Quantum cryptography with many users

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Outline

- No-cloning and quantum key distribution (QKD)

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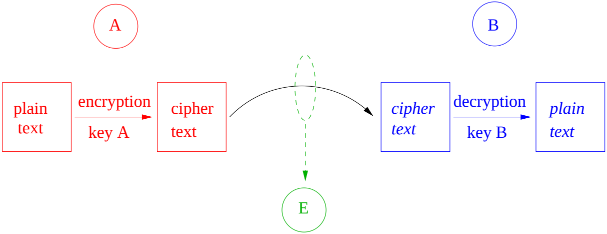
M. Epping, H. Kampermann, C. Macchiavello, and DB, New J. Phys. **19**, *093012 (2017)*

Cryptography



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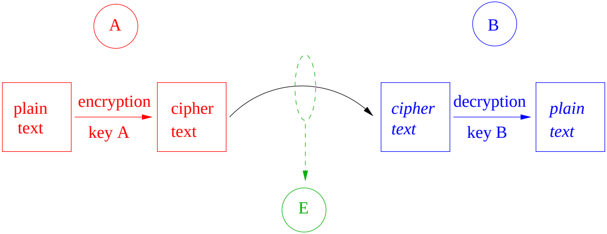
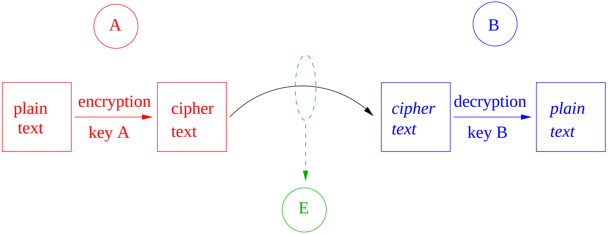


Vernam cipher ≡ "one-time pad" (1917):
Encoding with secret random key (only known to Alice and Bob, not to Eve). Proven to be secure.

Cryptography



Vernam cipher ≡ "one-time pad" (1917):
Encoding with secret random key (only known to Alice and Bob, not to Eve). Proven to be secure.

Problem:
How to establish secret random key?
↪ quantum cryptography ≡ quantum key distribution (QKD)

Quantum Mechanics and the No-Cloning Theorem

Perfect cloning of an unknown quantum state is impossible.

W.K. Wootters and W.H. Zurek, Nature **299**, *802 (1982)*

Reason: Quantum mechanics is linear!

Time evolution:

$$|\psi(t)\rangle = \mathcal{U}(t)|\psi(0)\rangle; \quad \mathcal{U}(t) = e^{-\frac{i}{\hbar}\mathcal{H}t}\ ; \quad \mathcal{U}^\dagger\mathcal{U} = \mathbb{1}$$

Action of copying transformation $\mathcal{U}$ on basis states (orthogonal):

$$\begin{aligned} \mathcal{U}|0\rangle|i\rangle &= |0\rangle|0\rangle\ , \\ \mathcal{U}|1\rangle|i\rangle &= |1\rangle|1\rangle\ . \end{aligned}$$

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Action of $\mathcal{U}$ on unknown state, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, with $|\,\alpha\,|^2 + |\,\beta\,|^2 = 1$:

$$\begin{aligned} \mathcal{U}|\psi\rangle|i\rangle &= \mathcal{U}(\alpha|0\rangle + \beta|1\rangle)|i\rangle \\ &= \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle \neq |\psi\rangle|\psi\rangle \end{aligned}$$

Approximate cloning: see e.g. *DB, D. DiVincenzo, A. Ekert, C. Fuchs, C. Macchiavello, and J. Smolin, Phys. Rev. A* **57**, *2368 (1998)*

The main idea of QKD:
use non-orthogonal quantum states to establish key

C. Bennett and G. Brassard; Proc. IEEE Conf. on Comp. Syst. Signal Proc., 175 (1984)

Remember: non-orthogonal states cannot be cloned perfectly

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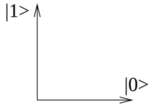

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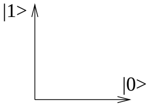
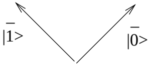


Basis 2: (e.g. rotated linear polarisation of photons)



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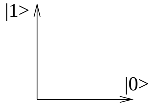

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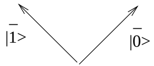

