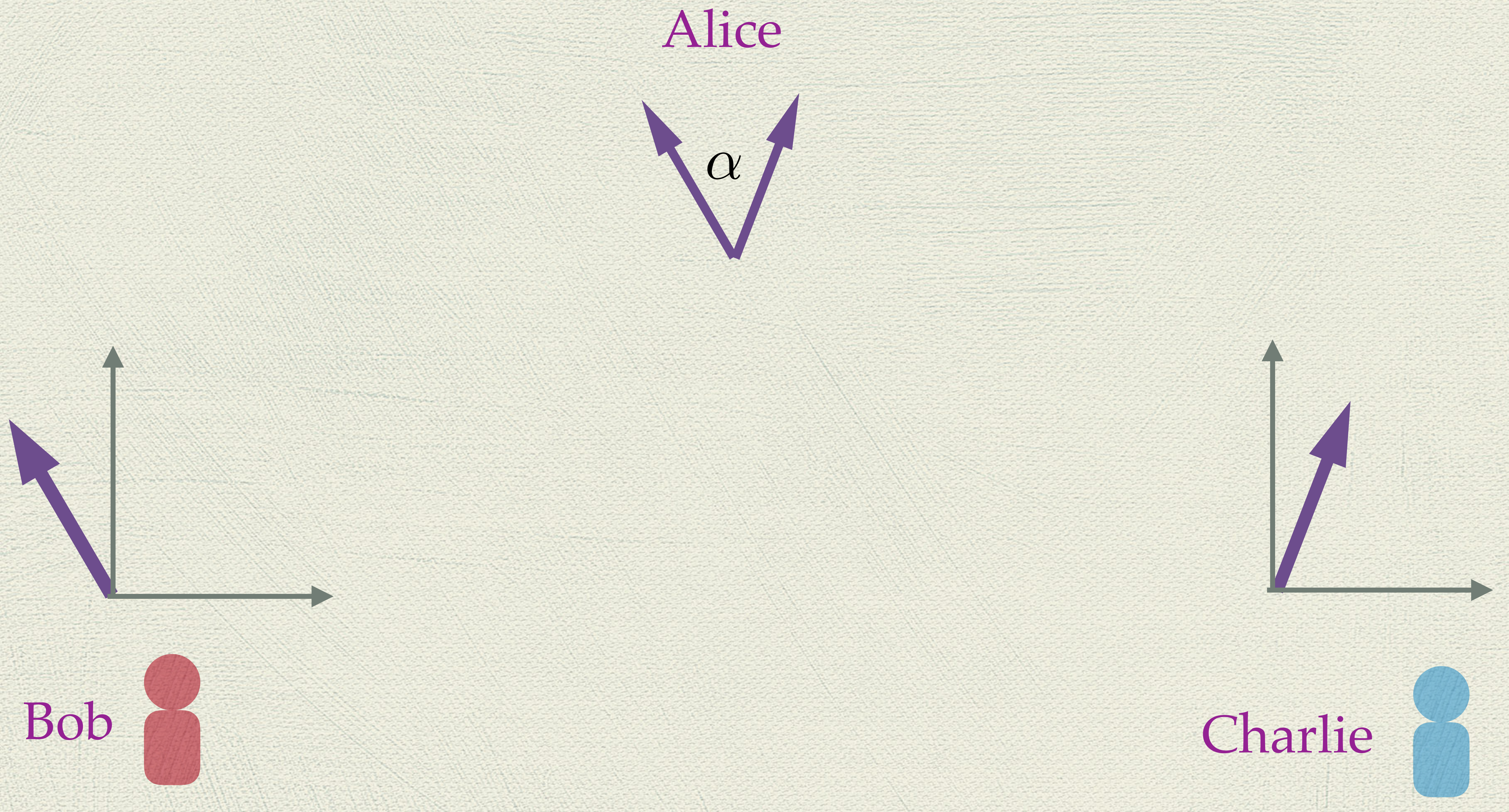
The main idea of resource theory:



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad \geq \qquad a|00\rangle + b|11\rangle$$

$\leq$

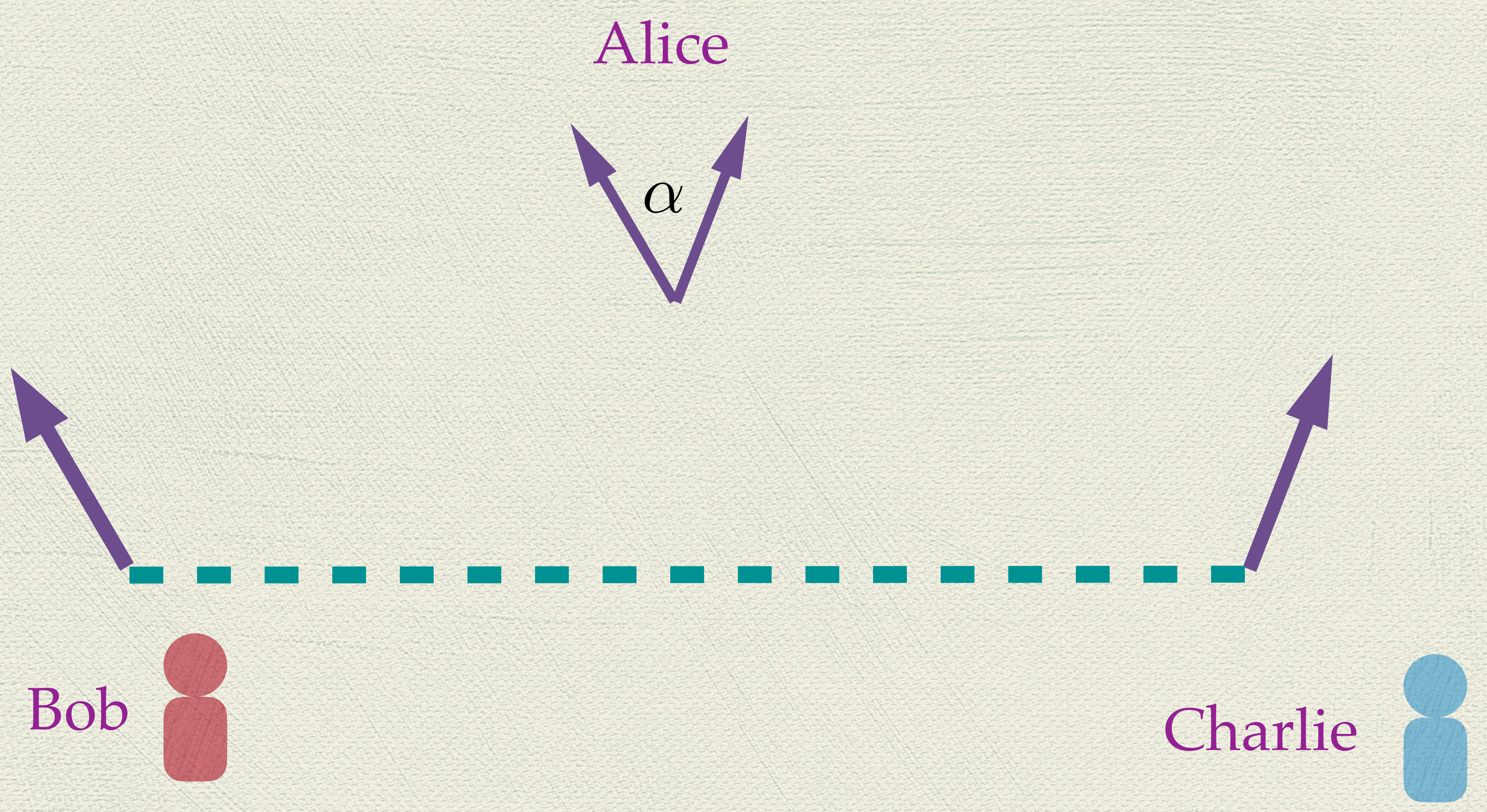
An example: Estimation of an angle



Information Gain= 0.0270

S. D. Bartlett, T. Rudolph and R. W. Spekkens, PRA (2004).

An example: Estimation of an angle



Information Gain= 0.0284

Lecture 2

Bartlett, Rudolph and Spekkens, Rev. of Mod. Phys., 2007

III- Doing QKD in the absence of shared reference frames

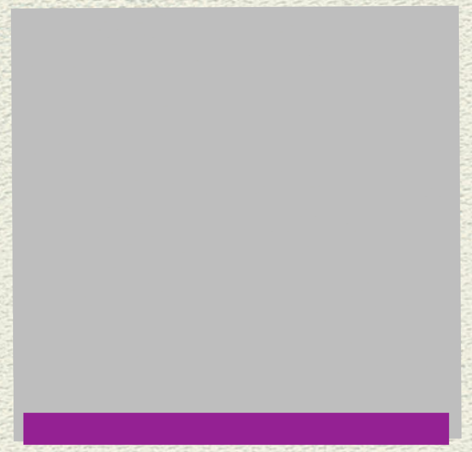
F. Rezazadeh, A. Mani, V. Karimipour,

Quantum Key Distribution, A simple reference frame independent method

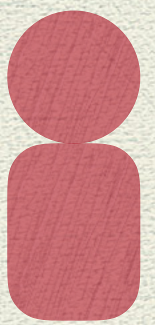
arXiv:1903.08279

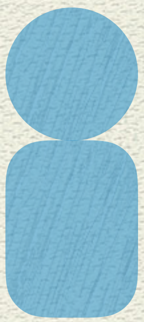
Bob measures exactly the same state that Alice has prepared.

Bob measures exactly the same state that Alice has prepared.