Basis 2: (e.g. rotated linear polarisation of photons)

A and B use both bases to establish secret key
(translate quantum states to classical 0's and 1's)

Quantum key distribution (BB84)

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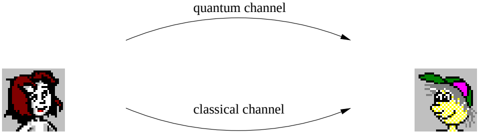
Aim: secret joint random key for Alice and Bob (Vernam cipher)



1) A sends random sequence (polar. photons): $\uparrow \; \nearrow \; \nearrow \; \rightarrow \; \nwarrow \; \uparrow \; \rightarrow \; \nwarrow$

   B measures randomly (two bases): $\uparrow\!\!\!\rightarrow \; \uparrow\!\!\!\rightarrow \; \nwarrow\!\!\nearrow \; \uparrow\!\!\!\rightarrow \; \nwarrow\!\!\nearrow \; \nwarrow\!\!\nearrow \; \uparrow\!\!\!\rightarrow \; \uparrow\!\!\!\rightarrow$

2) A and B exchange class. info about basis,

   keep matching cases: $1 \;\; r \;\; 0 \;\; 0 \;\; 1 \;\; r \;\; 0 \;\; r$

$\hookrightarrow$ Alice and Bob have established secret random key!

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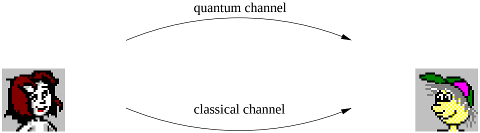

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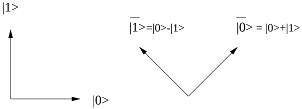
Security: no-cloning theorem!

Most simple strategy of the spy Eve:
"Intercept and resend" $\hookrightarrow$ corruption of $1/4$ bits of key;
discovery of Eve by comparison of parts of key!

Trade-off for winning information:

 information $\leftrightarrow$ disturbance of signal 

Non-orthogonal states:



Interaction of Eve introduces disturbance:

$$\begin{aligned}\mathcal{U}|0\rangle|E\rangle &= |0'\rangle|E_0\rangle \\ \mathcal{U}|\bar{1}\rangle|E\rangle &= |\bar{1}'\rangle|E_{\bar{1}}\rangle\end{aligned}$$

Unitarity:

$$\langle 0|\,\bar{1}\rangle\,\langle E|\,E\rangle = \langle 0'|\,\bar{1}'\rangle\,\langle E_0|\,E_{\bar{1}}\rangle$$

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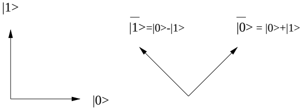

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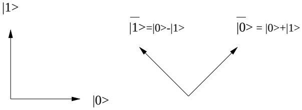

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$\hookrightarrow$ Always assume worst case: all noise is due to Eve.

A different view: Entanglement-based QKD

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What is entanglement of composite (pure) states?

$|\psi\rangle = |a\rangle \otimes |b\rangle \hookrightarrow$ separable

$|\psi\rangle \neq |a\rangle \otimes |b\rangle \hookrightarrow$ entangled

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$|\psi\rangle = |a\rangle \otimes |b\rangle \hookrightarrow$ separable

$|\psi\rangle \neq |a\rangle \otimes |b\rangle \hookrightarrow$ entangled

Example (separable): $|\psi\rangle = |00\rangle \equiv |0\rangle|0\rangle \equiv |0\rangle \otimes |0\rangle$

Example (entangled): Bell states

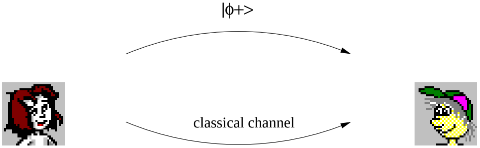
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Note: perfect correlations/anticorrelations for Bell states

A different view: Entanglement-based QKD

A. Ekert, Phys. Rev. Lett. **67**, *661 (1991)*

Aim: secret random key for Alice and Bob



1) A sends half of a Bell state to Bob: $|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$
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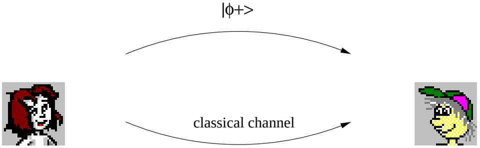

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Security: monogamy of entanglement

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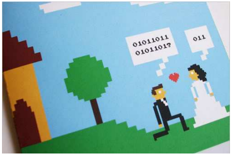

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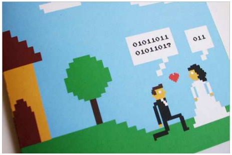

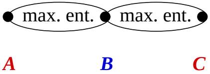

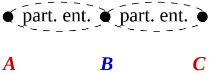

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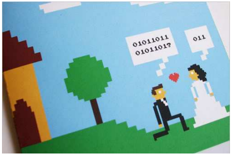

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$$E(B|A) + E(B|C) \leq E(B|AC)$$

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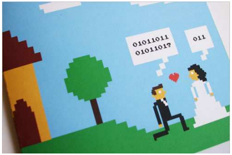

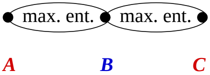

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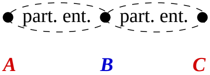
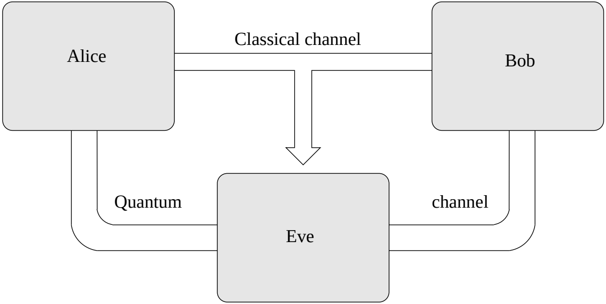


Possible

$$E(B|A) + E(B|C) \leq E(B|AC)$$

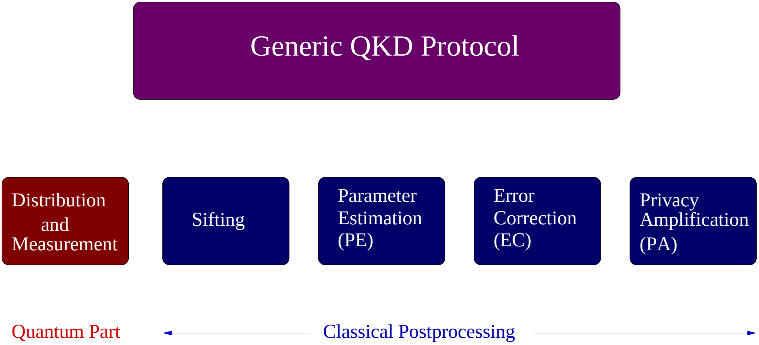
QKD in reality: noisy entangled state, $\rho = p|\phi^+\rangle\langle\phi^+| + (1-p)\frac{1}{4}\mathbb{1}$,
assume Eve to have purifying state (is partially correlated with A/B)
$\hookrightarrow$ security analysis

Quantum Key Distribution (QKD)



- Scenario: Alice und Bob have quantum channel (controlled by Eve) and classical channel (authenticated)
- Secure communication ⇔ Creation of a secret random key pair between Alice and Bob
- No restrictions on Eve

QKD: General description of a QKD protocol



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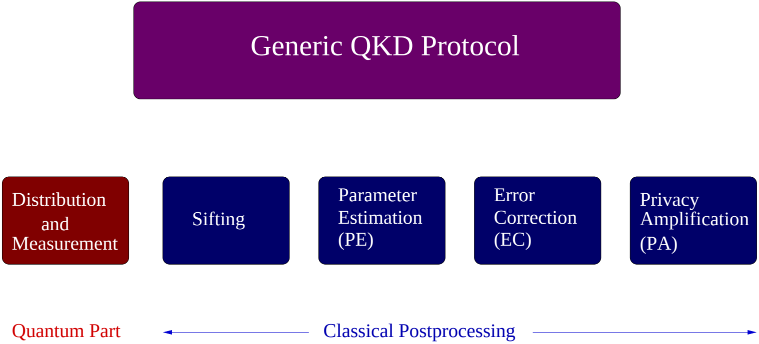