$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



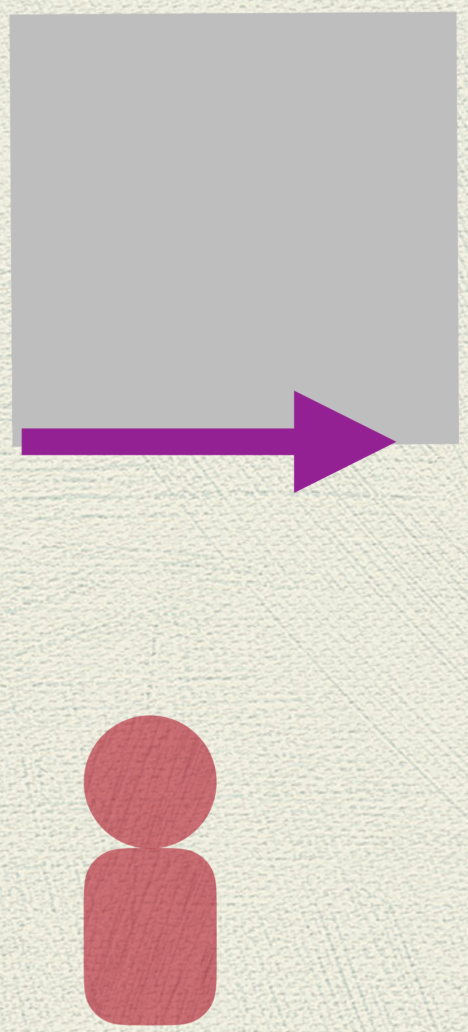


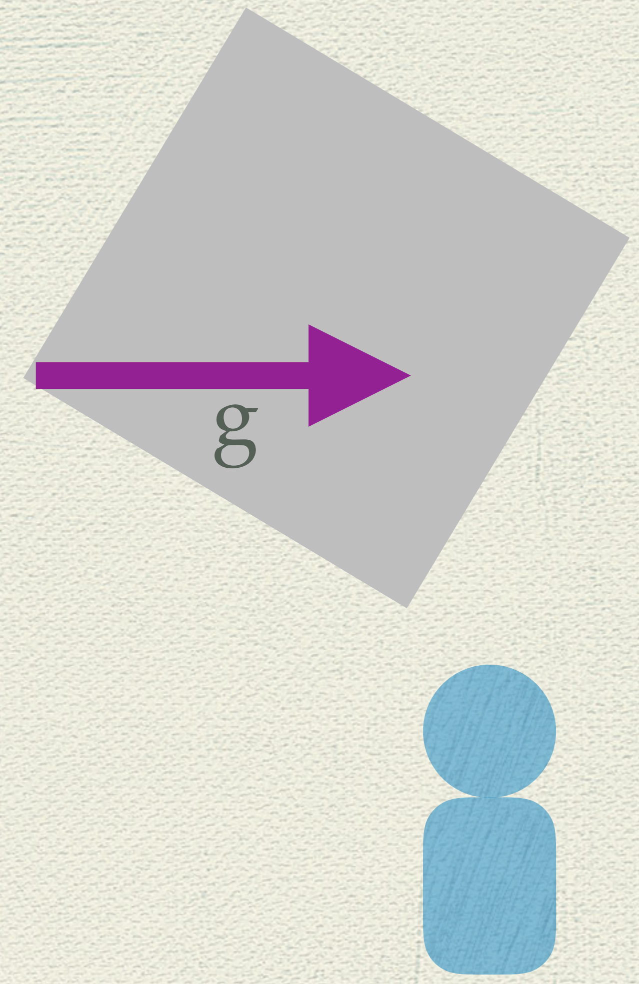
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Half of the time Bob receives the same bit as Alice has prepared.

Public Announcement of the bases

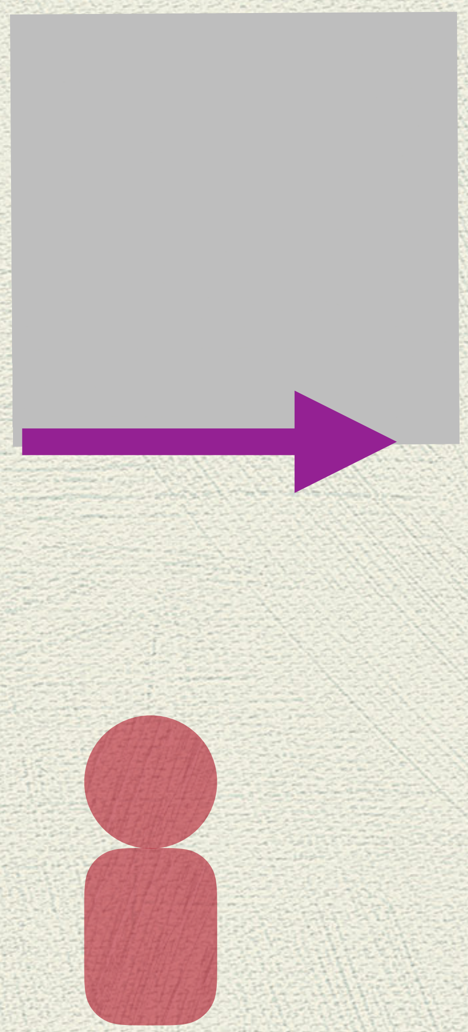
How do you do QKD
When you do not know your coordinate system?

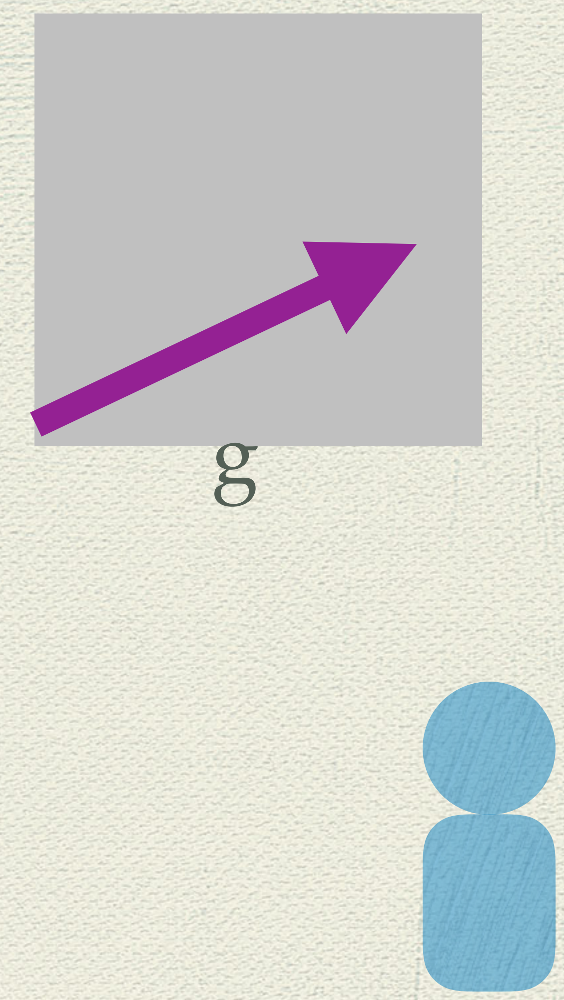




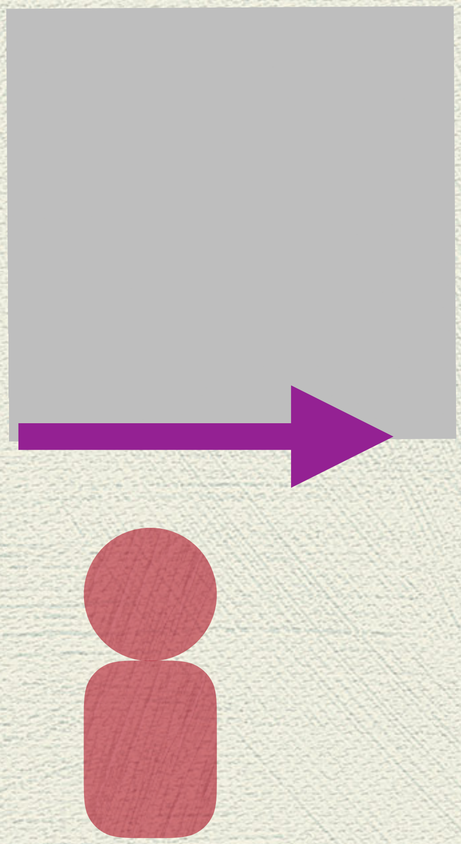
$$\rho_B = U(g)\rho_A U^\dagger(g)$$

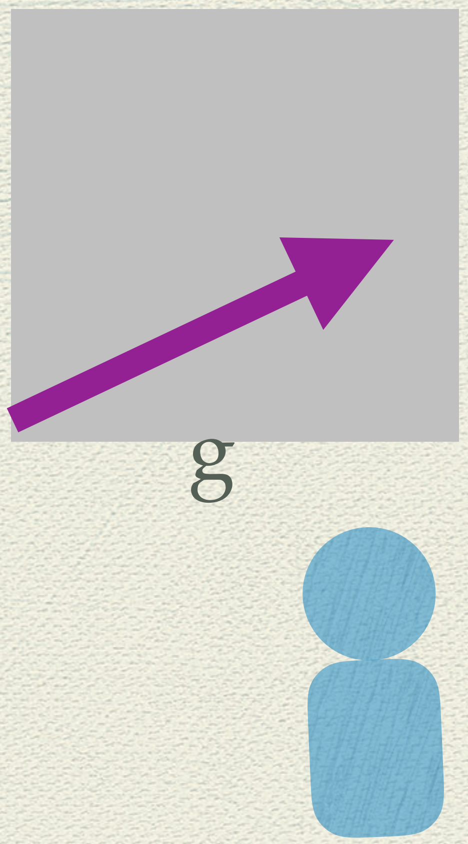
How do you do QKD
When you do not know your coordinate system?





$$\rho_B = U(g)\rho_A U^\dagger(g)$$

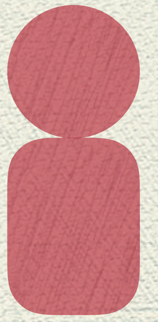




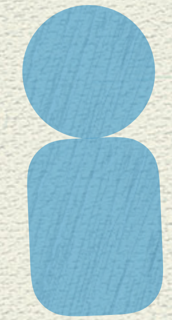
$$\rho_B = \int dg U(g) \rho_A U^\dagger(g)$$

$$\rho_B U(h) = U(h) \rho_B$$

With one particle we cannot send any information.



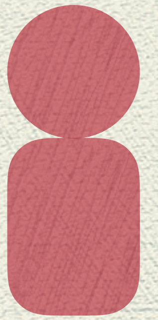
$$\rho_B U(h) = U(h)\rho_B$$

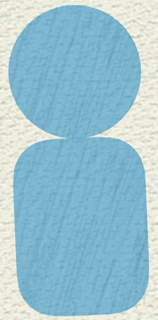


Shur's Lemma

↓

With one particle we cannot send any information.





$$\rho_B U(h) = U(h) \rho_B$$

Shur's Lemma

$\downarrow$

$$\rho_B \propto I$$

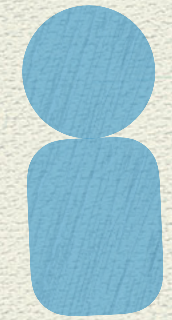
With one particle we cannot send any information.

$$\rho_B U(h) = U(h)\rho_B$$

Shur's Lemma

↓

$$\rho_B \propto I$$



With one particle we cannot send any information.

$$\rho_B U(h) = U(h) \rho_B$$

Shur's Lemma

$\downarrow$

$$\rho_B \propto I$$

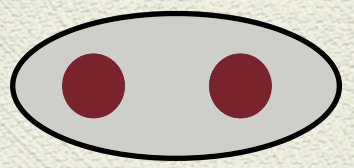
The z-component is different.

But the total spin is the same.

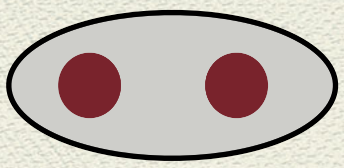
Toward QKD in the absence of SRF

$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

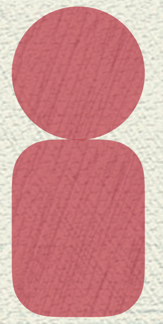
$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$

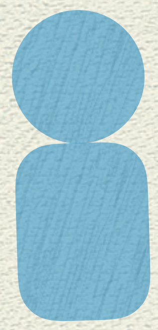


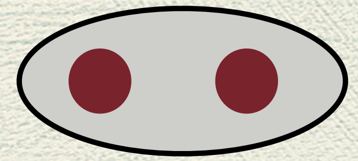
$|0\rangle$



$|1\rangle$

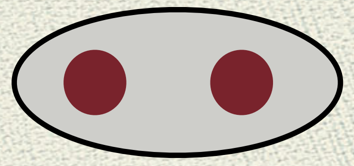




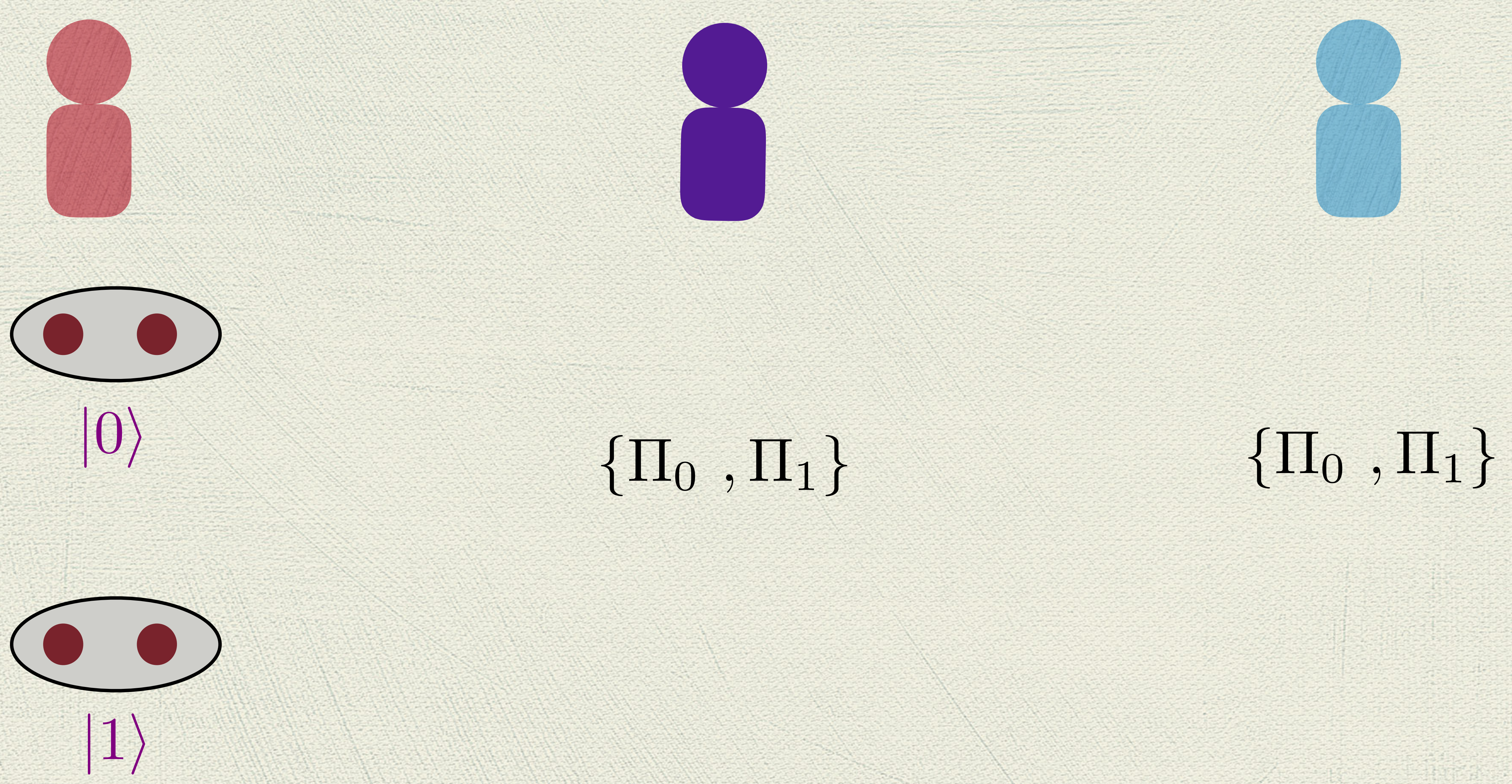


$|0\rangle$

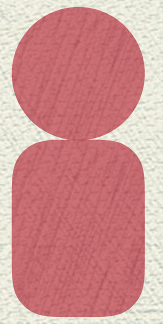
$$\text{POVM} = \{\Pi_0\ , \Pi_1\}$$

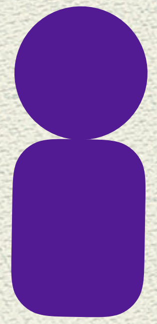


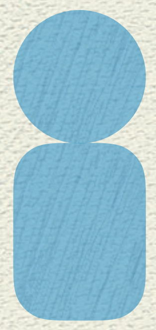
$|1\rangle$

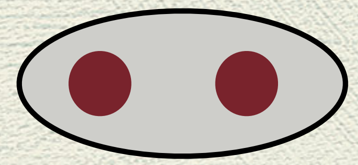

$|0\rangle$
$|1\rangle$
$\{\Pi_0\ ,\Pi_1\}$
$\{\Pi_0\ ,\Pi_1\}$

But we need two different bases!

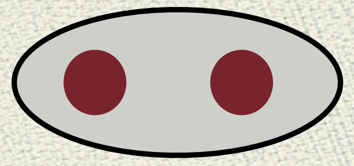








$|0\rangle$



$|1\rangle$

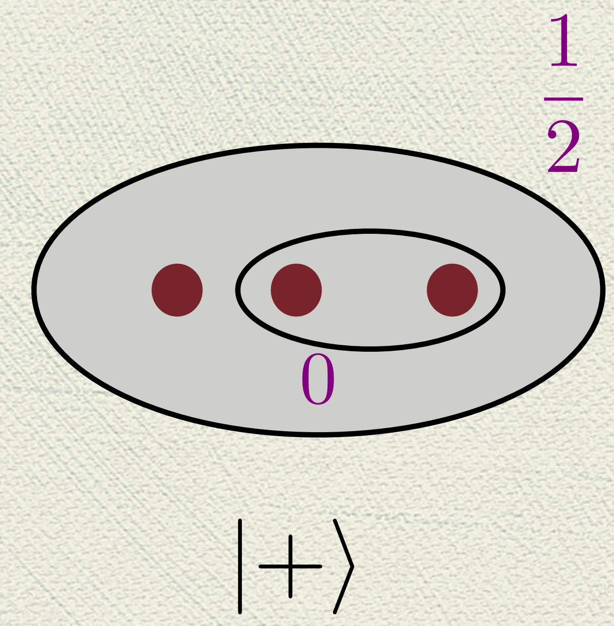
$\{\Pi_0\ ,\Pi_1\}$

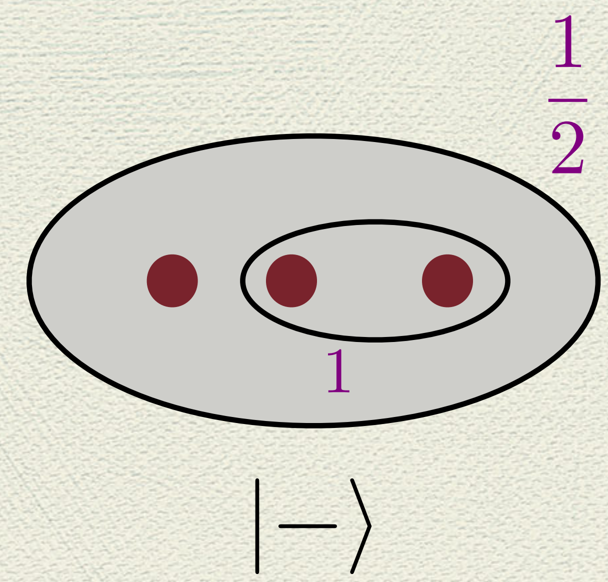
$\{\Pi_0\ ,\Pi_1\}$

How does Alice prepare her states?

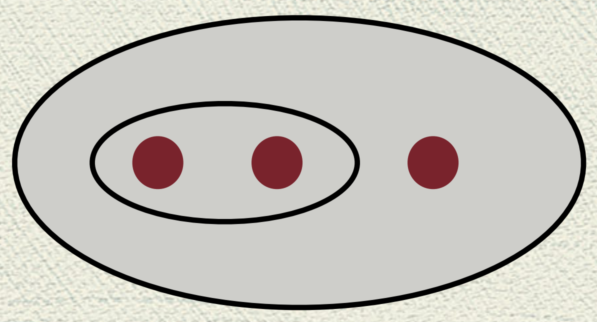
$$\left(\tfrac{1}{2}\times\tfrac{1}{2}\right)\times\tfrac{1}{2}=(0+1)\times\tfrac{1}{2}=\frac{1}{2}+\frac{1}{2}'+\frac{3}{2}$$

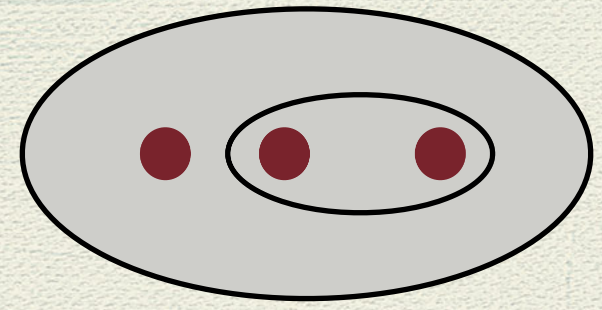
But they need two different bases.





How does Bob measure his states?





$\Pi_0 \otimes I$

$\Pi_1 \otimes I$

$I \otimes \Pi_0$

$I \otimes \Pi_1$

The two bases are related as follows.