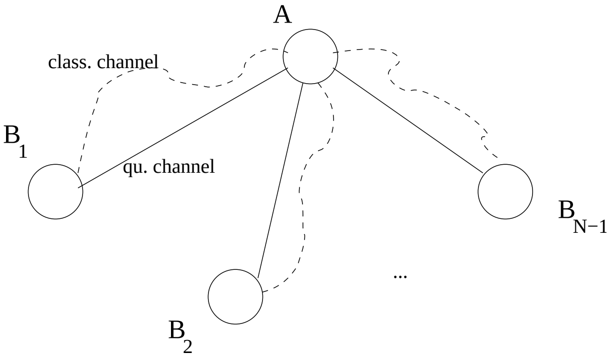
Equivalence of prepare+measure QKD with entanglement-based QKD

$\hookrightarrow$ In the following: use entanglement-based scheme

Generalisation of QKD to more than two parties

M. Epping, H. Kampermann, C. Macchiavello, and DB, New J. Phys. **19**, *093012 (2017)*

Aim: establish joint secret random key between N parties,
i.e. "conference key"



Establishing a conference key: Two possibilities

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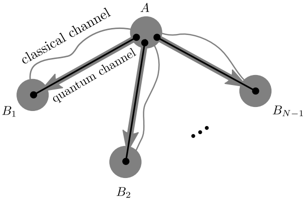
Using bipartite entanglement (2QKD):



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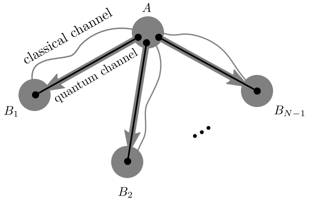
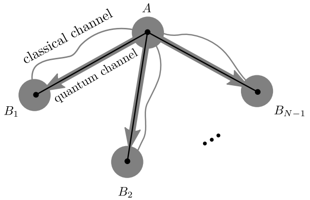


... or using multipartite entanglement (NQKD):



Multipartite entanglement

Multipartite entanglement

Multipartite entanglement of composite (pure) states of N parties:

$$|\psi\rangle = |a\rangle_{1,\ldots,k} \otimes |b\rangle_{k+1,\ldots,N} \quad \hookrightarrow \quad \text{separable across bipartite split}$$

$$|\psi\rangle \neq |a\rangle_{1,\ldots,k} \otimes |b\rangle_{k+1,\ldots,N} \quad \hookrightarrow \quad \text{multipartite entangled}$$

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Example (separable): $|\psi\rangle = |0\rangle|0\rangle...|0\rangle$

Example (entangled): GHZ states of N qubits

$$|\psi_j^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|j\rangle \pm |1\rangle|\bar{j}\rangle)$$

where j takes values $0, ..., 2^{N-1} - 1$ in binary notation;
$\bar{j}$ is negation of j, e.g. if $j = 010$ then $\bar{j} = 101$

Multipartite entanglement for QKD

Which types of multipartite entanglement can be used for QKD?

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Theorem (Perfect resource state for multipartite QKD)

For N qubits, with $N \geq 3$, the state $|\phi_{corr}\rangle = a_{0,...,0}|0,...,0\rangle + a_{1,...,1}|1,...,1\rangle$ with $|a_{0,...,0}|^2 + |a_{1,...,1}|^2 = 1$ leads to perfect classical correlations between any number of parties, if and only if each of them measures in the z-basis.

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leads to perfect classical correlations between any number of parties, if and only if each of them measures in the z-basis.

Proof: "$\Leftarrow$" clear;
"$\Rightarrow$": observable $\mathcal{M}_{ij}$ of two parties i and j:

$$\mathcal{M}_{ij} = (\vec{M}_i \cdot \vec{\sigma}) \otimes (\vec{M}_j \cdot \vec{\sigma}) = \sum_{\alpha,\beta \in \{x,y,z\}} M_i^\alpha M_j^\beta \sigma_i^\alpha \otimes \sigma_j^\beta,$$

$$\langle \phi_{corr} | \sigma_i^\alpha \otimes \sigma_j^\beta | \phi_{corr} \rangle = 0 \ \text{ unless } \ \alpha = \beta = z,$$

also $\langle \phi_{corr} | \sigma_i^\alpha \otimes \sigma_j^\beta | \phi_{corr} \rangle = 2[p_i^\alpha(+)p_j^\beta(+) + p_i^\alpha(-)p_j^\beta(-)] - 1$,
thus $p_i^\alpha(+)p_j^\beta(+) + p_i^\alpha(-)p_j^\beta(-) \neq 1$, unless $\alpha = \beta = z$.