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$$

Running the protocol

Exactly as in BB84

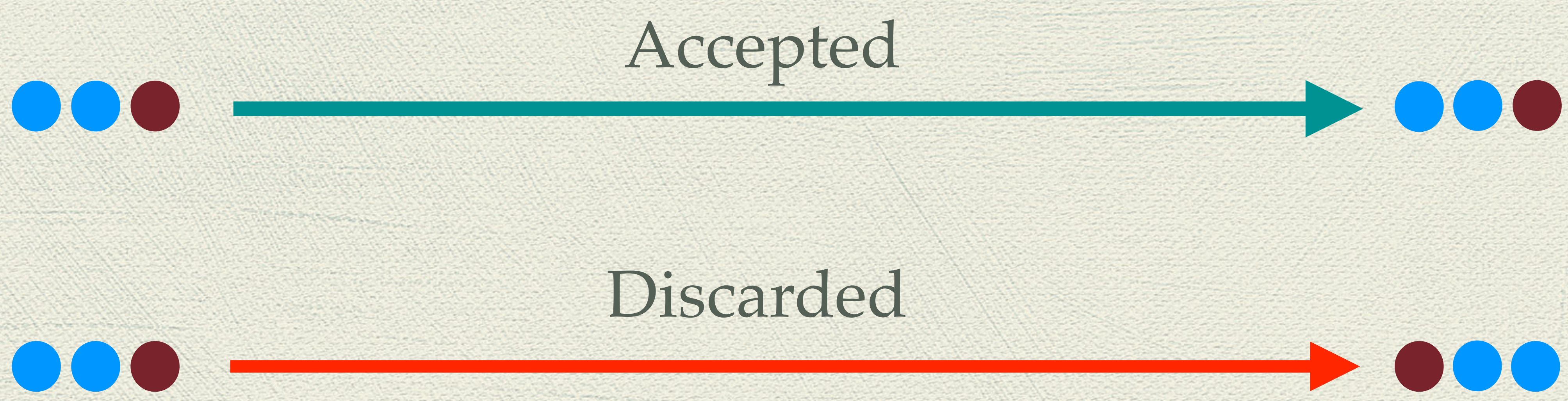
Alice and Bob only announce the pairs and not their values.

Running the protocol

Exactly as in BB84

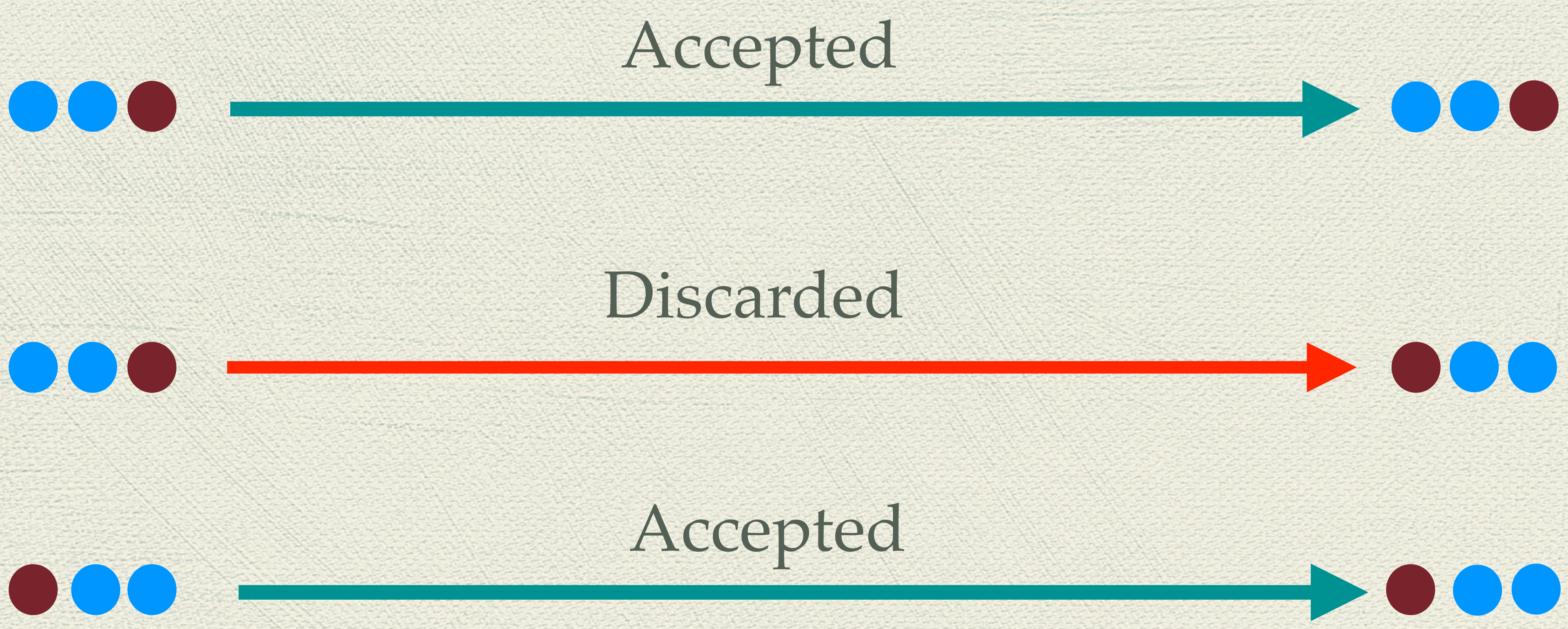
Running the protocol

Exactly as in BB84



Running the protocol

Exactly as in BB84



Running the protocol

Exactly as in BB84

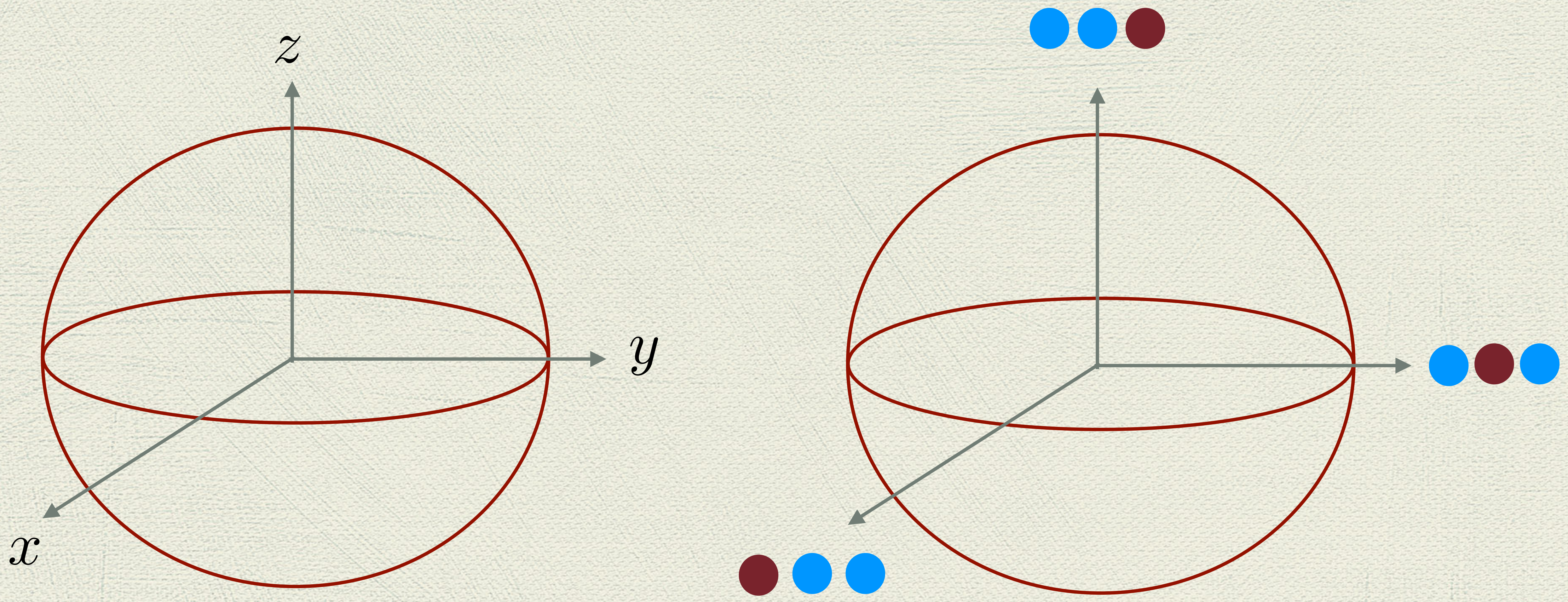
$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$$

Intervention of Eve leads to her detection with a probability of

$$\frac{3}{16}$$

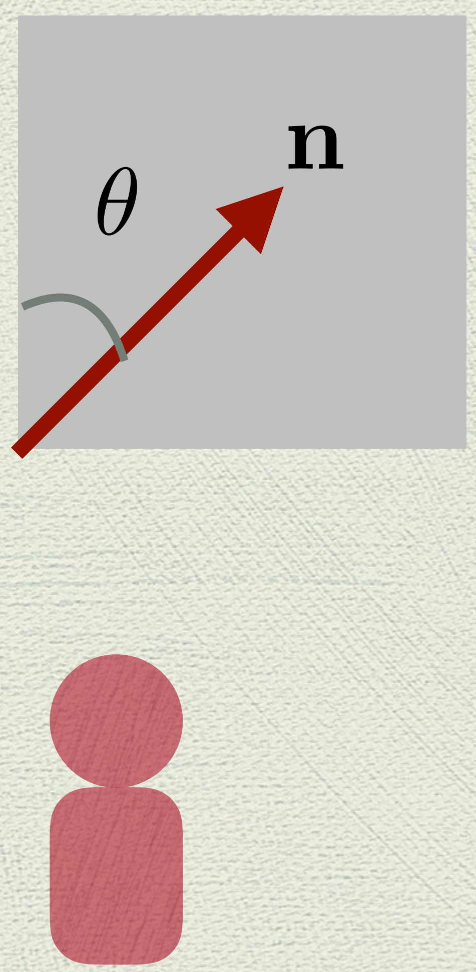
Which is only slightly lower than 1/4 for the BB84 protocol.

The six state protocol

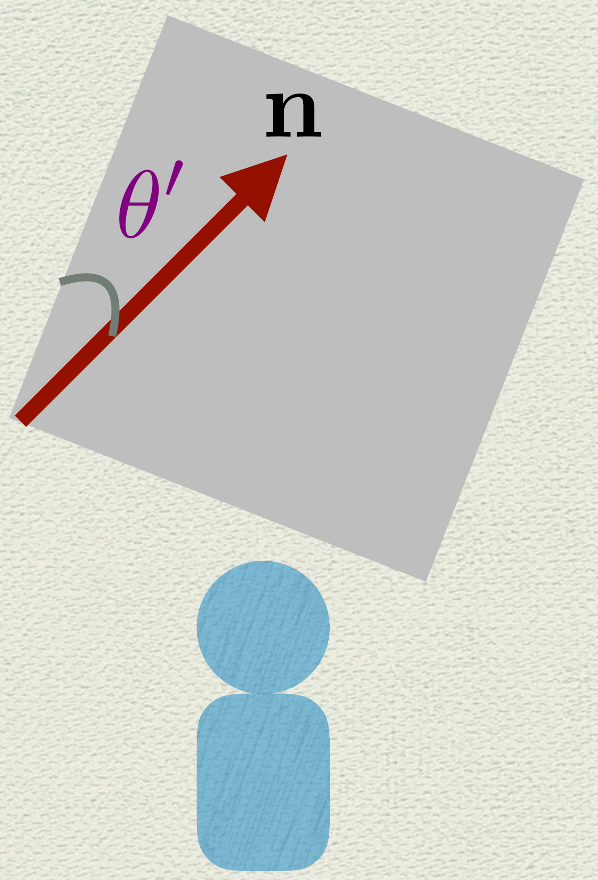


IV- Sending Relative information with Entangled States

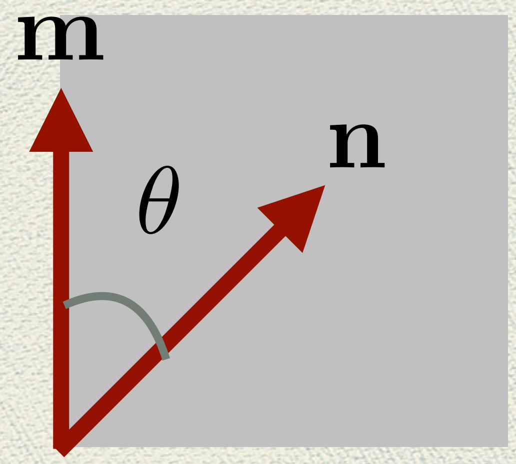
A Beheshti, S. Raeisi and V.Karimipour, Phys. Rev. A 99, 042330 (2019)

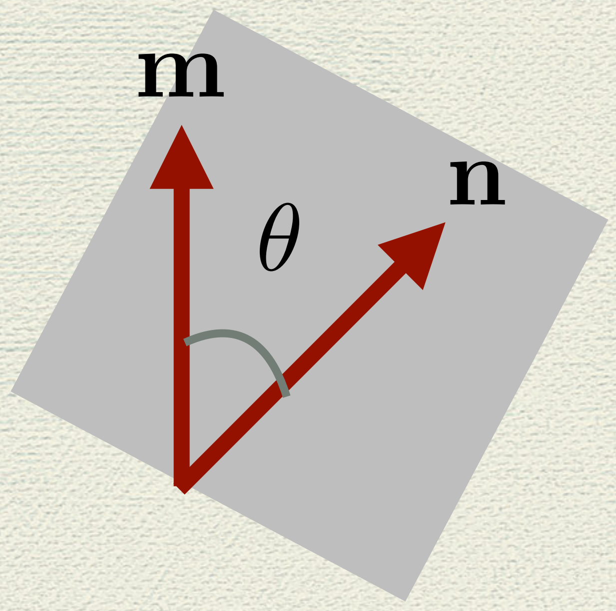


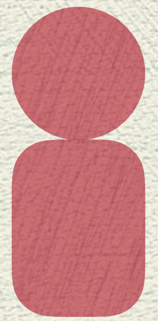
θ
$\mathbf{n}$



θ'
$\mathbf{n}$



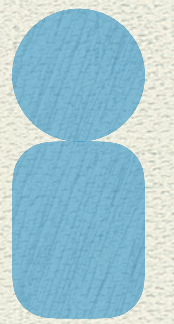




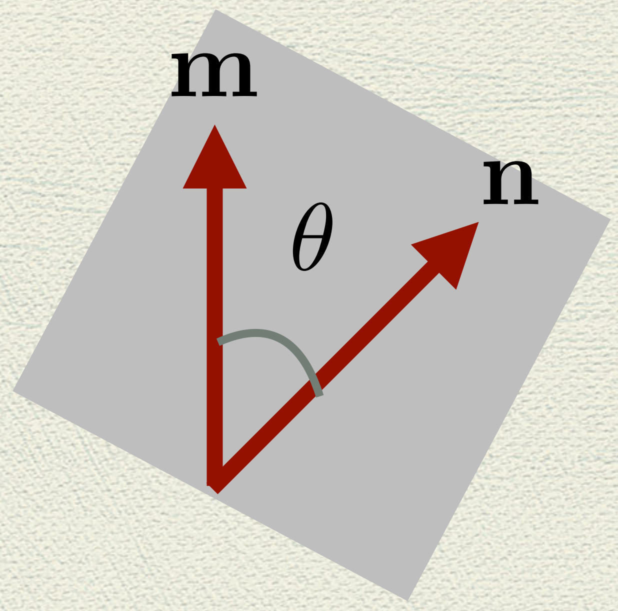
Relative parameters

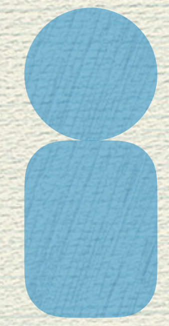
and

Global Parameters



Bartlett, Rudolph, and Spekkens Phys. Rev. A (2004)





Optimal Measurements by Bob

$$E_0 = \Pi_0 \qquad E_1 = \Pi_1$$

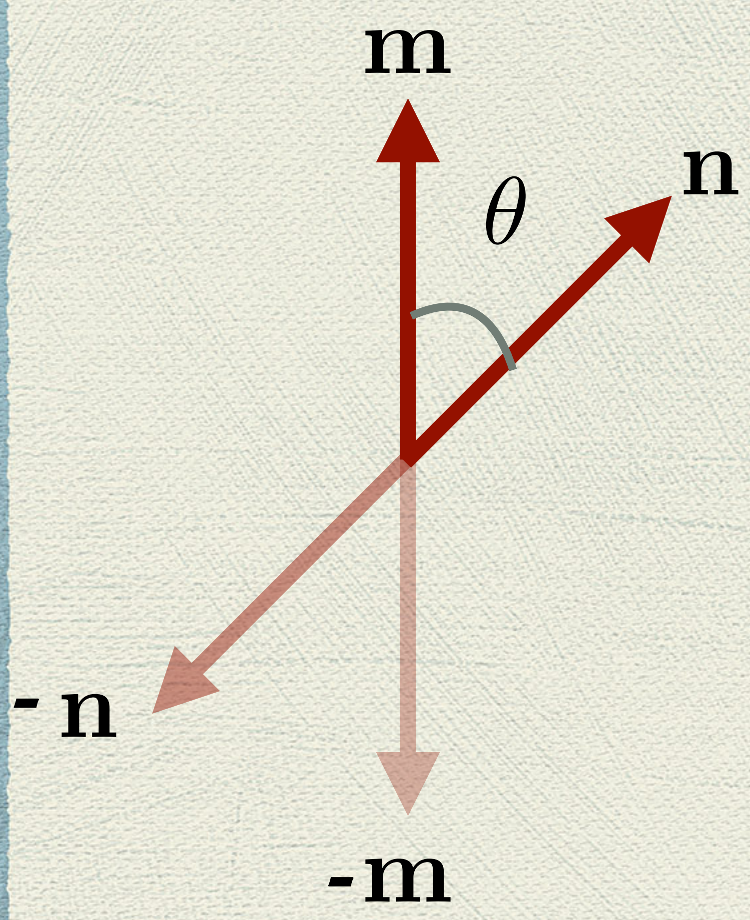
Total Spin Measurement

What if Alice sends entangled states to Bob?

No. of Parameters	
7	$$\|\psi\rangle = a\|0,0\rangle + b\|0,1\rangle + c\|1,0\rangle + d\|1,1\rangle$$
4	$$(U \otimes U) : \|\psi\rangle \longrightarrow \|\psi'\rangle$$

$$3 \quad Relative \quad Parameters$$

The relative parameters of an entangled state



$$|\Psi\rangle = \cos\alpha|\mathbf{m},\mathbf{n}\rangle + e^{i\psi}\sin\alpha|-\mathbf{m},-\mathbf{n}\rangle$$

$$(\theta \quad \alpha \quad \psi)$$

A Beheshti, S. Raeisi and V.Karimipour, Phys. Rev. A 99, 042330 (2019)

$$|\Psi\rangle = \cos\alpha|\mathbf{m},\mathbf{n}\rangle + e^{i\psi}\sin\alpha|-\mathbf{m},-\mathbf{n}\rangle$$

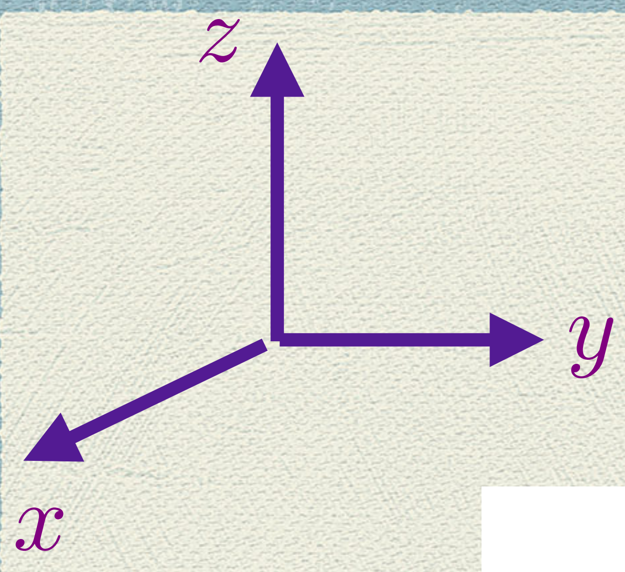
Question 1: Which of the three parameters

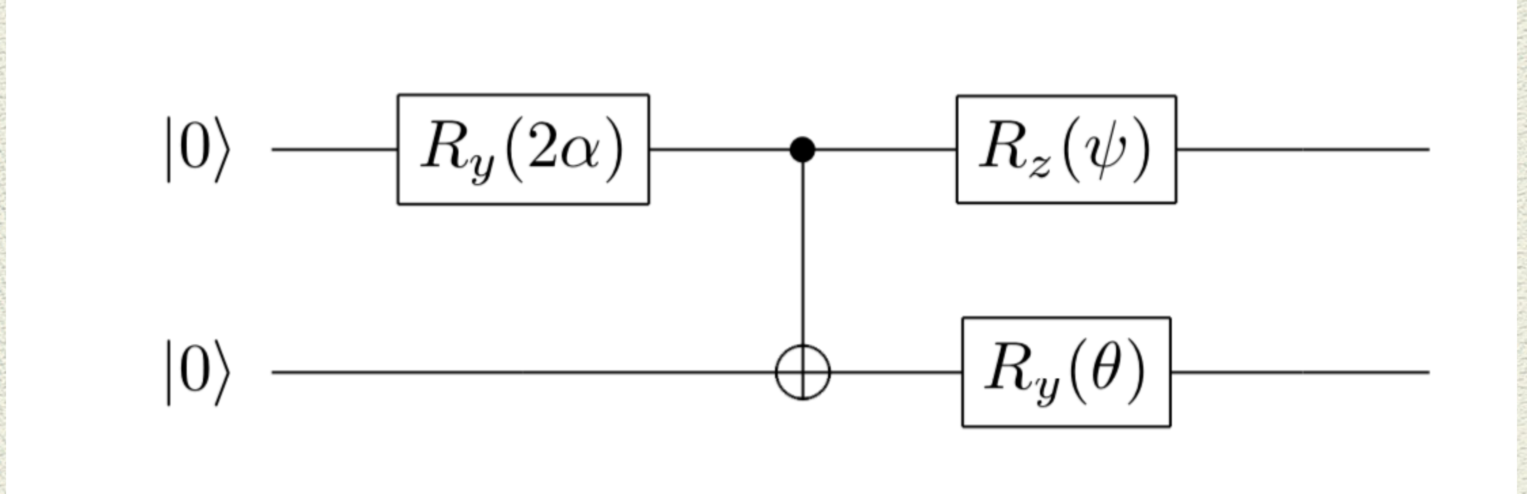
$$\alpha\ ,\ \theta\ ,\ \psi$$

carries more information?

$$|\Psi\rangle = \cos\alpha|\mathbf{m},\mathbf{n}\rangle + e^{i\psi}\sin\alpha| - \mathbf{m}, -\mathbf{n}\rangle$$

Question 2: At what value should we fix the other two parameters?