Multipartite QKD protocol

If one requires perfect correlations and uniformity of key,
the *only* possible resource state is $|GHZ\rangle = \frac{1}{\sqrt{2}}(|0,...,0\rangle + |1,...,1\rangle)$.

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Protocol for N-party quantum conference key distribution (NQKD):

1) *State preparation:* Parties A and B_i, $i = 1, 2, ..., N-1$
share $|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$.

Multipartite QKD protocol

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the *only* possible resource state is $|GHZ\rangle = \frac{1}{\sqrt{2}}(|0,...,0\rangle + |1,...,1\rangle)$.

Protocol for N-party quantum conference key distribution (NQKD):

1) *State preparation:* Parties A and B_i, $i = 1, 2, ..., N-1$ share $|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$.
2) *Measurement:* First type of measurement: All parties measure their respective qubits in z-basis ($\hookrightarrow$ key generation). Second type: parties measure randomly, with equal probability, in x- or y-basis (much less frequent).

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4) *Classical post-processing:* As in the bipartite protocol, error correction and privacy amplification is performed.

Secret key rate for NQKD

Security analysis:

- Analogous to bipartite case, with modifications in worst-case error correction and depolarisation

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$$r_\infty = \sup_{U \leftarrow K} \inf_{\sigma_{A\{B_i\}} \in \Gamma} \left[S(U|E) - \max_{i \in \{1, \ldots N-1\}} H(U|K_i) \right],$$

with $U \leftarrow K$: bitwise preprocessing channel on A's raw key bit K,
$S(U|E)$: conditional von-Neumann entropy of (class.) key variable and E,
$H(U|K_i)$: conditional Shannon entropy of U and B_i's guess of it,

Γ: set of all density matrices $\sigma_{A\{B_i\}}$ of A and B_i consistent with parameter estimation

Secret key rate: $R = r_\infty R_{\text{rep}}$ with repetition rate R_{rep}

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Introduce (extended) depolarisation procedure, $\hookrightarrow$ GHZ-diagonal state
$\hookrightarrow$ calculate secret fraction r_∞:

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Introduce (extended) depolarisation procedure, $\hookrightarrow$ GHZ-diagonal state $\hookrightarrow$ calculate secret fraction r_∞:

$$
\begin{aligned}
r_\infty = \quad & \left(1 - \frac{Q_Z}{2} - Q_X\right) \log_2 \left(1 - \frac{Q_Z}{2} - Q_X\right) \\
& + \left(Q_X - \frac{Q_Z}{2}\right) \log_2 \left(Q_X - \frac{Q_Z}{2}\right) \\
& + (1 - Q_Z)(1 - \log_2(1 - Q_Z)) - h(\max_{1 \le i \le N-1} Q_{AB_i})
\end{aligned}
$$

with Q_Z: probability that at least one B_i obtains different result than A in z-measurement,
with Q_X: probability that at least one B_i obtains in x-measurement a result that is incompatible with noiseless state,
binary entropy: $h(p) = -p \log_2 p - (1-p) \log_2(1-p)$,

Q_{AB_i}: probability that z-measurements of A and B_i disagree.

Example for explicit key rates

Noise model: mixture of GHZ-state and white noise (then $Q = Q_Z$)

$$r_\infty(Q,N) = 1 + h(Q) - h\left(Q\frac{2^N-1}{2^N-2}\right) - h\left(Q\frac{2^{N-1}}{2^N-2}\right) + \left(\log_2(2^{N-1}-1) - \frac{2^N-1}{2^N-2}\log_2(2^N-1)\right)Q,$$

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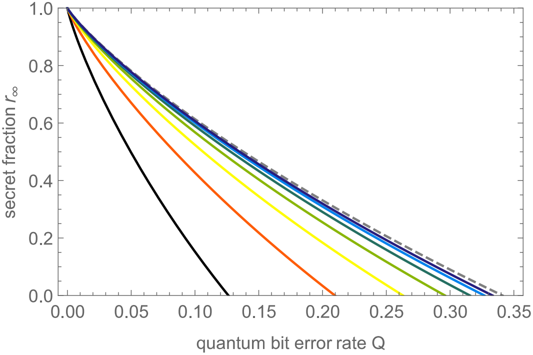

Key rates for $N = 2, 3, ..., 8$, from left to right.

Secret key rate as function of gate failure probability

Consider imperfect state preparation (depolarising noise): experimental creation of GHZ-state is more demanding with higher N!

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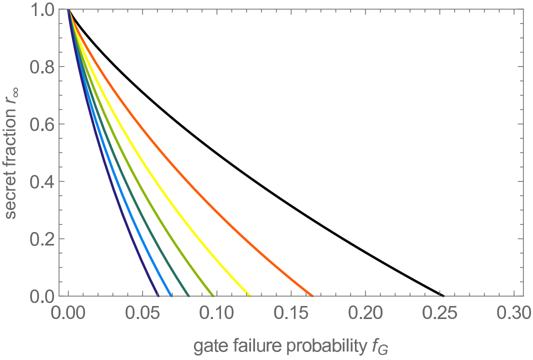
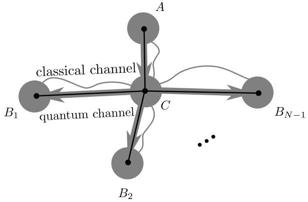


Key rates for $N = 2, 3, ..., 8$, from right to left.

Advantage of NQKD in quantum networks

Consider quantum networks with routers (can produce and entangle qubits), fixed channel capacity:



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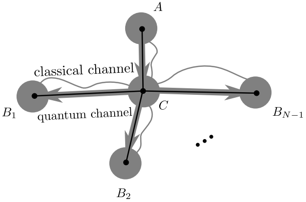
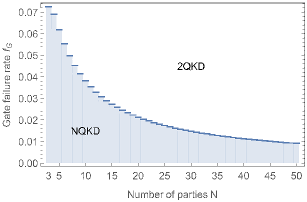
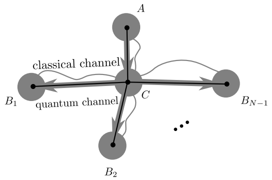


For small gate failure probability: NQKD is better than 2QKD!



Connection to quantum network coding

Distribution of GHZ-state in above network, with quantum operations at node C (router), and fixed channel capacities for all links:



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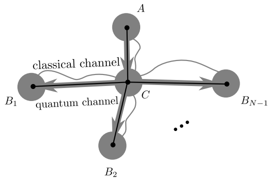

- A produces Bell state and sends only one qubit C to router: $|\!\!-\!\!\!-\!\!\rangle_{CA} = \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle)_{CA}$
- C produces $(N-1)$ qubits and entangles them with C via C_z gates: $|\psi_{\text{total}}\rangle = \frac{1}{\sqrt{2}}(|+\rangle_C |GHZ'\rangle_{AB_i} + |-\rangle_C X_{B_1} |GHZ'\rangle_{AB_i})$ where $|GHZ'\rangle$ is GHZ-state in X-basis.
- Router measures qubit C in X-basis and distributes qubits to B_i.
- Impossible to create $(N-1)$ Bell pairs by sending single qubit from A to router; need $(N-1)$ network uses.

M. Epping, H. Kampermann, and DB, New J. Phys. **18**, *103052 (2016)*

Further developments on multipartite QKD

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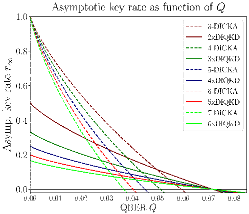
  J. Ribeiro, G. Murta, and S. Wehner, arXiv:1708.00798v2 [quant-ph]



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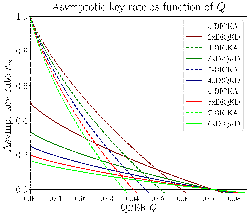
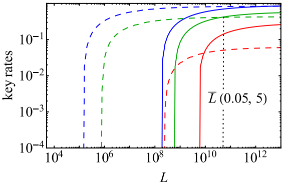


- Finite key effects:

  F. Grasselli, H. Kampermann, and DB, New J. Phys. **20**, *113014 (2018)*



Summary and open questions

- No-cloning $\hookrightarrow$ security in prepare-and-measure QKD

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Quantum Information Theory in Düsseldorf

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