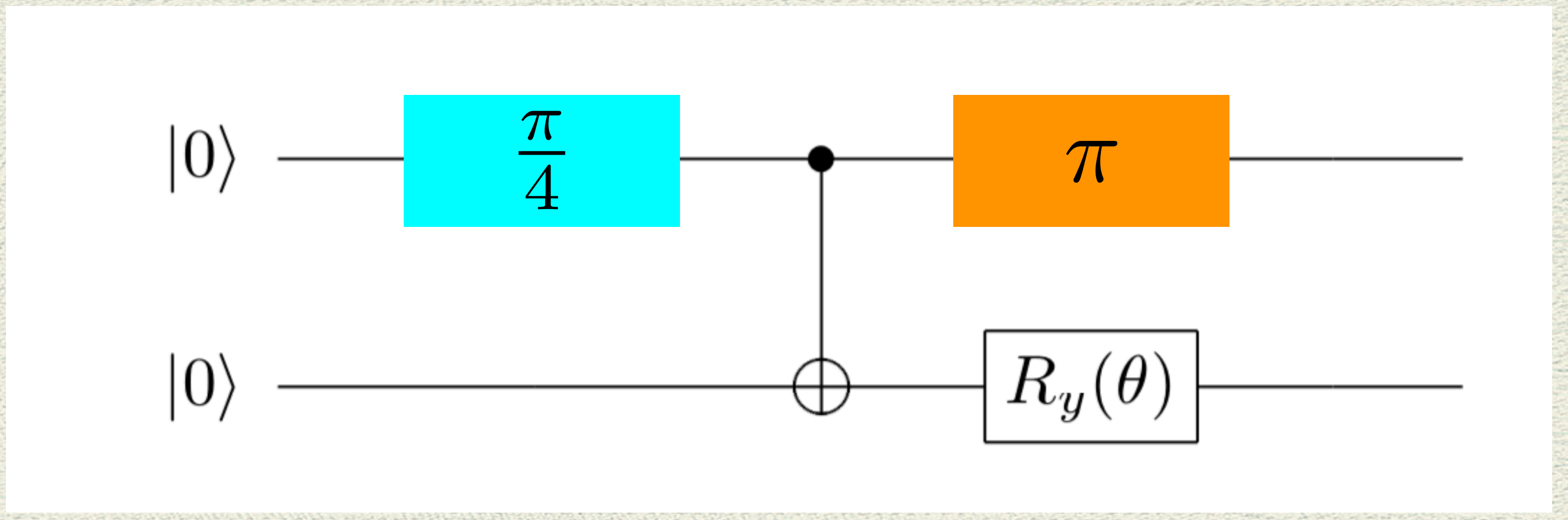


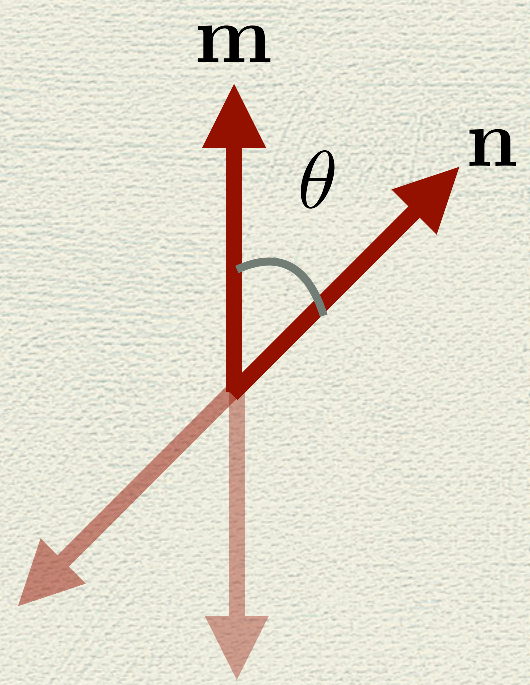


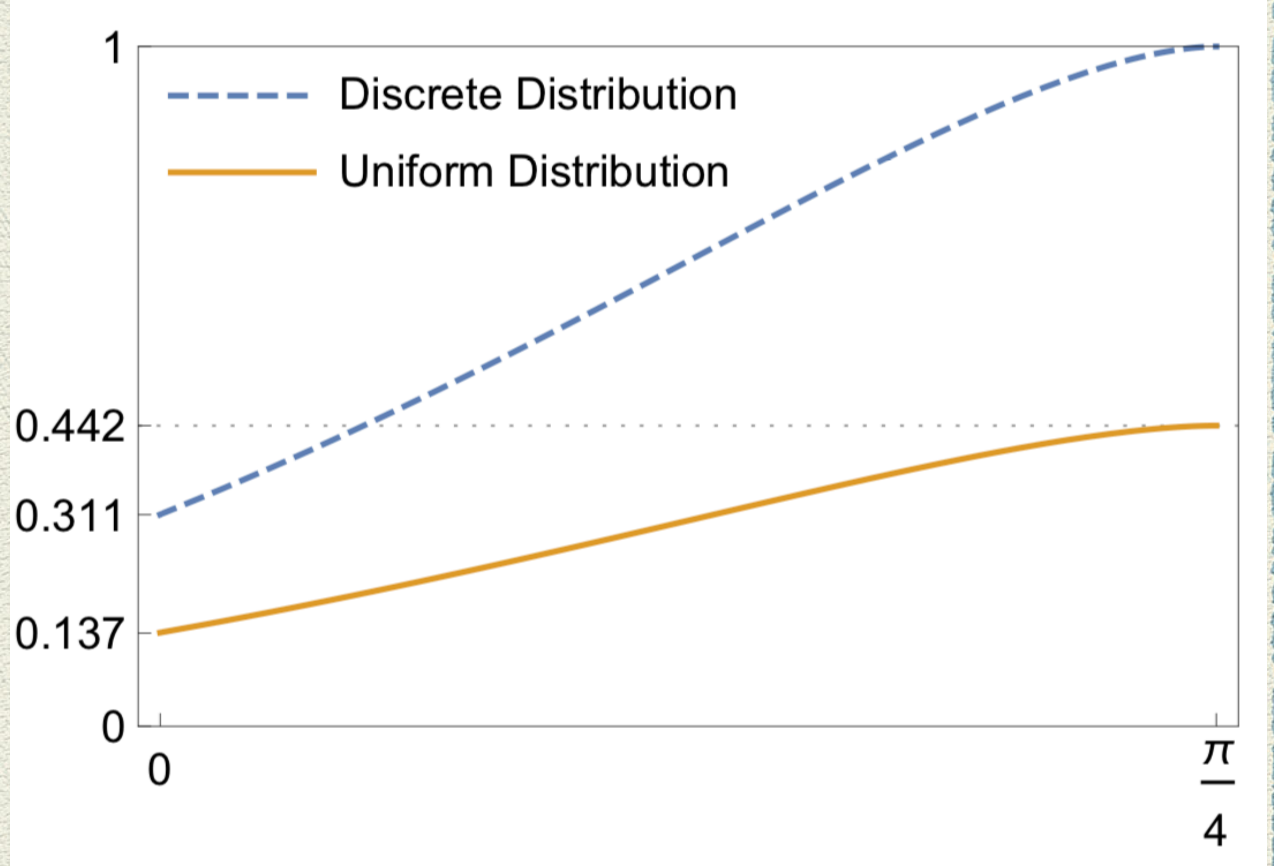
Question 3: How Alice tunes her relative parameters?

$$|\Psi\rangle = \cos\alpha|\mathbf{m},\mathbf{n}\rangle + e^{i\psi}\sin\alpha|-\mathbf{m},-\mathbf{n}\rangle$$

Average Information Gain in θ



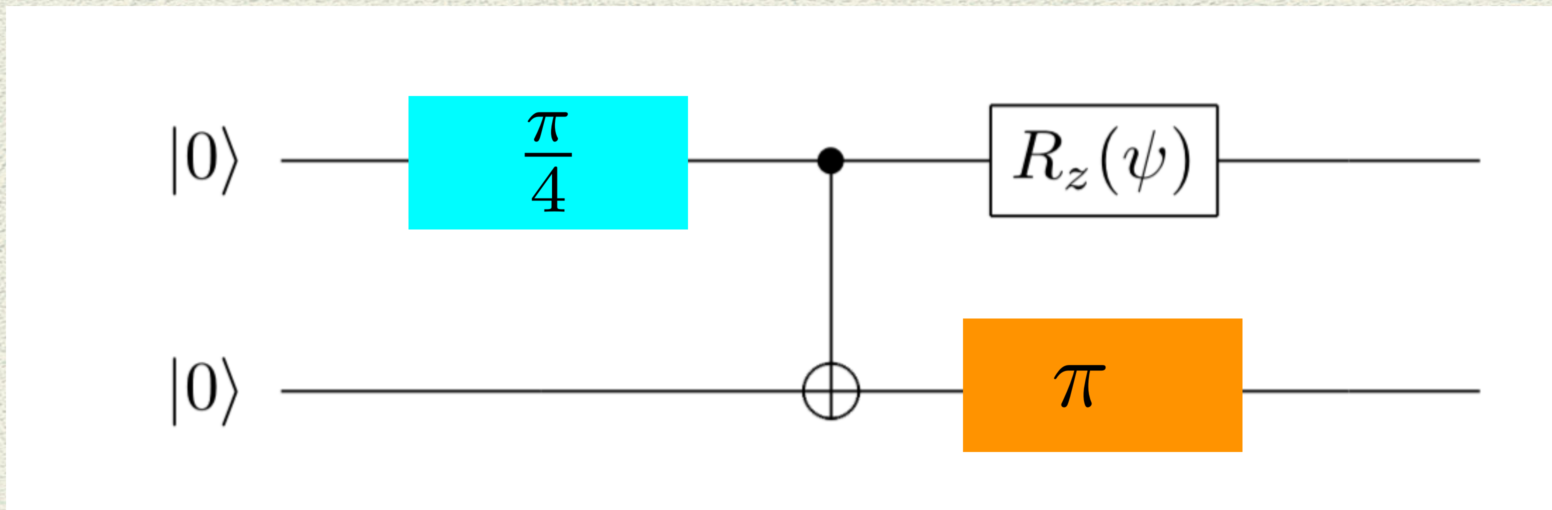


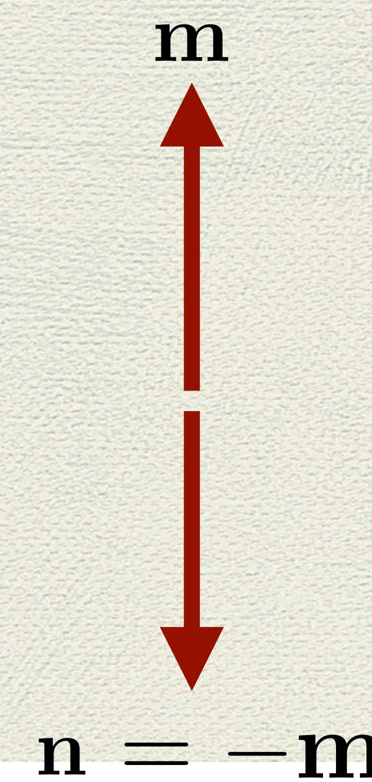


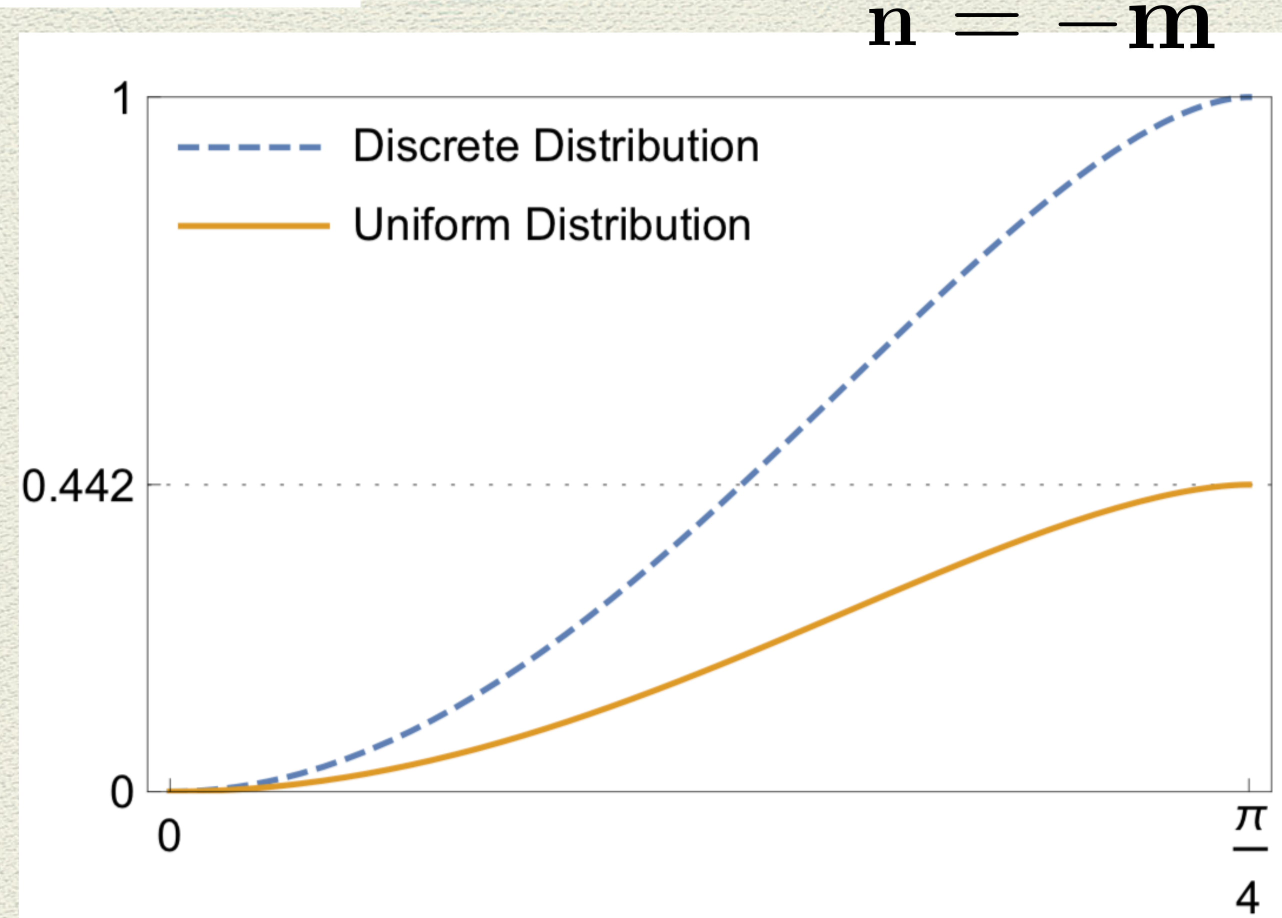
α_0

$$|\psi\rangle = \cos\alpha_0|\mathbf{m},\mathbf{n}\rangle + \sin\alpha_0|-\mathbf{m},-\mathbf{n}\rangle)$$

Average Information Gain in ψ

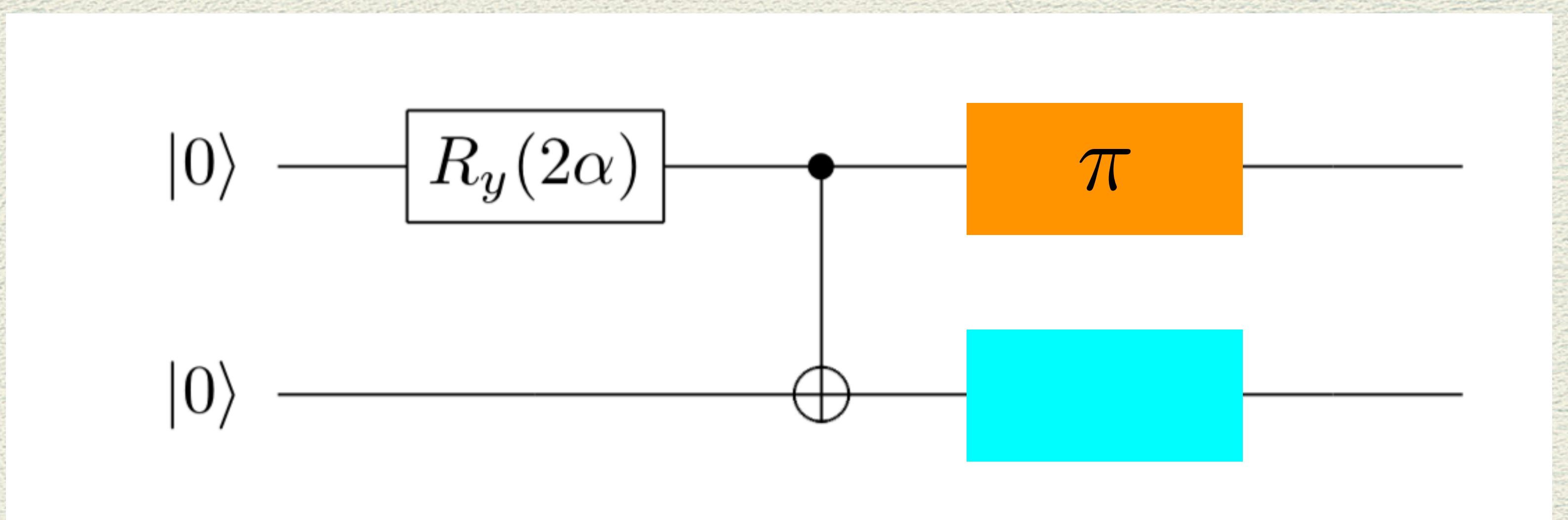


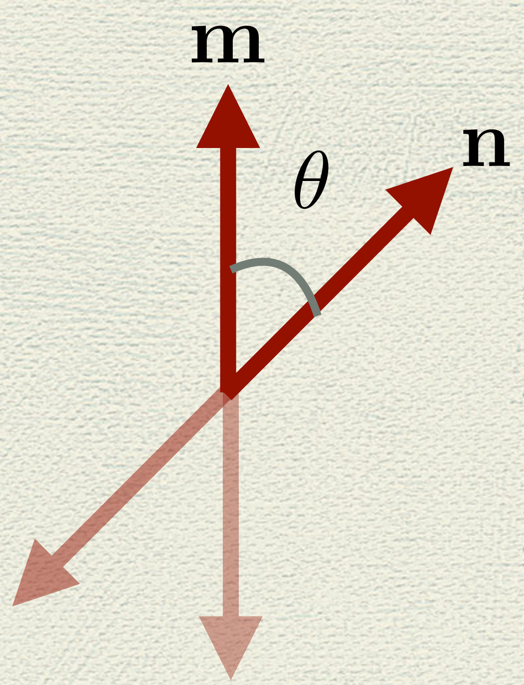


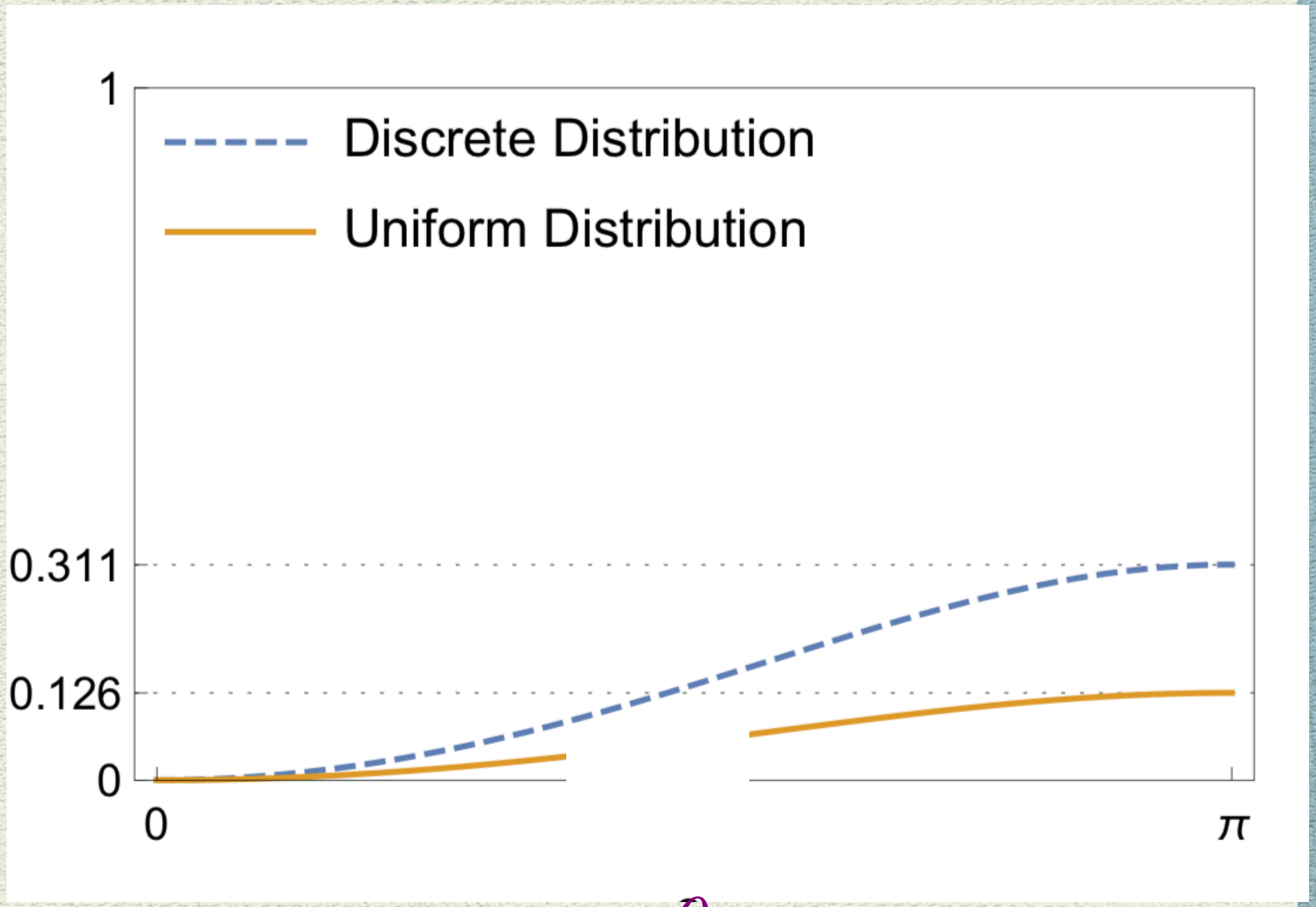


$$|\psi\rangle = \cos\alpha_0|\mathbf{m}, -\mathbf{m}\rangle + \sin\alpha_0 e^{I\psi}| - \mathbf{m}, \mathbf{m}\rangle)$$

Average Information Gain in α





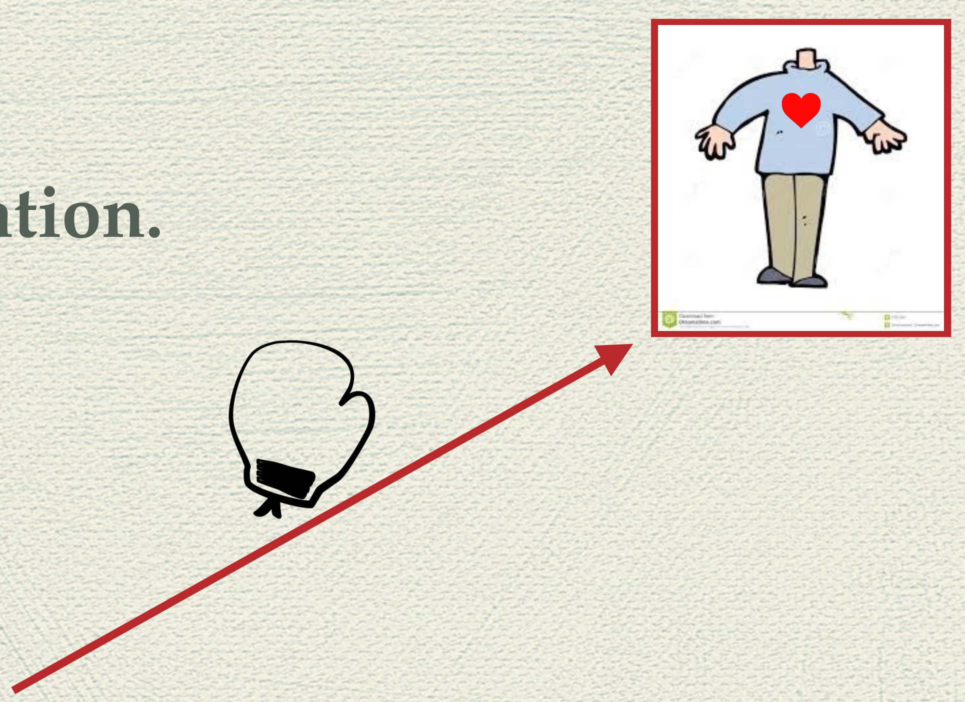


θ_0

$$|\psi\rangle = \cos\alpha|\mathbf{m},\mathbf{n}\rangle + \sin\alpha|-\mathbf{m},-\mathbf{n}\rangle)$$

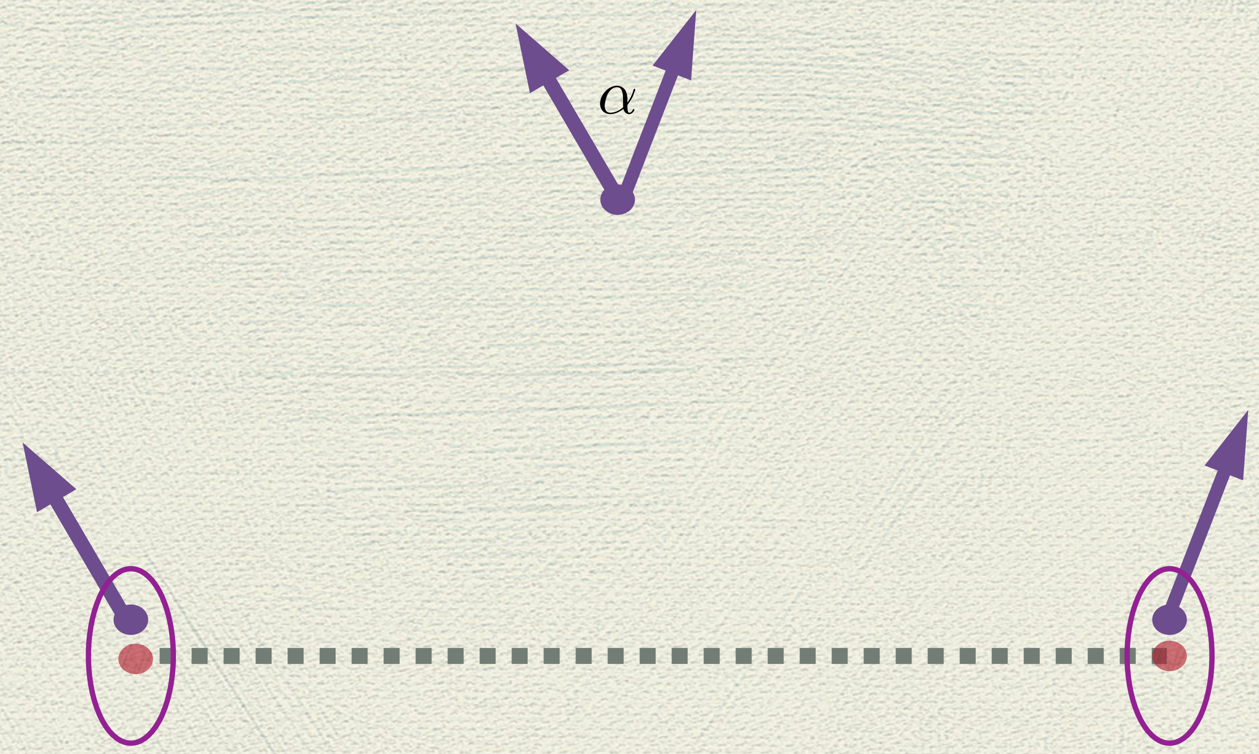
Summary of the two lectures:

1- There are unspeakable types of information.



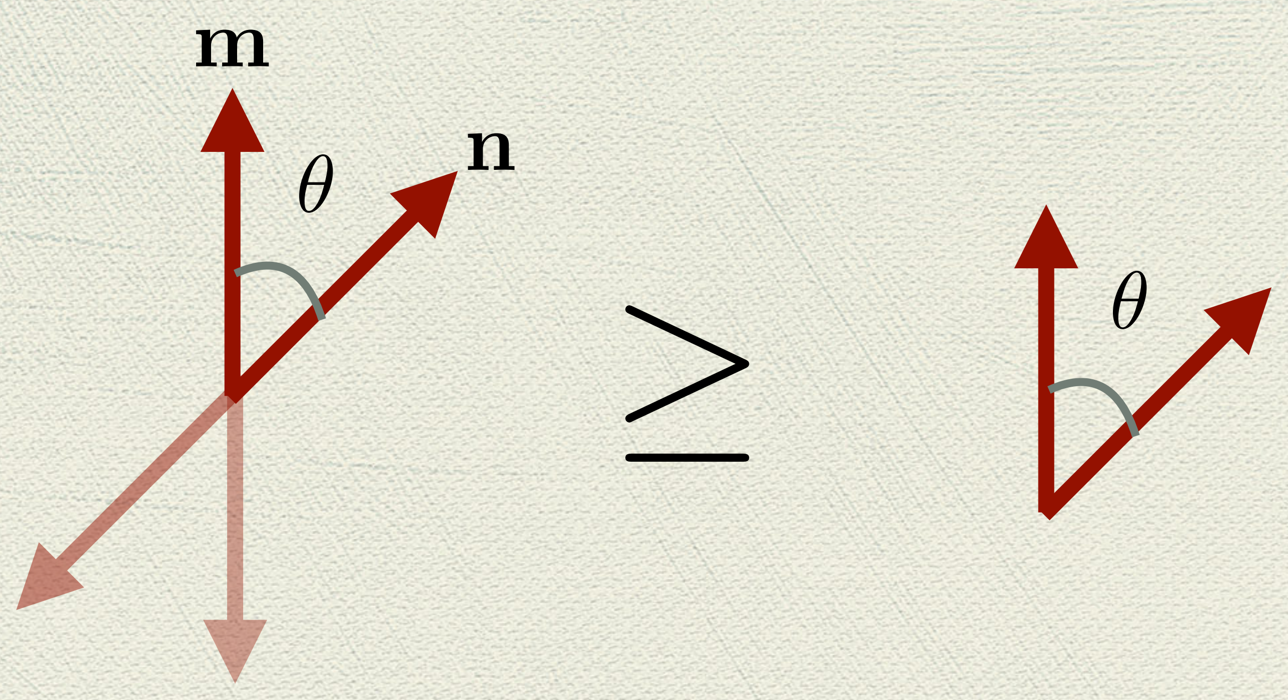
2- Entangled states can be used to set up secure shared reference frames.

3- For some tasks, shared singlet states are better than shared reference frames.



4- QKD can be done in complete absence of any shared coordinate system.

5- Entangled states can carry a higher amount of relative information.



Thank you for your attention